ALGEBRA Review w/ Answer Key

Quiz 3.1 to 3.4

Solving and Graphing Linear Equations,
Rate of Change and Slope
Direct Variation Equations
3.1 Solving Linear Equations

Determine whether each equation is a linear equation. Write yes or no. If yes, write the equation in standard form. If no, please explain.

1. $4xy + 2y = 9$
2. $8x - 3y = 6 - 4x$
3. $7x + y + 3 = y$

4. $5 - 2y = 3x$
5. $\frac{x}{4} - \frac{y}{3} = 1$
6. $\frac{5}{x} - \frac{2}{y} = 7$

Graph each equation by making a table and finding the intercepts. (Only graph once—either by table or intercepts.)

$y = x + 4$

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

x-intercept:

y-intercept:

$5x - 2y = 7$

<table>
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x-intercept:

y-intercept:
A telephone company charges $4.95 per month for long distance calls plus $0.05 per minute. The monthly cost $c$ of long distance calls can be described by the equation $c = 0.05m + 4.95$, where $m$ is the number of minutes.

a. Find the y-intercept of the graph of the equation.

b. Graph the equation.

c. If you talk 140 minutes, what is the monthly cost?

Suppose a migrating killer whale is swimming at an average rate of 4.5 kilometers per hour. The distance $d$ the whale has traveled in $t$ hours can be predicted by the equation $d = 4.5t$.

a. Graph the equation. (Make a table)

b. Use the graph to predict the time it takes the killer whale to travel 30 kilometers.
3.2 Graphing Linear Equations

A bus is driving at 60 miles per hour toward a bus station that is 250 miles away. The function \( d = 250 - 60t \) represents the distance \( d \) from the bus station the bus is \( t \) hours after it has started driving. Find the zero of this function. Describe what this value means in this context.

You receive a gift card for trading cards from a local store. The function \( d = 20 - 1.95c \) represents the remaining dollars \( d \) on the gift card after obtaining \( c \) packages of cards. Find the zero of this function. Describe what this value means in this context.
3.3 Rate of Change and Slope

Find the slope of the line that passes through each pair of points.

2. \( (0, 0), (3, 1) \)

3. \( (0, 1), (1, 2) \)

2. \( (-2, -3), (3, 1) \)

3. \( (-2, 3), (3, 3) \)

Find the slope of the line that passes through each pair of points.

\((6, 3), (7, -4)\) \hspace{1cm} \((5, 9), (3, 9)\)

\((-2, -5), (7, 8)\) \hspace{1cm} \((-7, 8), (-7, 5)\)
Find the value of \( r \) so the line that passes through each pair of points has the given slope.

\[ (-2, r), (6, 7), \quad m = \frac{1}{2} \]

\[ (-7, 2), (-8, r), \quad m = -5 \]

A daily newspaper had 12,125 subscribers when it began publication in the year 2011. Five years later it had 10,100 subscribers. What is the average yearly rate of change in the number of subscribers for the five-year period?
3.4 Direct Variation

Name the constant of variation for each equation. Then determine the slope of the line that passes through each pair of points.

2. \[ y = \frac{4}{3}x \]

3. \[ y = -\frac{5}{2}x \]

Graph each equation. Make a table of values.

4. \[ y = -2x \]

5. \[ y = \frac{6}{5}x \]
Write a direct variation equation that relates $x$ and $y$. Assume that $y$ varies directly as $x$. Then solve.

7. If $y = 7.5$ when $x = 0.5$, find $y$ when $x = -0.3$.

8. If $y = 80$ when $x = 32$, find $x$ when $y = 100$.

Write a direct variation equation that relates the variables. Then graph the equation.

10. **MEASURE** The width $W$ of a rectangle is two thirds of the length $\ell$.

\[
\text{equation:}
\]

<table>
<thead>
<tr>
<th>make a table of values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{array}{c}
\text{Rectangle Dimensions} \\
\begin{array}{c}
W \\
10 \\
8 \\
6 \\
4 \\
2 \\
0 \\
\end{array} \\
\begin{array}{c}
\text{Width} \\
0 \\
2 \\
4 \\
6 \\
8 \\
10 \\
12 \\
\end{array} \\
\begin{array}{c}
\text{Length} \\
0 \\
2 \\
4 \\
6 \\
8 \\
10 \\
12 \\
\end{array}
\end{array}
\]
ALGEBRA

Review
w/ Answer Key

Quiz 3.1 to 3.4
Solving and Graphing Linear Equations,
Rate of Change and Slope
Direct Variation Equations
3.1 Solving Linear Equations

Determine whether each equation is a linear equation. Write yes or no. If yes, write the equation in standard form. If no, please explain.

1. $4xy + 2y = 9$
   
   no, you do not multiply the variables

2. $8x - 3y = 6 - 4x$
   
   $\frac{+4x}{+4x} \frac{-3y}{12x - 3y = 6}$
   
   yes

3. $7x + y + 3 = y$
   
   $\frac{-y}{-y} \frac{-3}{7x + 3 = 0}$
   
   $\frac{-3}{-3 - 3}$
   
   $7x = -3$
   
   yes

4. $5 - 2y = 3x$
   
   $\frac{+2y}{+2y} \frac{5}{3x + 2y}$

5. $\frac{x}{4} - \frac{y}{3} = 1$
   
   $12 \left( \frac{x}{4} - \frac{y}{3} = 1 \right)$

   $3x - 4y = 12$
   
   yes

6. $\frac{5}{x} - \frac{2}{y} = 7$
   
   no, you do not divide by variables

Graph each equation by making a table and finding the intercepts. (Only graph once—either by table or intercepts.)

**y = x + 4**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

x-intercept: $0 = x + 4$

y-intercept: $y = 0 + 4$

(0,4)

**5x - 2y = 7**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-1/2</td>
</tr>
<tr>
<td>-1</td>
<td>-6</td>
</tr>
<tr>
<td>0</td>
<td>-7/2</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>3/2</td>
</tr>
</tbody>
</table>

x-intercept: $5x - 2(0) = 7$

$5x = 7$

$x = \frac{7}{5}$

($\frac{7}{5}$, 0)

y-intercept: $5(0) - 2y = 7$

$-2y = 7$

$y = -\frac{7}{2}$

(0, -\frac{7}{2})
A telephone company charges $4.95 per month for long distance calls plus $0.05 per minute. The monthly cost \( c \) of long distance calls can be described by the equation \( c = 0.05m + 4.95 \), where \( m \) is the number of minutes.

a. Find the \( y \)-intercept of the graph of the equation.

\[
\text{let } x = 0 \text{ or } m = 0
\]

\[
C = 0.05(0) + 4.95 = 4.95
\]

b. Graph the equation.

\[
\text{let } m = 40 \text{ as an example point to connect}
\]

\[
C = 0.05(40) + 4.95 = 20 + 4.95 = 24.95
\]

\[
C = 4.95
\]

\[
C = 11.95
\]

\[
\frac{40 \times 0.05}{7.00}
\]

Suppose a migrating killer whale is swimming at an average rate of 4.5 kilometers per hour. The distance \( d \) the whale has traveled in \( t \) hours can be predicted by the equation \( d = 4.5t \).

a. Graph the equation. (Make a table)

\[
\begin{array}{c|c}
\text{negative not needed} & \text{distance in km} \\
\hline
0 & 0 \\
1 & 4.5 \\
2 & 9 \\
3 & 13.5 \\
4 & 18 \\
5 & 22.5 \\
6 & 27
\end{array}
\]

b. Use the graph to predict the time it takes the killer whale to travel 30 kilometers.

\[
30 = 4.5t
\]

\[
\frac{30}{4.5} = 6.666...
\]

\[
6.\overline{6} \text{ hours}
\]

\[
t = \frac{2}{3}
\]
3.2 Graphing Linear Equations

A bus is driving at 60 miles per hour toward a bus station that is 250 miles away. The function \( d = 250 - 60t \) represents the distance \( d \) from the bus station the bus is \( t \) hours after it has started driving. Find the zero of this function. Describe what this value means in this context.

\[
\begin{align*}
0 &= 250 - 60t \\
250 - 250 &= 60t \\
-250 &= -60t \\
-\frac{250}{-60} &= t \\
4.16 &\approx t \\
t &\approx 4 \text{ hours}
\end{align*}
\]

Just after 4 hours, the bus will be at the station.

You receive a gift card for trading cards from a local store. The function \( d = 20 - 1.95c \) represents the remaining dollars \( d \) on the gift card after obtaining \( c \) packages of cards. Find the zero of this function. Describe what this value means in this context.

\[
\begin{align*}
0 &= 20 - 1.95c \\
-20 &= -1.95c \\
-\frac{20}{-1.95} &\approx c \\
10.25 &\approx c
\end{align*}
\]

\[
\begin{array}{c|c}
\text{c} & \text{d} \\
\hline
0 & 20 \\
1 & 18.05 \\
2 & 16.10 \\
3 & 14.15 \\
4 & 12.20 \\
5 & 10.25 \\
10 & 0.50
\end{array}
\]

After 10 packages of cards, the balance will be $0.
3.3 Rate of Change and Slope

Find the slope of the line that passes through each pair of points.

1. \( \frac{\text{rise}}{\text{run}} = \frac{1}{3} \)
2. \( \frac{\text{rise}}{\text{run}} = \frac{4}{5} \)
3. \( \frac{\text{rise}}{\text{run}} = \frac{-3}{-1} = -3 \)
   \( \frac{\text{rise}}{\text{run}} = \frac{0}{5} = 0 \)

---

Find the slope of the line that passes through each pair of points.

\[
\begin{align*}
\text{for } (6, 3), (7, -4) & \quad m &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 3}{7 - 6} = -7 \\
& \quad m = -7 \\
\text{for } (-2, -5), (7, 8) & \quad m &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - (-5)}{7 - (-2)} = \frac{13}{9} \\
& \quad m = \frac{13}{9}
\end{align*}
\]

\[
\begin{align*}
\text{for } (5, 9), (3, 9) & \quad m &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 9}{3 - 5} = \frac{0}{-2} = 0 \\
& \quad m = 0 \\
\text{for } (-7, 8), (-7, 5) & \quad m &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 8}{-7 - (-7)} = \frac{-3}{0} \quad (\text{undefined})
\end{align*}
\]
Find the value of \( r \) so the line that passes through each pair of points has the given slope.

\[
\begin{align*}
(-2, r), (6, 7), \quad m &= \frac{1}{2} \\
\frac{1}{2} &= \frac{7-r}{6-(-2)} \\
\frac{1}{2} &= \frac{7-r}{8} \\
1(8) &= 2(7-r) \\
8 &= 14 - 2r \\
-14 &= -2r \\
-7 &= -2r \\
\frac{-7}{-2} &= r \\
\boxed{3.5 &= r}
\end{align*}
\]

\[
\begin{align*}
(-7, 2), (-8, r), \quad m &= -5 \\
\frac{-5}{1} &= \frac{r-2}{-1} \\
-5 &= 1(r-2) \\
5 &= r-2 \\
+2 &= +2 \\
\boxed{7 &= r}
\end{align*}
\]

A daily newspaper had 12,125 subscribers when it began publication in the year 2011. Five years later it had 10,100 subscribers. What is the average yearly rate of change in the number of subscribers for the five-year period?

\[
\begin{align*}
(2011, 12,125) \quad \text{and} \quad (2016, 10,100) \\
\frac{y_2 - y_1}{x_2 - x_1} &= m \\
\frac{10,100 - 12,125}{2016 - 2011} &= m \\
\frac{-2025}{5} &= m \\
\boxed{-405 &= m}
\end{align*}
\]

The paper lost an average of 405 subscribers each year.
3.4 Direct Variation

Name the constant of variation for each equation. Then determine the slope of the line that passes through each pair of points.

2. \[ y = \frac{4}{3}x \]
   \[ \text{CoV} = \frac{4}{3} \]
   \[ \text{slope} \Rightarrow \frac{\text{rise}}{\text{run}} = \frac{4}{3} \]

3. \[ y = -\frac{5}{2}x \]
   \[ \text{CoV} = -\frac{5}{2} \]
   \[ \text{slope} \Rightarrow \frac{\text{rise}}{\text{run}} = -\frac{5}{2} \]

Graph each equation. Make a table of values.

4. \( y = -2x \)

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<td>-2</td>
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5. \( y = \frac{6}{5}x \)

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Write a direct variation equation that relates \( x \) and \( y \). Assume that \( y \) varies directly as \( x \). Then solve.

7. If \( y = 7.5 \) when \( x = 0.5 \), find \( y \) when \( x = -0.3 \).

\[
\begin{align*}
    y &= kx \\
    7.5 &= k(0.5) \\
    0.5 &= k \\
    15 &= k
\end{align*}
\]

\[
\begin{align*}
    y &= 15x \\
    y &= 15(-0.3) \\
    y &= -4.5
\end{align*}
\]

8. If \( y = 80 \) when \( x = 32 \), find \( x \) when \( y = 100 \).

\[
\begin{align*}
    y &= kx \\
    80 &= k(32) \\
    32 &= k \\
    2.5 &= k
\end{align*}
\]

\[
\begin{align*}
    y &= 2.5x \\
    100 &= 2.5x \\
    40 &= x
\end{align*}
\]

Write a direct variation equation that relates the variables. Then graph the equation.

10. **MEASURE** The width \( W \) of a rectangle is two thirds of the length \( \ell \).

\[
W = \frac{2}{3} \ell
\]

**equation:**

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<td>( \ell )</td>
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**Rectangle Dimensions**

<table>
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</tr>
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<tbody>
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<td>0</td>
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