In Chapter 3, you solved systems of linear equations algebraically and graphically.

In Chapter 10, you will:
- Use the Midpoint and Distance Formulas.
- Write and graph equations of parabolas, circles, ellipses, and hyperbolas.
- Identify conic sections.
- Solve systems of quadratic equations and inequalities.

SPACE Conic sections are evident in many aspects of space. Equations of circles are used to pilot spacecraft and satellites in circular orbits around Earth and the Moon. Planets travel in elliptical paths, not circular ones as previously thought. Comets travel along one branch of a hyperbola, which can help us to predict when they will appear again.
Diagnose Readiness  |  You have two options for checking prerequisite skills.

1  **Textbook Option**  Take the Quick Check below. Refer to the Quick Review for help.

<table>
<thead>
<tr>
<th><strong>Quick Check</strong></th>
<th><strong>Quick Review</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve each equation by completing the square. (Lesson 5-5)</td>
<td>Example 1</td>
</tr>
<tr>
<td>1. $x^2 + 8x + 7 = 0$</td>
<td>Solve $x^2 + 6x - 16 = 0$ by completing the square. $x^2 + 6x = 16$</td>
</tr>
<tr>
<td>2. $x^2 + 5x - 6 = 0$</td>
<td>$x^2 + 6x + 9 = 16 + 9$</td>
</tr>
<tr>
<td>3. $x^2 - 8x + 15 = 0$</td>
<td>$(x + 3)^2 = 25$</td>
</tr>
<tr>
<td>4. $x^2 + 2x - 120 = 0$</td>
<td>$x + 3 = \pm 5$</td>
</tr>
<tr>
<td>5. $2x^2 + 7x - 15 = 0$</td>
<td>$x + 3 = 5$ or $x + 3 = -5$</td>
</tr>
<tr>
<td>6. $2x^2 + 3x - 5 = 0$</td>
<td>$x = 2$</td>
</tr>
<tr>
<td>7. $x^2 - \frac{3}{2}x - \frac{23}{16} = 0$</td>
<td>$x = -8$</td>
</tr>
<tr>
<td>8. $3x^2 - 4x = 2$</td>
<td></td>
</tr>
</tbody>
</table>

Find the coordinates of the vertices of the preimage $\triangle$ and image $\triangle'$ after the given translation. Then graph the preimage and image. (Lesson 4-4)

9. quadrilateral $ABCD$ with vertices $A(-5, -1), B(-4, 3), C(2, 3),$ and $D(1, -1)$, translated 3 units right and 4 units down

10. triangle $EFG$ with vertices $E(-2, 0), F(5, 2),$ and $G(4, -3)$, translated 1 unit left and 2 units up

11. triangle $JKL$ with vertices $J(1, 4), K(2, -5),$ and $L(-6, -6)$, translated 4 units left and 2 units up

12. Triangle $XYZ$ with vertices $X(-2, 2), Y(3, 5),$ and $Z(5, -2)$ is translated so that $X'$ is at $(1, -5)$. Find the coordinates of $Y'$ and $Z'$.

13. **LANDSCAPING**  Laura plots her shed plans on a grid with each unit equal to 1 foot. She places the corners at (100, 50), (110, 50), (110, 40), and (100, 40). She decides to move the shed up 10 feet and to the right 15 feet. What will be the new coordinates of the shed?

**Example 2**

Find the coordinates of the vertices of the image of triangle $RST$ with $R(1, 4), S(4, 2),$ and $T(2, 0)$ if it is moved 2 units to the left and 1 unit up. Then graph $\triangle RST$ and its image $\triangle R'S'T'$.

Write the vertex matrix for $\triangle RST$.

$$\begin{bmatrix} 1 & 4 & 2 \\ 4 & 2 & 0 \end{bmatrix}$$

Add the translation matrix $\begin{bmatrix} -2 & -2 & -2 \\ -1 & 1 & 1 \end{bmatrix}$ to the vertex matrix.

$$\begin{bmatrix} 1 & 4 & 2 \\ 4 & 2 & 0 \end{bmatrix} + \begin{bmatrix} -2 & -2 & -2 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 0 \\ 5 & 3 & 1 \end{bmatrix}$$

The vertices of $\triangle R'S'T'$ are $R'(-1, 5), S'(2, 3),$ and $T'(0, 1)$.

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2  **Online Option**  Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 10. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

### Foldable Study Organizer

**Conic Sections** Make this Foldable to help you organize your Chapter 10 notes about conic sections. Begin with eight sheets of grid paper.

1. **Staple** the stack of grid paper along the top to form a booklet.

2. **Cut** seven lines from the bottom of the top sheet, six lines from the second sheet, and so on.

3. **Label** with lesson numbers as shown.

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### New Vocabulary

<table>
<thead>
<tr>
<th>English</th>
<th>Español</th>
</tr>
</thead>
<tbody>
<tr>
<td>parabola</td>
<td>parábola</td>
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<tr>
<td>focus</td>
<td>foco</td>
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<tr>
<td>directrix</td>
<td>directriz</td>
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<tr>
<td>circle</td>
<td>círculo</td>
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<tr>
<td>center of a circle</td>
<td>centro de un círculo</td>
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<td>radius</td>
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<td>foci</td>
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<td>eje mayor</td>
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<tr>
<td>minor axis</td>
<td>eje menor</td>
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<tr>
<td>center of an ellipse</td>
<td>centro de una elipse</td>
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<tr>
<td>vertices</td>
<td>vértices</td>
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<tr>
<td>co-vertices</td>
<td>co-vértices</td>
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<tr>
<td>constant sum</td>
<td>suma constante</td>
</tr>
<tr>
<td>hyperbola</td>
<td>hipérbola</td>
</tr>
<tr>
<td>transverse axis</td>
<td>eje transversal</td>
</tr>
<tr>
<td>conjugate axis</td>
<td>eje conjugado</td>
</tr>
<tr>
<td>constant difference</td>
<td>diferencia constante</td>
</tr>
</tbody>
</table>

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### Review Vocabulary

- **quadratic equation** p. 259 ecuación cuadrática a quadratic function in the form $ax^2 + bx + c = 0$, where $a \neq 0$
- **system of equations** p. 135 sistema de ecuaciones a set of equations with the same variables
- **x- and y-intercepts** p. 71 intersecciones $x$ $y$ $y$ the $x$- or $y$-coordinate of the point at which a graph crosses the $x$- or $y$-axis
1 The Midpoint Formula Recall that point $M$ is the midpoint of segment $PQ$ if $M$ is between $P$ and $Q$ and $PM = MQ$. There is a formula for the coordinates of the midpoint of a segment in terms of the coordinates of the endpoints.

Key Concept Midpoint Formula

Words If a line segment has endpoints $P(x_1, y_1)$ and $Q(x_2, y_2)$, then the midpoint of the segment has coordinates

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Model

Example 1 Find a Midpoint

Find the coordinates of $M$, the midpoint of $JK$, for $J(-1, 2)$ and $K(6, 1)$.

Let $J$ be $(x_1, y_1)$ and $K$ be $(x_2, y_2)$.

$$M\left(\frac{-1 + 6}{2}, \frac{2 + 1}{2}\right) = M\left(\frac{5}{2}, \frac{3}{2}\right)$$

Guided Practice

1A. Find the coordinates of the midpoint of $AB$ for $A(5, 12)$ and $B(-4, 8)$.

1B. Find the coordinates of the midpoint of $CD$ for $C(4, 5)$ and $D(14, 13)$.

2 The Distance Formula The distance between two points, $a$ and $b$, on a number line is $|a - b|$ or $|b - a|$. You can use this fact and the Pythagorean Theorem to derive a formula for the distance between two points on a coordinate plane.
Let \( d \) represent the distance between \((x_1, y_1)\) and \((x_2, y_2)\).

\[
\begin{align*}
\text{By the Pythagorean Theorem,} \\
\quad d^2 &= a^2 + b^2 \\
\text{Substitute.} \\
\quad d^2 &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \\
\text{Find the nonnegative square root of each side.} \\
\quad d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\end{align*}
\]

**Key Concept** *Distance Formula*

Words  
The distance between two points with coordinates \((x_1, y_1)\) and \((x_2, y_2)\) is given by \(d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\).

**Real-World Example 2**  
Find the Distance Between Two Points

**DISC GOLF**  
Troy’s disc is 20 feet short and 8 feet to the right of the basket. On his first putt, the disc lands 2 feet to the left and 3 feet beyond the basket. If the disc went in a straight line, how far did it go?

Model the situation. If the basket is at \((0, 0)\), then the location of the disc is \((8, -20)\). The location after the first putt is \((-2, 3)\).

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
= \sqrt{(-2 - 8)^2 + (3 - (-20))^2} \\
= \sqrt{(-10)^2 + 23^2} \\
= \sqrt{100 + 529} \\
= \sqrt{629} \text{ or about } 25
\]

The disc traveled about 25 feet on his first putt.

**Guided Practice**

2. Sharon hits a golf ball 12 feet above the hole and 3 feet to the left. Her first putt traveled to 2 feet above the cup and 1 foot to the right. How far did the ball travel on her first putt?
There most likely will be problems involving the Midpoint and Distance Formulas on standardized tests you will have to take.

**Test Example 3**

A coordinate grid is placed over a Florida map. St. Augustine is located at (3, 13), and Rockledge is located at (8, -1). If Port Orange is halfway between St. Augustine and Rockledge, which is closest to the distance in coordinate units from St. Augustine to Port Orange?

A 4.75  
B 7.43  
C 14.9  
D 19

**Read the Test Item**
The question asks you to find the distance between one city and the midpoint. Find the midpoint, and then use the Distance Formula.

**Solve the Test Item**
Use the Midpoint Formula to find the coordinates of Port Orange.

\[
\text{midpoint} = \left( \frac{3 + 8}{2}, \frac{13 + (-1)}{2} \right) = (5.5, 6)
\]

Use the Distance Formula to find the distance between St. Augustine (3, 13) and Port Orange (5.5, 6).

\[
\text{distance} = \sqrt{(3 - 5.5)^2 + (13 - 6)^2} = \sqrt{(-2.5)^2 + 7^2} = \sqrt{55.25} \text{ or about 7.43}
\]

The answer is B.

**Guided Practice**
3. The coordinates for points A and B are (-4, -5) and (10, -7), respectively. Find the distance between the midpoint of A and B and point B.

F \(\sqrt{10}\) units  
G \(5\sqrt{10}\) units  
H \(\sqrt{50}\) units  
J \(10\sqrt{5}\) units

---

**Check Your Understanding**

**Example 1**
Find the midpoint of the line segment with endpoints at the given coordinates.

1. (-4, 7), (3, 9)  
2. (8, 2), (-1, -5)  
3. (11, 6), (18, 13.5)  
4. (-12, -2), (-10.5, -6)

**Example 2**
Find the distance between each pair of points with the given coordinates.

5. (3, -5), (13, -11)  
6. (8, 1), (-2, 9)  
7. (0.25, 1.75), (3.5, 2.5)  
8. (-4.5, 10.75), (-6.25, -7)

**Example 3**
9. **MULTIPLE CHOICE** The map of a mall is overlaid with a numeric grid. The kiosk for the cell phone store is halfway between The Ice Creamery and the See Clearly eyeglass store. If the ice cream store is at (2, 4) and the eyeglass store is at (78, 46), find the distance the kiosk is from the eyeglass store.

A 43.4 units  
B 47.2 units  
C 62.4 units  
D 94.3 units
Example 1  Find the midpoint of the line segment with endpoints at the given coordinates.
10. (20, 3), (15, 5)  11. (−27, 4), (19, −6)  12. (−0.4, 7), (11, −1.6)
13. (5.4, −8), (9.2, 10)  14. (−5.3, −8.6), (−18.7, 1)  15. (−6.4, −8.2), (−9.1, −0.8)

Example 2  Find the distance between each pair of points with the given coordinates.
16. (1, 2), (6, 3)  17. (3, −4), (0, 12)  18. (−6, −7), (11, −12)  19. (−10, 8), (−8, −8)
20. (4, 0), (5, −6)  21. (7, 9), (−2, −10)  22. (−4, −5), (15, 17)  23. (14, −20), (−18, 25)

Example 3 24. TRACK AND FIELD A shot put is thrown from the inside of a circle. A coordinate grid is placed over the shot put circle. The toe board is located at the front of the circle at (−4, 1), and the back of the circle is at (5, 2). If the center of the circle is halfway between these two points, what is the distance from the toe board to the center of the circle?

Find the midpoint of the line segment with endpoints at the given coordinates. Then find the distance between the points.
25. (−93, 15), (90, −15)  26. (−22, 42), (57, 2)  27. (−70, −87), (59, −14)  28. (−98, 5), (−77, 64)
29. (41, −45), (−25, 75)  30. (90, 60), (−3, −2)  31. (−1.2, 2.5), (0.34, −7)  32. (−7.54, 3.89), (4.04, −0.38)
33. \( \left( -\frac{5}{12}, \frac{1}{3} \right), \left( -\frac{17}{2}, -\frac{5}{3} \right) \)  34. \( \left( -\frac{5}{4}, -\frac{13}{2} \right), \left( -\frac{4}{3}, -\frac{5}{6} \right) \)
35. \( (-3\sqrt{2}, -4\sqrt{5}), (-3\sqrt{3}, 9) \)  36. \( \left( \frac{\sqrt{3}}{3}, \frac{\sqrt{2}}{4} \right), \left( -\frac{2\sqrt{3}}{3}, \frac{\sqrt{2}}{4} \right) \)

37. SPACE Use the labeled points on the outline of the circular crater on Mars to estimate its diameter in kilometers. Assume each unit on the coordinate system is 1 kilometer.

38. GEOMETRY Triangle ABC has vertices A(2, 1), B(−6, 5), and C(−2, −3).
   a. An isosceles triangle has two sides with equal length. Is triangle ABC isosceles? Explain.
   b. An equilateral triangle has three sides of equal length. Is triangle ABC equilateral? Explain.
   c. Triangle EFG is formed by joining the midpoints of the sides of triangle ABC. What type of triangle is EFG? Explain.
   d. Describe any relationship between the lengths of the sides of the two triangles.
39. **PACKAGE DELIVERY** To determine the mileage between cities for their overnight delivery service, a package delivery service superimposes a coordinate grid over the United States. Each side of a grid unit is equal to 0.316 mile. Suppose the locations of two distribution centers are at (132, 428) and (254, 105). Find the actual distance between these locations to the nearest mile.

40. **HIKING** Orlando wants to hike from his camp to a waterfall. The waterfall is 5 miles south and 8 miles east of his campsite.
   a. Use the Distance Formula to determine how far the waterfall is from the campsite.
   b. Verify your answer in part a by using the Pythagorean Theorem to determine the distance between the campsite and the waterfall.
   c. Orlando wants to stop for lunch halfway to the waterfall. If the camp is at the origin, where should he stop?

41. **MULTIPLE REPRESENTATIONS** Triangle XYZ has vertices X(4, 9), Y(8, -9), and Z(-6, 5).
   a. **Concrete** Draw △XYZ on a coordinate plane.
   b. **Numerical** Find the coordinates of the midpoint of each side of the triangle.
   c. **Geometric** Find the perimeter of △XYZ and the perimeter of the triangle with vertices at the points found in part b.
   d. **Analytical** How do the perimeters in part c compare?

**H.O.T. Problems** Use Higher-Order Thinking Skills

42. **CHALLENGE** Find the coordinates of the point that is three fourths of the way from P(-1, 12) to Q(5, -10).

43. **REASONING** Identify all the points in a plane that are 3 units or less from the point (5, 6). What figure does this make?

44. **REASONING** Triangle ABC is a right triangle.
   a. Find the midpoint of the hypotenuse. Call it point Q.
   b. Classify △BQC according to the lengths of its sides. Include sufficient evidence to support your conclusion.
   c. Classify △BQA according to its angles.

45. **OPEN ENDED** Plot two points, and find the distance between them. Does it matter which ordered pair is first when using the Distance Formula? Explain.

46. **WRITING IN MATH** Explain how the Midpoint Formula can be used to approximate the halfway point between two locations on a map.
47. SHORT RESPONSE You currently earn $8.10 per hour and your boss gives you a 10% raise. What is your new hourly wage?

48. SAT/ACT A right circular cylinder has a radius of 3 and a height of 5. Which of the following dimensions of a rectangular solid will have a volume closest to that of the cylinder?

A 5, 5, 6
B 5, 6, 6
C 5, 5, 5

49. GEOMETRY If the sum of the lengths of the two legs of a right triangle is 49 inches and the hypotenuse is 41 inches, find the longer of the two legs.

F 9 in.
H 42 in.
G 40 in.
J 49 in.

50. Five more than 3 times a number is 17. Find the number.

A 3
C 5
B 4
D 6

Spiral Review

Solve each equation. Check your solutions. (Lesson 9-6)

51. \( \frac{12}{v^2 - 16} - \frac{24}{v - 4} = 3 \)

52. \( \frac{w}{w - 1} + w = \frac{4w - 3}{w - 1} \)

53. \( \frac{4n^2 - 9}{n^2} - \frac{2n}{n + 3} = \frac{3}{n - 3} \)

54. SWIMMING When a person swims underwater, the pressure in his or her ears varies directly with the depth at which he or she is swimming. (Lesson 9-5)

a. Write a direct variation equation that represents this situation.

b. Find the pressure at 60 feet.

c. It is unsafe for amateur divers to swim where the water pressure is more than 65 pounds per square inch. How deep can an amateur diver safely swim?

d. Make a table showing the number of pounds of pressure at various depths of water. Use the data to draw a graph of pressure versus depth.

Solve each equation or inequality. Round to the nearest ten-thousandth. (Lesson 8-6)

55. \( 9^2 - 4 = 6.28 \)

56. \( 8.2^2 - 3 = 42.5 \)

57. \( 2.1^2 - 5 = 9.32 \)

58. \( 8\sqrt{n} > 52^n + 3 \)

59. \( 7^n + 2 \leq 13^n - n \)

60. \( 3^n + 2 \geq 8^n \)

Solve each equation. (Lesson 7-7)

61. \( (6n - 5)^3 + 3 = -2 \)

62. \( (5x + 7)^3 + 3 = 5 \)

63. \( (3x - 2)^\frac{1}{3} + 6 = 5 \)

Skills Review

Write each quadratic equation in vertex form. Then identify the vertex, axis of symmetry, and direction of opening. (Lesson 5-7)

64. \( y = -x^2 - 4x + 8 \)

65. \( y = x^2 - 6x + 1 \)

66. \( y = -2x^2 + 20x - 35 \)
A parabola can be defined as the set of all points in a plane that are the same distance from a given point called the focus and a given line called the directrix.

The line segment through the focus of a parabola and perpendicular to the axis of symmetry is called the latus rectum. The endpoints of the latus rectum lie on the parabola.

Equations of Parabolas

The standard form of the equation of a parabola with vertex \((h, k)\) and axis of symmetry \(x = h\) is \(y = a(x - h)^2 + k\).

- If \(a > 0\), \(k\) is the minimum value of the related function and the parabola opens upward.
- If \(a < 0\), \(k\) is the maximum value of the related function and the parabola opens downward.

An equation of a parabola in the form \(y = ax^2 + bx + c\) is the general form. Any equation in general form can be written in standard form. The shape of a parabola and the distance between the focus and directrix depend on the value of \(a\) in the equation.
Example 1  Analyze the Equation of a Parabola

Write \( y = 2x^2 - 12x + 6 \) in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

\[
\begin{align*}
y &= 2x^2 - 12x + 6 \quad &\text{Original equation} \\
&= 2(x^2 - 6x) + 6 \quad &\text{Factor 2 from the } x- \text{ and } x^2-\text{terms.} \\
&= 2(x^2 - 6x + \frac{9}{2}) + 6 - 2(\frac{9}{2}) \quad &\text{Complete the square on the right side.} \\
&= 2(x^2 - 6x + 9) + 6 - 2(9) \quad &\text{The 9 added when you complete the square is multiplied by 2.}
&= 2(x - 3)^2 - 12 \quad &\text{Factor.}
\end{align*}
\]

The vertex of this parabola is located at \((3, -12)\), and the equation of the axis of symmetry is \(x = 3\). The parabola opens upward.

Guided Practice

1. Write \( y = 4x^2 + 16x + 34 \) in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

2 Graph Parabolas  In Chapter 5, you learned that the graph of the quadratic equation \( y = a(x - h)^2 + k \) is a transformation of the parent graph of \( y = x^2 \) translated \( h \) units horizontally and \( k \) units vertically, and reflected and/or dilated depending on the value of \( a \).

Example 2  Graph Parabolas

Graph each equation.

a. \( y = -3x^2 \)

For this equation, \( h = 0 \) and \( k = 0 \). The vertex is at the origin. Since the equation of the axis of symmetry is \( x = 0 \), substitute some small positive integers for \( x \) and find the corresponding \( y \)-values.

Since the graph is symmetric about the \( y \)-axis, the points at \((-1, -3), (-2, -12), \) and \((-3, -27)\) are also on the parabola. Use all of these points to draw the graph.

b. \( y = -3(x - 4)^2 + 5 \)

The equation is of the form \( y = a(x - h)^2 + k \), where \( h = 4 \) and \( k = 5 \). The graph of this equation is the graph of \( y = -3x^2 \) in part a translated 4 units to the right and up 5 units. The vertex is now at \((4, 5)\).

Guided Practice

2A. \( y = 2x^2 \)

2B. \( y = 2(x - 1)^2 - 4 \)
Equations of parabolas with vertical axes of symmetry have the parent function \( y = x^2 \) and are of the form \( y = a(x - h)^2 + k \). These are functions. Equations of parabolas with horizontal axes of symmetry are of the form \( x = a(y - k)^2 + h \) and are not functions. The parent graph for these equations is \( x = y^2 \).

### Example 3 Graph an Equation in General Form

Graph each equation.

**a.** \(2x - y^2 = 4y + 10\)

**Step 1** Write the equation in the form \( x = a(y - k)^2 + h \).

\[
2x - y^2 = 4y + 10 \\
2x = y^2 + 4y + 10 \\
2x = (y^2 + 4y + 4) + 10 - 4 \\
2x = (y + 2)^2 + 6 \\
x = \frac{1}{2}(y + 2)^2 + 3
\]

**Step 2** Use the equation to find information about the graph. Then draw the graph based on the parent graph, \( x = y^2 \).

- **vertex:** (3, −2)
- **axis of symmetry:** \( y = -2 \)
- **focus:** \( \left(3 + \frac{1}{4}, -2\right) \) or (3.5, −2)
- **directrix:** \( x = 3 - \frac{1}{4} \) or 2.5
- **direction of opening:** right, since \( a > 0 \)
- **length of latus rectum:** \( \left|\frac{1}{\left(\frac{1}{2}\right)}\right| \) or 2 units

**b.** \(y + 2x^2 + 32 = -16x - 1\)

**Step 1** \( y + 2x^2 + 32 = -16x - 1 \)

\[
y = -2x^2 - 16x - 33 \\
y = -2(x^2 + 8x + 16) - 33 - 32 \\
y = -2(x + 4)^2 - 1
\]

**Step 2**

- **vertex:** (−4, −1)
- **axis of symmetry:** \( x = -4 \)
- **focus:** \( (-4, -\frac{9}{8}) \)
- **directrix:** \( y = -\frac{7}{8} \)
- **length of latus rectum:** \( \frac{1}{2} \) unit
- **opens down**

### Guided Practice

3A. \(3x - y^2 = 4x + 25\)  
3B. \(y = x^2 + 6x - 4\)
You can use specific information about a parabola to write an equation and draw a graph.

**Example 4 Write an Equation of a Parabola**

Write an equation for a parabola with vertex at \((-2, -4)\) and directrix \(y = 1\). Then graph the equation.

The directrix is a horizontal line, so the equation of the parabola is of the form \(y = a(x - h)^2 + k\). Find \(a\), \(h\), and \(k\).

- The vertex is at \((-2, -4)\), so \(h = -2\) and \(k = -4\).
- Use the equation of the directrix to find \(a\).

\[
\begin{align*}
1 &= -4 - \frac{1}{4a} \\
5 &= -\frac{1}{4a} \\
20a &= -1 \\
a &= -\frac{1}{20}
\end{align*}
\]

So, the equation of the parabola is \(y = -\frac{1}{20}(x + 2)^2 - 4\).

**Guided Practice**

Write an equation for each parabola described below. Then graph the equation.

4A. vertex \((1, 3)\), focus \((1, 5)\)

4B. focus \((5, 6)\), directrix \(x = -2\)

Parabolas are often used in the real world.

**Real-World Example 5 Write an Equation for a Parabola**

**ENVIRONMENT** Solar energy may be harnessed by using parabolic mirrors. The mirrors reflect the rays from the Sun to the focus of the parabola. The focus of each parabolic mirror at the facility described at the left is 6.25 feet above the vertex. The latus rectum is 25 feet long.

a. Assume that the focus is at the origin. Write an equation for the parabola formed by each mirror.

In order for the mirrors to collect the Sun’s energy, the parabola must open upward. Therefore, the vertex must be below the focus.

focus: \((0, 0)\)  vertex: \((0, -6.25)\)

The measure of the latus rectum is 25. So \(25 = \left|\frac{1}{a}\right|\), and \(a = \frac{1}{25}\).

Using the form \(y = a(x - h)^2 + k\), an equation for the parabola formed by each mirror is \(y = \frac{1}{25}x^2 - 6.25\).

b. Graph the equation.

Now use all of the information to draw a graph.

**Guided Practice**

5. Write and graph an equation for a parabolic mirror that has a focus 4.5 feet above the vertex and a latus rectum that is 18 feet long, when the focus is at the origin.
Example 1 Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

1. \( y = 2x^2 - 24x + 40 \)  
2. \( y = 3x^2 - 6x - 4 \)  
3. \( x = y^2 - 8y - 11 \)  
4. \( x + 3y^2 + 12y = 18 \)

Examples 2–3 Graph each equation.

5. \( y = (x - 4)^2 - 6 \)  
6. \( y = 4(x + 5)^2 + 3 \)  
7. \( y = -3x^2 - 4x - 8 \)  
8. \( x = 3y^2 - 6y + 9 \)

Example 4 Write an equation for each parabola described below. Then graph the equation.

9. vertex (0, 2), focus (0, 4)  
10. vertex (-2, 4), directrix x = -1  
11. focus (3, 2), directrix y = 8  
12. vertex (-1, -5), focus (-5, -5)

Example 5 13. ASTRONOMY Consider a parabolic mercury mirror like the one described at the beginning of the lesson. The focus is 6 feet above the vertex and the latus rectum is 24 feet long.

a. Assume that the focus is at the origin. Write an equation for the parabola formed by the parabolic microphone.

b. Graph the equation.

Practice and Problem Solving Extra Practice begins on page 947.

Example 1 Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

14. \( y = x^2 - 8x + 13 \)  
15. \( y = 3x^2 + 42x + 149 \)  
16. \( y = -6x^2 - 36x - 8 \)  
17. \( y = -3x^2 - 9x - 6 \)  
18. \( x = \frac{1}{3}y^2 - 3y + 4 \)  
19. \( x = \frac{2}{3}y^2 - 4y + 12 \)

Examples 2–3 Graph each equation.

20. \( y = \frac{1}{3}x^2 \)  
21. \( y = -2x^2 \)  
22. \( y = -2(x - 2)^2 + 3 \)  
23. \( y = 3(x - 3)^2 - 5 \)  
24. \( x = \frac{1}{2}y^2 \)  
25. \( 4x - y^2 = 2y + 13 \)

Example 4 Write an equation for each parabola described below. Then graph the equation.

26. vertex (0, 1), focus (0, 4)  
27. vertex (1, 8), directrix y = 3  
28. focus (-2, -4), directrix x = -6  
29. focus (2, 4), directrix x = 10  
30. vertex (-6, 0), directrix x = 2  
31. vertex (9, 6), focus (9, 5)

Example 5 32. BASEBALL When a ball is thrown, the path it travels is a parabola. Suppose a baseball is thrown from ground level, reaches a maximum height of 50 feet, and hits the ground 200 feet from where it was thrown. Assuming this situation could be modeled on a coordinate plane with the focus of the parabola at the origin, find the equation of the parabolic path of the ball. Assume the focus is on ground level.

33. SPACE Ground antennas and satellites are used to relay signals between the NASA Mission Operations Center and the spacecraft it controls. One such dish is 146 feet in diameter. Its focus is 48 feet from the vertex.

a. Sketch two options for the dish, one that opens up and one that opens left.

b. Write two equations that model the sketches in part a.

c. If you wanted to find the depth of the dish, does it matter which equation you use? Why or why not?
34. **UMBRELLAS** A beach umbrella has an arch in the shape of a parabola that opens downward. The umbrella spans 6 feet across and is $1 \frac{1}{2}$ feet high. Write an equation of a parabola to model the arch, assuming that the origin is at the point where the pole and umbrella meet at the vertex of the arch.

35. **AUTOMOBILES** An automobile headlight contains a parabolic reflector. The light coming from the source bounces off the parabolic reflector and shines out the front of the headlight. The equation of the cross section of the reflector is $y = \frac{1}{12}x^2$. How far from the vertex should the filament for the high beams be placed?

36. **MULTIPLE REPRESENTATIONS** Start with a sheet of wax paper that is about 15 inches long and 12 inches wide.

a. **Concrete** Make a line that is perpendicular to the sides of the sheet by folding the sheet near one end. Open up the paper again. This line is the directrix. Mark a point about midway between the sides of the sheet so that the distance from the directrix is about 1 inch. This is the focus.

b. **Concrete** Start with a new sheet of wax paper. Form another outline of a parabola with a focus that is about 3 inches from the directrix.

c. **Concrete** On a new sheet of a wax paper, form a third outline of a parabola with a focus that is about 5 inches from the directrix.

d. **Verbal** Compare the shapes of the three parabolas. How does the distance between the focus and the directrix affect the shape of a parabola?

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**H.O.T. Problems** Use Higher-Order Thinking Skills

37. **REASONING** How do you change the equation of the parent function $y = x^2$ to shift the graph to the right?

38. **OPEN ENDED** Two different parabolas have their vertex at $(-3, 1)$ and contain the point with coordinates $(-1, 0)$. Write two possible equations for these parabolas.

39. **ERROR ANALYSIS** Brianna and Russell are graphing $\frac{1}{4}y^2 + x = 0$. Is either of them correct? Explain your reasoning.

40. **WRITING IN MATH** How are parabolas used in televising sporting events? Explain why a televised sporting event filmed with a parabolic microphone is better than a televised sporting event filmed with a standard microphone.
41. A gardener is placing a fence around a 1320-square-foot rectangular garden. He ordered 148 feet of fencing. If he uses all the fencing, what is the length of the longer side of the garden?
A 30 ft  C 44 ft  B 34 ft  D 46 ft

42. SAT/ACT When a number is divided by 5, the result is 7 more than the number. Find the number.
F \(-\frac{35}{4}\)  J \(\frac{28}{4}\)
G \(-\frac{35}{6}\)  K \(\frac{35}{4}\)
H \(\frac{35}{6}\)

43. GEOMETRY What is the area of the following square, if the length of \(\overline{BD}\) is \(2\sqrt{2}\)?

44. SHORT RESPONSE The measure of the smallest angle of a triangle is two thirds the measure of the middle angle. The measure of the middle angle is three sevenths of the measure of the largest angle. Find the largest angle's measure.

Spiral Review

45. GEOMETRY Find the perimeter of a triangle with vertices at (2, 4), (−1, 3), and (1, −3). (Lesson 10-1)

46. WORK A worker can powerwash a wall of a certain size in 5 hours. Another worker can do the same job in 4 hours. If the workers work together, how long would it take to do the job? Determine whether your answer is reasonable. (Lesson 9-6)

Solve each equation or inequality. Round to the nearest ten-thousandth. (Lesson 8-7)

47. \(\ln (x + 1) = 1\)
48. \(\ln (x - 7) = 2\)
49. \(e^x > 1.6\)
50. \(e^{5x} \geq 25\)

Simplify. (Lesson 7-4)

51. \(\sqrt{0.25}\)
52. \(\sqrt{-0.064}\)
53. \(\sqrt[4]{z^8}\)
54. \(-\sqrt[4]{x^6}\)

List all of the possible rational zeros of each function. (Lesson 6-8)

55. \(h(x) = x^3 + 8x + 6\)
56. \(p(x) = 3x^3 - 5x^2 - 11x + 3\)
57. \(h(x) = 9x^6 - 5x^3 + 27\)

Skills Review

Simplify each expression. (Concepts and Skills Bank 2)

58. \(\sqrt{24}\)
59. \(\sqrt[4]{45}\)
60. \(\sqrt{252}\)
61. \(\sqrt{512}\)
You can use TI-Nspire™ or TI-Nspire CAS™ technology to examine characteristics of circles and the relationship with an equation of the circle.

**Activity**

**Step 1** Draw a circle.
- From the Home screen, select **New Document**. Select **Add Graphs & Geometry**. Then press **menu** and select **Shapes**, and then select **Circle**. Place the pointer at the origin and press to set the center of the circle. Move the pointer out, creating a circle like the one shown.
- From **menu**, select **Points & Lines**, and then **Point On** to place a point on the circle.
- Then, draw a radius by selecting **menu**, **Points & Lines**, and then **Segment**.

**Step 2** Add labels.
- Under **menu**, select **Actions**, then **Coordinates and Equations**. Use the pointer to select the center of the circle and display its coordinates. Move the coordinates out of the way.
- Display the length of the radius using **menu**, then **Measurement**, and then **Length**.
- Use **menu**, **Actions**, and **Coordinates and Equations** to display an equation of the circle.

**Step 3** Change the radius.
Move the pointer so that a point on the circle is highlighted, then press and hold until it is selected. Examine the equation of the circle. Then move the edge of the circle in. Make note of changes in the equation.

**Step 4** Move the center of the circle.
Move the pointer so that the center of the circle is highlighted, then press and hold until it is selected. Move the center of the circle. Again, examine the equation of the circle.

**Analyze the Results**

1. How does moving the edge of the circle in or out affect the equation of the circle?
2. What effect does moving the center of the circle have on the equation?
3. Repeat the activity by placing the center of a circle in Quadrant I. Move the center to each of the other three quadrants. How does the equation change?
4. **MAKE A CONJECTURE** Without graphing, write an equation of each circle.
   - **a.** center: $(4, 2)$, radius: $3$
   - **b.** center: $(-1, 1)$, radius: $8$
   - **c.** center: $(-6, -5)$, radius: $2.5$
   - **d.** center: $(h, k)$, radius: $r$
1 Equations of Circles

A circle is the set of all points in a plane that are equidistant from a given point in the plane, called the center. Any segment with endpoints at the center and a point on the circle is a radius of the circle.

Assume that \((x, y)\) are the coordinates of a point on the circle at the right. The center is at \((h, k)\), and the radius is \(r\). You can find an equation of the circle by using the Distance Formula.

\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d \\
\sqrt{(x - h)^2 + (y - k)^2} = r
\]

Key Concept Equations of Circles

<table>
<thead>
<tr>
<th>Standard Form of Equation</th>
<th>(x^2 + y^2 = r^2)</th>
<th>((x - h)^2 + (y - k)^2 = r^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center</td>
<td>((0, 0))</td>
<td>((h, k))</td>
</tr>
<tr>
<td>Radius</td>
<td>(r)</td>
<td>(r)</td>
</tr>
</tbody>
</table>

You can use the standard form of the equation of a circle to write an equation for a circle given the center and the radius or diameter.

Real-World Example 1 Write an Equation Given the Radius

DELIVERY Appliances + More offers free delivery within 35 miles of the store. The Jacksonville store is located 100 miles north and 45 miles east of the corporate office. Write an equation to represent the delivery boundary of the Jacksonville store if the origin of the coordinate system is the corporate office.

Since the corporate office is at \((0, 0)\), the Jacksonville store is at \((45, 100)\). The boundary of the delivery region is the circle centered at \((45, 100)\) with radius 35 miles.

\[
(x - 45)^2 + (y - 100)^2 = 35^2
\]

\[
(x - 45)^2 + (y - 100)^2 = 1225
\]

Guided Practice

1. **WI-FI** A certain wi-fi phone has a range of 30 miles in any direction. If the phone is 4 miles south and 3 miles west of headquarters, write an equation to represent the area within which the phone can operate via the Wi-Fi system.
You can write the equation of a circle when you know the location of the center and a point on the circle.

**Example 2 Write an Equation from a Graph**

Write an equation for each graph.

**Guided Practice**

You can use the Midpoint and Distance Formulas when you know the endpoints of the radius or diameter of a circle.

**Example 3 Write an Equation Given a Diameter**

Write an equation for a circle if the endpoints of a diameter are at (7, 6) and (−1, −8).

**Step 1** Find the center.

\[
(h, k) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{7 + (-1)}{2}, \frac{6 + (-8)}{2}\right) = (3, -1)
\]

**Step 2** Find the radius.

\[
r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
= \sqrt{(3 - 7)^2 + (-1 - 6)^2} = \sqrt{(-4)^2 + (-7)^2} = \sqrt{65}
\]

The radius of the circle is \(\sqrt{65}\) units, so \(r^2 = 65\). Substitute \(h, k,\) and \(r^2\) into the standard form of the equation of a circle. An equation of the circle is \((x - 3)^2 + (y + 1)^2 = 65\).

**Guided Practice**

3. Write an equation for a circle if the endpoints of a diameter are at (3, −3) and (1, 5).
Graph Circles You can use symmetry to help you graph circles.

Example 4 Graph an Equation in Standard Form

Find the center and radius of the circle with equation \(x^2 + y^2 = 100\). Then graph the circle.

- The center of the circle is at \((0, 0)\), and the radius is 10.
- The table lists some integer values for \(x\) and \(y\) that satisfy the equation.
- Because the circle is centered at the origin, it is symmetric about the \(y\)-axis. Therefore, the points at \((-6, 8)\), \((-8, 6)\), and \((-10, 0)\) lie on the graph.
- The circle is also symmetric about the \(x\)-axis, so the points \((-6, -8)\), \((-8, -6)\), \((0, -10)\), \((6, -8)\), and \((8, -6)\) lie on the graph.
- Plot all of these points and draw the circle that passes through them.

Guided Practice

4. Find the center and radius of the circle with equation \(x^2 + y^2 = 81\). Then graph the circle.

Circles with centers that are not \((0, 0)\) can be graphed by using translations. The graph of \((x - h)^2 + (y - k)^2 = r^2\) is the graph of \(x^2 + y^2 = r^2\) translated \(h\) units horizontally and \(k\) units vertically.

Example 5 Graph an Equation Not in Standard Form

Find the center and radius of the circle with equation \(x^2 + y^2 - 8x + 12y - 12 = 0\). Then graph the circle.

Complete the squares.

\[
x^2 + y^2 - 8x + 12y - 12 = 0
\]
\[
x^2 - 8x + \square + y^2 + 12y + \square = 12 + \square + \square
\]
\[
x^2 - 8x + 16 + y^2 + 12y + 36 = 12 + 16 + 36
\]
\[
(x - 4)^2 + (y + 6)^2 = 64
\]

The center of the circle is at \((4, -6)\) and the radius is 8. The graph of \((x - 4)^2 + (y + 6)^2 = 64\) is the same as \(x^2 + y^2 = 64\) translated 4 units to the right and down 6 units.

Guided Practice

5. Find the center and radius of the circle with equation \(x^2 + y^2 + 4x - 10y - 7 = 0\). Then graph the circle.
Example 1

1. **WEATHER** On average, the eye of a tornado is about 200 feet across. Suppose the center of the eye is at the point (72, 39). Write an equation to represent the boundary of the eye.

Write an equation for each circle given the center and radius.

2. center: (−2, −6), \( r = 4 \) units
3. center: (1, −5), \( r = 3 \) units

Example 2

Write an equation for each graph.

4.

5.

Example 3

Write an equation for each circle given the endpoints of a diameter.

6. (−1, −7) and (0, 0)
7. (4, −2) and (−4, −6)

Examples 4–5

Find the center and radius of each circle. Then graph the circle.

8. \( x^2 + y^2 = 16 \)
9. \( x^2 + (y − 7)^2 = 9 \)
10. \( (x − 4)^2 + (y − 4)^2 = 25 \)
11. \( x^2 + y^2 − 4x + 8y − 5 = 0 \)

Practice and Problem Solving

Extra Practice begins on page 947.

Example 1

Write an equation for each circle given the center and radius.

12. center: (4, 9), \( r = 6 \)
13. center: (−3, 1), \( r = 4 \)
14. center: (−7, −3), \( r = 13 \)
15. center: (−2, −1), \( r = 9 \)
16. center: (1, 0), \( r = \sqrt{15} \)
17. center: (0, −6), \( r = \sqrt{35} \)

18. **AIR TRAFFIC CONTROL** The radar for a county airport control tower is located at (5, 10) on a map. It can detect a plane up to 20 miles away. Write an equation for the outer limits of the detection area.

Example 2

Write an equation for each graph.

19.

20.

21.

22.
**Example 3** Write an equation for each circle given the endpoints of a diameter.

23. (2, 1) and (2, −4)  
24. (−4, −10) and (4, −10)  
25. (5, −7) and (−2, −9)
26. (−6, 4) and (4, 8)  
27. (2, −5) and (6, 3)  
28. (18, 11) and (−19, −13)

29. **LAWN CARE** A sprinkler waters a circular section of lawn.
   a. Write an equation to represent the boundary of the sprinkler area if the endpoints of a diameter are at (−12, 16) and (12, −16).
   b. What is the area of the lawn that the sprinkler waters?

30. **SPACE** Apollo 8 was the first manned spacecraft to orbit the Moon at an average altitude of 185 kilometers above the Moon’s surface. Write an equation to model a single circular orbit of the command module if the endpoints of a diameter of the Moon are at (1740, 0) and (−1740, 0). Let the center of the Moon be at the origin of the coordinate system measured in kilometers.

31. \( x^2 + y^2 = 75 \)
32. \((x - 3)^2 + y^2 = 4 \)
33. \((x - 1)^2 + (y - 4)^2 = 34 \)
34. \( x^2 + (y - 14)^2 = 144 \)
35. \((x - 5)^2 + (y + 2)^2 = 16 \)
36. \( x^2 + y^2 = 256 \)
37. \((x - 4)^2 + y^2 = \frac{8}{9} \)
38. \( \left(x + \frac{2}{3}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{16}{25} \)
39. \( x^2 + y^2 + 4x = 9 \)
40. \( x^2 + y^2 - 6y + 8x = 0 \)
41. \( x^2 + y^2 + 2x + 4y = 9 \)
42. \( x^2 + y^2 - 3x + 8y = 20 \)
43. \( x^2 + y^2 + 6y = -50 - 14x \)
44. \( x^2 - 18x + 53 = 18y - y^2 \)
45. \( 2x^2 + 2y^2 - 4x + 8y = 32 \)
46. \( 3x^2 + 3y^2 - 6y + 12x = 24 \)

47. **SPACE** A satellite is in a circular orbit 25,000 miles above Earth.
   a. Write an equation for the orbit of this satellite if the origin is at the center of Earth. Use 8000 miles as the diameter of Earth.
   b. Draw a sketch of Earth and the orbit to scale. Label your sketch.

48. **COMMUNICATIONS** Suppose an unobstructed radio station broadcast could travel 120 miles. Assume the station is centered at the origin.
   a. Write an equation to represent the boundary of the broadcast area with the origin as the center.
   b. If the transmission tower is relocated 40 miles east and 10 miles south of the current location, and an increased signal will transmit signals an additional 80 miles, what is an equation to represent the new broadcast area?

49. **GEOMETRY** Concentric circles are circles with the same center but different radii. Refer to the graph at the right where \( AB \) is a diameter of the circle.
   a. Write an equation of the circle concentric with the circle at the right, with radius 4 units greater.
   b. Write an equation of the circle concentric with the circle at the right, with radius 2 units less.
   c. Graph the circles from parts a and b on the same coordinate plane.

50. **EARTHQUAKES** The Rose Bowl is located about 35 miles west and 40 miles north of downtown Los Angeles. Suppose an earthquake occurs with its epicenter about 55 miles from the stadium. Assume that the origin of a coordinate plane is located at the center of downtown Los Angeles. Write an equation for the set of points that could be the epicenter of the earthquake.
Write an equation for the circle that satisfies each set of conditions.

51. center (9, −8), passes through (19, 22)
52. center \((-\sqrt{15}, 30)\), passes through the origin
53. center at (8, −9), tangent to y-axis
54. center at (2, 4), tangent to x-axis
55. center in the first quadrant; tangent to \(x = 5\), the \(x\)-axis, and the \(y\)-axis
56. center in the second quadrant; tangent to \(y = 1\), \(y = 5\), and the \(y\)-axis

57. **MULTIPLE REPRESENTATIONS** Graph \(y = \sqrt[ ]{9 - x^2}\) and \(y = -\sqrt[ ]{9 - x^2}\) on the same graphing calculator screen.

a. **Verbal** Describe the graph formed by the union of these two graphs.

b. **Algebraic** Write an equation for the union of the two graphs.

c. **Verbal** Most graphing calculators cannot graph the equation \(x^2 + y^2 = 49\) directly. Describe a way to use a graphing calculator to graph the equation. Then graph the equation.

d. **Analytical** Solve \((x - 2)^2 + (y + 1)^2 = 4\) for \(y\). Why do you need two equations to graph a circle on a graphing calculator?

e. **Verbal** Do you think that it is easier to graph the equation in part d using graph paper and a pencil or using a graphing calculator? Explain.

Find the center and radius of each circle. Then graph the circle.

58. \(x^2 - 12x + 84 = -y^2 + 16y\)
59. \(4x^2 + 4y^2 + 36y + 5 = 0\)
60. \((x + \sqrt{5})^2 + y^2 - 8y = 9\)
61. \(x^2 + 2\sqrt{7}x + 7 + (y - \sqrt{11})^2 = 11\)

**H.O.T. Problems** **Use Higher-Order Thinking Skills**

62. **ERROR ANALYSIS** Heather says that \((x - 2)^2 + (y + 3)^2 = 36\) and \((x - 2) + (y + 3) = 6\) are equivalent equations. Carlota says that the equations are *not* equivalent. Is either of them correct? Explain your reasoning.

63. **OPEN ENDED** Consider graphs with equations of the form \((x - 3)^2 + (y - a)^2 = 64\).

Assign three different values for \(a\), and graph each equation. Describe all graphs with equations of this form.

64. **REASONING** Explain why the phrase “in a plane” is included in the definition of a circle. What would be defined if the phrase were *not* included?

65. **OPEN ENDED** Concentric circles have the same center, but most often, not the same radius. Write equations of two concentric circles. Then graph the circles.

66. **REASONING** Assume that \((x, y)\) are the coordinates of a point on a circle. The center is at \((h, k)\), and the radius is \(r\). Find an equation of the circle by using the Distance Formula.

67. **WRITING IN MATH** The circle with equation \((x - a)^2 + (y - b)^2 = r^2\) lies in the first quadrant and is tangent to both the \(x\)-axis and the \(y\)-axis. Sketch the circle. Describe the possible values of \(a\), \(b\), and \(r\). Do the same for a circle in Quadrants II, III, and IV. Discuss the similarities among the circles.
68. **GRIDDED RESPONSE**  Two circles, both with radii 6, have exactly one point in common. If $A$ is a point on one circle and $B$ is a point on the other circle, what is the maximum possible length for the line segment $AB$?

69. In the senior class, there are 20% more girls than boys. If there are 180 girls, how many more girls than boys are there among the seniors?

   A 30
   B 36
   C 90
   D 144

70. A $1000 deposit is made at a bank that pays 2% compounded weekly. How much will you have in your account at the end of 10 years?

   F $1200.00
   H $1221.36
   G $1218.99
   J $1224.54

71. The mean of six numbers is 20. If one of the numbers is removed, the average of the remaining numbers is 15. What is the number that was removed?

   A 42
   B 43
   C 45
   D 48

**Spiral Review**

Graph each equation. (Lesson 10-2)

72. $y = -\frac{1}{2}(x - 1)^2 + 4$
73. $4(x - 2) = (y + 3)^2$
74. $(y - 8)^2 = -4(x - 4)$

Find the midpoint of the line segment with endpoints at the given coordinates. Then find the distance between the points. (Lesson 10-1)

75. $\left(-3, -\frac{2}{11}\right)$, $\left(5, \frac{9}{11}\right)$
76. $\left(2\sqrt{3}, -5\right)$, $\left(-3\sqrt{3}, 9\right)$
77. $(2.5, 4)$, $(-2.5, 2)$

78. If $y$ varies directly as $x$ and $y = 8$ when $x = 6$, find $y$ when $x = 15$. (Lesson 9-5)
79. If $y$ varies jointly as $x$ and $z$ and $y = 80$ when $x = 5$ and $z = 8$, find $y$ when $x = 16$ and $z = 2$. (Lesson 9-5)
80. If $y$ varies inversely as $x$ and $y = 16$ when $x = 5$, find $y$ when $x = 20$. (Lesson 9-5)

Evaluate each expression. (Lesson 8-3)

81. $\log_9 243$
82. $\log_2 \frac{1}{32}$
83. $\log_3 \frac{1}{81}$
84. $\log_{10} 0.001$

85. **AMUSEMENT PARKS**  The velocity v in feet per second of a roller coaster at the bottom of a hill is related to the vertical drop h in feet and the velocity $v_0$ in feet per second of the coaster at the top of the hill by the formula $v_0 = \sqrt{v^2 - 64h}$. (Lesson 7-5)

   a. Explain why $v_0 = v - 8\sqrt{h}$ is not equivalent to the given formula.
   b. What velocity must the coaster have at the top of the hill to achieve a velocity of 125 feet per second at the bottom?

**Skills Review**

Solve each equation by completing the square. (Lesson 5-5)

86. $x^2 + 3x - 18 = 0$
87. $2x^2 - 3x - 3 = 0$
88. $x^2 + 2x + 6 = 0$
Follow the steps below to construct a type of conic section.

**Activity** Make an Ellipse

**Step 1** Place two thumbtacks in a piece of cardboard, about 1 foot apart.

**Step 2** Tie a knot in a piece of string and loop it around the thumbtacks. Place your pencil in the string.

**Step 3** Keep the string tight and draw a curve. Continue drawing until you return to your starting point.

The curve you have drawn is called an **ellipse**. The points where the thumbtacks are located are called the **foci** of the ellipse. *Foci* is the plural of *focus*.

**Model and Analyze**

Place a large piece of grid paper on a piece of cardboard.

1. Place the thumbtacks at (7, 0) and (−7, 0). Choose a string long enough to loop around both thumbtacks. Draw an ellipse.

2. Repeat Exercise 1, but place the thumbtacks at (4, 0) and (−4, 0). Use the same loop of string and draw an ellipse. How does this ellipse compare to the one in Exercise 1?

Place the thumbtacks at each set of points and draw an ellipse. You may change the length of the loop of string if you like.

3. (11, 0), (−11, 0) 4. (3, 0), (−3, 0) 5. (13, 3), (−9, 3)

**Make a Conjecture**

Describe what happens to the shape of an ellipse when each change is made.

6. The thumbtacks are moved closer together.

7. The thumbtacks are moved farther apart.

8. The length of the loop of string is increased.

9. The thumbtacks are arranged vertically.

10. One thumbtack is removed, and the string is looped around the remaining thumbtack.

11. Pick a point on one of the ellipses you have drawn. Use a ruler to measure the distances from that point to the points where the thumbtacks were located. Add the distances. Repeat for other points on the same ellipse. What relationship do you notice?

12. Could this activity be done with a rubber band instead of a piece of string? Explain.
Mercury, like all of the planets of our solar system, does not orbit the Sun in a perfect circular path. At its farthest point, Mercury is about 43 million miles from the Sun. At its closest point, it is only about 28.5 million miles from the Sun. This orbit is in the shape of an ellipse with the Sun at a focus.

Ellipses

1. **Equations of Ellipses**

An ellipse is the set of all points in a plane such that the sum of the distances from two fixed points is constant. These two points are called the foci of the ellipse.

Every ellipse has two axes of symmetry, the major axis and the minor axis. The axes are perpendicular at the center of the ellipse.

The foci of an ellipse always lie on the major axis. The endpoints of the major axis are the vertices of the ellipse and the endpoints of the minor axis are the co-vertices of the ellipse.

Key Concept: Equations of Ellipses Centered at the Origin

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$</th>
<th>$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orientation</td>
<td>horizontal</td>
<td>vertical</td>
</tr>
<tr>
<td>Foci</td>
<td>$(c, 0), (-c, 0)$</td>
<td>$(0, c), (0, -c)$</td>
</tr>
<tr>
<td>Length of Major Axis</td>
<td>$2a$ units</td>
<td>$2a$ units</td>
</tr>
<tr>
<td>Length of Minor Axis</td>
<td>$2b$ units</td>
<td>$2b$ units</td>
</tr>
</tbody>
</table>

There are several important relationships among the many parts of an ellipse.

- The length of the major axis, $2a$ units, equals the sum of the distances from the foci to any point on the ellipse.
- The values of $a$, $b$, and $c$ are related by the equation $c^2 = a^2 - b^2$.
- The distance from a focus to either co-vertex is $a$ units.
The sum of the distances from the foci to any point on the ellipse, or the constant sum, must be greater than the distance between the foci.

**Example 1** Write an Equation Given Vertices and Foci

**Write an equation for the ellipse.**

**Step 1** Find the center.

The foci are equidistant from the center.

The center is at (0, 0).

**Step 2** Find the value of $a$.

The vertices are (0, 9) and (0, -9), so the length of the major axis is 18.

The value of $a$ is 18/2 or 9, and $a^2 = 81$.

**Step 3** Find the value of $b$.

We can use $c^2 = a^2 - b^2$ to find $b$.

The foci are 7 units from the center, so $c = 7$.

$c^2 = a^2 - b^2$ 
$49 = 81 - b^2$ 
$a = 9$ and $c = 7$ 
$b^2 = 32$ Solve for $b^2$.

**Step 4** Write the equation.

Because the major axis is vertical, $a^2$ goes with $y$ and $b^2$ goes with $x$.

The equation for the ellipse is $\frac{y^2}{81} + \frac{x^2}{32} = 1$.

**Guided Practice**

1. Write an equation for an ellipse with vertices at (-4, 0) and (4, 0) and foci at (2, 0) and (-2, 0).

Like other graphs, the graph of an ellipse can be translated. When the graph is translated $h$ units right and $k$ units up, the center of the translation is $(h, k)$. This is equivalent to replacing $x$ with $x - h$ and replacing $y$ with $y - k$ in the parent function.

**Key Concept** Equations of Ellipses Centered at $(h, k)$

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Horizontal $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$</th>
<th>Vertical $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orientation</td>
<td>$h$ units right and $k$ units up</td>
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</tr>
<tr>
<td>Foci</td>
<td>$(h \pm c, k)$</td>
<td>$(h, k \pm c)$</td>
</tr>
<tr>
<td>Vertices</td>
<td>$(h \pm a, k)$</td>
<td>$(h, k \pm a)$</td>
</tr>
<tr>
<td>Co-vertices</td>
<td>$(h, k \pm b)$</td>
<td>$(h \pm b, k)$</td>
</tr>
</tbody>
</table>

We can use this information to determine the equations for ellipses. The original ellipse at the right is horizontal and has a major axis of 10 units, so $a = 5$.

The length of the minor axis is 6 units, so $b = 3$.

The ellipse is translated 4 units right and 5 units down. So, the value of $h$ is 4 and the value of $k$ is -5.

The equation for the original ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

The equation for the translation is $\frac{(x-4)^2}{25} + \frac{(y+5)^2}{9} = 1$. 

**Study Tip**

**Major Axis** In standard form, if the $x^2$-term has the greater denominator, then the major axis is horizontal. If the $y^2$-term has the greater denominator, then it is vertical.
You can also determine the equation for an ellipse if you are given all four vertices.

**Example 2** Write an Equation Given the Lengths of the Axes

Write an equation for the ellipse with vertices at \((6, -8)\) and \((6, 4)\) and co-vertices at \((3, -2)\) and \((9, -2)\).

The \(x\)-coordinate is the same for both vertices, so the ellipse is vertical.

The center of the ellipse is at \(\left( \frac{6 + 6}{2}, \frac{-8 + 4}{2} \right)\) or \((6, -2)\).

The length of the major axis is \(4 - (-8)\) or 12 units, so \(a = 6\).

The length of the minor axis is \(9 - 3\) or 6 units, so \(b = 3\).

The equation for the ellipse is \(\frac{(y + 2)^2}{36} + \frac{(x - 6)^2}{9} = 1\). \(a^2 = 36, b^2 = 9\)

**Guided Practice**

2. Write an equation for the ellipse with vertices at \((-3, 8)\) and \((9, 8)\) and co-vertices at \((3, 12)\) and \((3, 4)\).

Many real-world phenomena can be represented by ellipses.

**Real-World Example 3** Write an Equation for an Ellipse

**SPACE** Refer to the application at the beginning of the lesson. Mercury’s greatest distance from the Sun, or **aphelion**, is about 43 million miles. Mercury’s closest distance, or **perihelion**, is about 28.5 million miles. The diameter of the Sun is about 870,000 miles. Use this information to determine an equation relating Mercury’s elliptical orbit around the Sun in millions of miles.

**Understand** We need to determine an equation representing Mercury’s orbit around the Sun.

**Plan** Including the diameter of the Sun, the sum of the perihelion and aphelion equals the length on the major axis of the ellipse. We can use this information to determine the values of \(a, b,\) and \(c\).

**Solve** Find the value of \(a\).

The value of \(a\) is one half the length of the major axis.

\(a = 0.5(43 + 28.5 + 0.87)\) or 36.185

Find the value of \(c\).

The value of \(c\) is the distance from the center of the ellipse to the focus. This distance is equal to \(a\) minus the perihelion and the radius of the Sun.

\(c = 36.185 - 28.5 - 0.435\) or 7.25
Find the value of $b$.

\[
\begin{align*}
c^2 &= a^2 - b^2 \\
(7.25)^2 &= (36.185)^2 - b^2 \\
52.5625 &= 1309.3542 - b^2 \\
b^2 &= 1256.828 \\
b &= 35.4518
\end{align*}
\]

So, with the center of the orbit at the origin, the equation relating Mercury’s orbit around the Sun can be modeled by

\[
\frac{x^2}{1309.3542} + \frac{y^2}{1256.828} = 1.
\]

**Check** Use your answer to recalculate $a$, $b$, and $c$. Then determine the aphelion and perihelion based on your answer. Compare to the actual values.

**Guided Practice**

3. **SPACE** Pluto’s distance from the Sun is 2.757 billion miles at perihelion and about 4.583 billion miles at aphelion. Determine an equation relating Pluto’s orbit around the Sun in billions of miles with the center of the horizontal ellipse at the origin.

**Graph Ellipses** When you are given an equation for an ellipse that is not in standard form, you can write it in standard form by completing the square for both $x$ and $y$. Once the equation is in standard form, you can use it to graph the ellipse.

**Example 4 Graph an Ellipse**

Find the coordinates of the center and foci, and the lengths of the major and minor axes of an ellipse with equation $25x^2 + 9y^2 + 250x - 36y + 436 = 0$. Then graph the ellipse.

**Step 1** Write in standard form. Complete the square for each variable to write this equation in standard form.

\[
\begin{align*}
25x^2 + 9y^2 + 250x - 36y + 436 &= 0 \\
25(x^2 + 10x) + 9(y^2 - 4y) &= -436 \\
25(x^2 + 10x + 25) + 9(y^2 - 4y + 4) &= -436 + 25(25) + 9(4) \\
25(x + 5)^2 + 9(y - 2)^2 &= 225 \\
\frac{(x + 5)^2}{9} + \frac{(y - 2)^2}{25} &= 1
\end{align*}
\]

**Step 2** Find the center. 

$h = -5$ and $k = 2$, so the center of the ellipse is at $(-5, 2)$.

**Step 3** Find the lengths of the axes and graph.

The ellipse is vertical.

$a^2 = 25$, so $a = 5$. $b^2 = 9$, so $b = 3$.

The length of the major axis is $2 \cdot 5$ or 10.

The length of the minor axis is $2 \cdot 3$ or 6.

The vertices are at $(-5, 7)$ and $(-5, -3)$.

The co-vertices are at $(-2, 2)$ and $(-8, 2)$. 

**Real-World Link**

Earth’s orbit around the Sun is nearly circular, with only about a 3% difference between perihelion and aphelion.

**Source:** The Astronomer
Check Your Understanding

Example 1 Write an equation for each ellipse.

1. 

Example 2 Write an equation for an ellipse that satisfies each set of conditions.

3. vertices at (−2, −6) and (−2, 4), co-vertices at (−5, −1) and (1, −1)

4. vertices at (−2, 5) and (14, 5), co-vertices at (6, 1) and (6, 9)

Example 3

5. ARCHITECTURE An architectural firm sent a proposal to a city for building a coliseum, shown at the right.

a. Determine the values of a and b.

b. Assuming that the center is at the origin, write an equation to represent the ellipse.

c. Determine the coordinates of the foci.

6. SPACE Earth’s orbit is about 91.4 million miles at perihelion and about 94.5 million miles at aphelion. Determine an equation relating Earth’s orbit around the Sun in millions of miles with the center of the horizontal ellipse at the origin.

Example 4 Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

7. \( \frac{(y + 1)^2}{64} + \frac{(x - 5)^2}{28} = 1 \)

8. \( \frac{(x + 2)^2}{48} + \frac{(y - 1)^2}{20} = 1 \)

9. \( 4x^2 + y^2 - 32x - 4y + 52 = 0 \)

10. \( 9x^2 + 25y^2 + 72x - 150y + 144 = 0 \)
Example 1  Write an equation for each ellipse.

11. \( \frac{x^2}{36} + \frac{(y + 8)^2}{64} = 1 \)

12. \( \frac{(x + 9)^2}{100} + \frac{(y + 8)^2}{64} = 1 \)

13. \( \frac{(x + 12)^2}{100} + \frac{(y + 4)^2}{25} = 1 \)

14. \( \frac{(x - 1)^2}{4} + \frac{(y - 7)^2}{9} = 1 \)

15. \( \frac{(x + 5)^2}{9} + \frac{(y - 1)^2}{4} = 1 \)

16. \( \frac{(x - 13)^2}{25} + \frac{(y + 6)^2}{64} = 1 \)

Example 2  Write an equation for an ellipse that satisfies each set of conditions.

17. Vertices at \((-6, 4)\) and \((12, 4)\), co-vertices at \((3, 12)\) and \((3, -4)\)

18. Vertices at \((-1, 11)\) and \((-1, 1)\), co-vertices at \((-4, 6)\) and \((2, 6)\)

19. Center at \((-2, 6)\), vertex at \((-2, 16)\), co-vertex at \((1, 6)\)

20. Center at \((3, -4)\), vertex at \((8, -4)\), co-vertex at \((3, -2)\)

21. Vertices at \((4, 12)\) and \((4, -4)\), co-vertices at \((1, 4)\) and \((7, 4)\)

22. Vertices at \((-11, 2)\) and \((-1, 2)\), co-vertices at \((-6, 0)\) and \((-6, 4)\)

Example 3  TUNNELS The opening of a tunnel in the mountains can be modeled by semielipses, or halves of ellipses. If the opening is 14.6 meters wide and 8.6 meters high, determine an equation to represent the opening with the center at the origin.

Example 4  Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

24. \( \frac{(x - 3)^2}{36} + \frac{(y - 2)^2}{128} = 1 \)

25. \( \frac{(x + 6)^2}{50} + \frac{(y - 3)^2}{72} = 1 \)

26. \( \frac{x^2}{27} + \frac{(y - 5)^2}{64} = 1 \)

27. \( \frac{(x + 4)^2}{16} + \frac{y^2}{75} = 1 \)

28. \( 3x^2 + y^2 - 6x - 8y - 5 = 0 \)

29. \( 3x^2 + 4y^2 - 18x + 24y + 3 = 0 \)

30. \( 7x^2 + y^2 - 56x + 6y + 93 = 0 \)

31. \( 3x^2 + 2y^2 + 12x - 20y + 14 = 0 \)

32. SPACE Like the planets, Halley’s Comet travels around the Sun in an elliptical orbit. The aphelion is 3282.9 million miles and the perihelion is 54.87 million miles. Determine an equation relating the comet’s orbit around the Sun in millions of miles with the center of the horizontal ellipse at the origin.
Write an equation for an ellipse that satisfies each set of conditions.

33. center at \((-5, -2)\), focus at \((-5, 2)\), co-vertex at \((-8, -2)\)
34. center at \((4, -3)\), focus at \((9, -3)\), co-vertex at \((4, -5)\)
35. foci at \((-2, 8)\) and \((6, 8)\), co-vertex at \((2, 10)\)
36. foci at \((4, 4)\) and \((4, 14)\), co-vertex at \((0, 9)\)

37. **GOVERNMENT** The Oval Office is located in the West Wing of the White House. It is an elliptical shaped room used as the main office by the President of the United States. The long axis is 10.9 meters long and the short axis is 8.8 meters long. Write an equation to represent the outer walls of the Oval Office. Assume that the center of the room is at the origin.

38. **SOUND** A whispering gallery is an elliptical room in which a faint whisper at one focus that cannot be heard by other people in the room, can easily be heard by someone at the other focus. Suppose an ellipse is 400 feet long and 120 feet wide. What is the distance between the foci?

39. **MULTIPLE REPRESENTATIONS** The eccentricity of an ellipse measures how circular the ellipse is.
   a. **Graphical** Graph \(\frac{x^2}{81} + \frac{y^2}{36} = 1\) and \(\frac{x^2}{81} + \frac{y^2}{9} = 1\) on the same graph.
   b. **Verbal** Describe the difference between the two graphs.
   c. **Algebraic** The eccentricity of an ellipse is \(\frac{c}{a}\). Find the eccentricity for each.
   d. **Analytical** Make a conjecture about the relationship between the value of an ellipse’s eccentricity and the shape of the ellipse as compared to a circle.

**H.O.T. Problems** Use Higher-Order Thinking Skills

40. **ERROR ANALYSIS** Serena and Karissa are determining the equation for an ellipse with foci at \((-4, -11)\) and \((-4, 5)\) and co-vertices at \((2, -3)\) and \((-10, -3)\). Is either of them correct? Explain your reasoning.

<table>
<thead>
<tr>
<th>Serena</th>
<th>Karissa</th>
</tr>
</thead>
<tbody>
<tr>
<td>((x - 4)^2) + ((y + 3)^2) = 1</td>
<td>((x + 4)^2) + ((y + 3)^2) = (\frac{100}{36})</td>
</tr>
</tbody>
</table>

41. **OPEN ENDED** Write an equation for an ellipse with a focus at the origin.

42. **CHALLENGE** When the values of \(a\) and \(b\) are equal, an ellipse is a circle. Use this information and your knowledge of ellipses to determine the formula for the area of an ellipse in terms of \(a\) and \(b\).

43. **CHALLENGE** Determine an equation for an ellipse with foci at \((2, \sqrt{6})\) and \((2, -\sqrt{6})\) that passes through \((3, \sqrt{6})\).

44. **REASONING** What happens to the location of the foci as an ellipse becomes more circular? Explain your reasoning.

45. **REASONING** An ellipse has foci at \((-7, 2)\) and \((18, 2)\). If \((2, 14)\) is a point on the ellipse, show that \((2, -10)\) is also a point on the ellipse.

46. **WRITING IN MATH** Explain why the domain is \(\{x \mid -a \leq x \leq a\}\) and the range is \(\{y \mid -b \leq y \leq b\}\) for an ellipse with equation \(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\).
47. Multiply.

\[(2 + 3i)(4 + 7i)\]

A 8 + 21i  
B -13 + 26i  
C -6 + 10i  
D 13 + 12i

48. The average lifespan of American women has been tracked, and the model for the data is

\[y = 0.2t + 73,\]  
where \(t = 0\) corresponds to 1960. What is the meaning of the \(y\)-intercept?

F In 2007, the average lifespan was 60.  
G In 1960, the average lifespan was 58.  
H In 1960, the average lifespan was 73.  
J The lifespan is increasing 0.2 years every year.

49. GRIDDED RESPONSE If we decrease a number by 6 and then double the result, we get 5 less than the number. What is the number?

50. SAT/ACT The length of a rectangular prism is one inch greater than its width. The height is three times the length. Find the volume of the prism.

A \[3x^3 + x^2 + 3x\]  
B \[x^3 + x^2 + x\]  
C \[3x^3 + 6x^2 + 3x\]  
D \[3x^3 + 3x^2 + 3x\]  
E \[3x^3 + 3x^2\]

### Spiral Review

Write an equation for the circle that satisfies each set of conditions. **(Lesson 10-3)**

51. center \((8, -9)\), passes through \((21, 22)\)

52. center at \((4, 2)\), tangent to \(x\)-axis

53. center in the second quadrant; tangent to \(y = -1, y = 9,\) and the \(y\)-axis

54. **ENERGY** A parabolic mirror is used to collect solar energy. The mirrors reflect the rays from the Sun to the focus of the parabola. The focus of a particular mirror is 9.75 feet above the vertex, and the latus rectum is 39 feet long. **(Lesson 10-2)**

a. Assume that the focus is at the origin. Write an equation for the parabola formed by the mirror.

b. One foot is exactly 0.3048 meter. Rewrite the equation for the mirror in meters.

c. Graph one of the equations for the mirror.

d. Which equation did you choose to graph? Explain why.

Simplify each expression. **(Lesson 9-2)**

55. \[\frac{6}{d^2 + 4d + 4} + \frac{5}{d + 2}\]

56. \[\frac{a}{a^2 - a - 20} + \frac{2}{a + 4}\]

57. \[\frac{x}{x + 1} + \frac{3}{x^2 - 4x - 5}\]

Solve each equation. **(Lesson 8-4)**

58. \(\log_{10} (x^2 + 1) = 1\)

59. \(\log_b 64 = 3\)

60. \(\log_b 121 = 2\)

Simplify. **(Lesson 6-1)**

61. \(-5ab^2(-3a^2b + 6a^3b - 3a^4b^4)\)

62. \(2xy(3xy^3 - 4xy + 2y^4)\)

63. \((4x^2 - 3y^2 + 5xy) - (8xy + 3y^2)\)

64. \((10x^2 - 3xy + 4y^2) - (3x^2 + 5xy)\)

### Skills Review

Write an equation of the line passing through each pair of points. **(Lesson 2-4)**

65. \((-2, 5)\) and \((3, 1)\)

66. \((7, 1)\) and \((7, 8)\)

67. \((-3, 5)\) and \((2, 2)\)
Find the midpoint of the line segment with endpoints at the given coordinates. (Lesson 10-1)

1. $\left(7, 4\right), \left(-1, -5\right)$  
2. $\left(-2, -9\right), \left(-6, 0\right)$

Find the distance between each pair of points with the given coordinates. (Lesson 10-1)

3. $\left(0, 6\right), \left(-2, 5\right)$  
4. $\left(10, 1\right), \left(0, -4\right)$

5. **Hiking** Carla and Lance left their campsite and hiked 6 miles directly north and then turned and hiked 7 miles east to view a waterfall. (Lesson 10-1)
   a. How far is the waterfall from their campsite?
   b. Let the campsite be located at the origin on a coordinate grid. At the waterfall they decide to head directly back to the campsite. If they stop halfway between the waterfall and the campsite for lunch, at what coordinates will they stop for lunch?

Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola. (Lesson 10-2)

6. $y = 3x^2 - 12x + 21$  
7. $x - 2y^2 = 4y + 6$
8. $y = \frac{1}{2}x^2 + 12x - 8$  
9. $x = 3y^2 + 5y - 9$

10. **Bridges** Write an equation of a parabola to model the shape of the suspension cable of the bridge shown. Assume that the origin is at the lowest point of the cables. (Lesson 10-2)

![Bridge Diagram](image)

11. $y = x^2 + 6x + 5$
12. $x = -2y^2 + 4y + 1$
13. Find the center and radius of the circle with equation $(x - 1)^2 + y^2 = 9$. Then graph the circle. (Lesson 10-3)
14. Write an equation for a circle if the endpoints of a diameter are at $(8, 31)$ and $(32, 49)$. (Lesson 10-3)

15. Write an equation for a circle if the endpoints of a diameter are at $(0, 6)$, $(3, 4)$. (Lesson 10-3)
16. **Multiple Choice** What is the radius of the circle with equation $x^2 + 2x + y^2 + 14y + 34 = 0$? (Lesson 10-3)
   A. 2  
   B. 4  
   C. 8  
   D. 16

Identify the coordinates of the vertex and focus, the equation of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum. (Lesson 10-2)

17. $(x + 4)^2 + \frac{(y - 2)^2}{9} = 1$
18. $(x - 1)^2 + \frac{(y + 2)^2}{4} = 1$

Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with the given equation. Then graph the ellipse. (Lesson 10-4)

19. $4y^2 + 9x^2 + 14y - 90x + 205 = 0$

20. **Multiple Choice** Which equation represents an ellipse with endpoints of the major axis at $(-4, 10)$ and $(-4, -6)$ and foci at about $(-4, 7.3)$ and $(-4, -3.3)$? (Lesson 10-4)
   F. $\frac{(x - 2)^2}{36} + \frac{(y + 4)^2}{64} = 1$
   G. $\frac{(x + 4)^2}{64} + \frac{(y - 2)^2}{36} = 1$
   H. $\frac{(y - 2)^2}{64} + \frac{(x + 4)^2}{36} = 1$
   J. $\frac{(x - 2)^2}{64} + \frac{(y + 4)^2}{36} = 1$
**Equations of Hyperbolas**

Similar to an ellipse, a **hyperbola** is the set of all points in a plane such that the absolute value of the differences of the distances from the foci is constant.

Every hyperbola has two axes of symmetry, the **transverse axis** and the **conjugate axis**. The axes are perpendicular at the center of the hyperbola.

The **foci** of a hyperbola always lie on the transverse axis. The **vertices** are the endpoints of the transverse axis. The **co-vertices** are the endpoints of the conjugate axis.

As a hyperbola recedes from the center, both halves approach asymptotes.

**Key Concept: Equations of Hyperbolas Centered at the Origin**

<table>
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<tr>
<th>Standard Form</th>
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</tr>
<tr>
<td>Length of Transverse Axis</td>
<td>2a units</td>
<td>2a units</td>
</tr>
<tr>
<td>Length of Conjugate Axis</td>
<td>2b units</td>
<td>2b units</td>
</tr>
<tr>
<td>Equations of Asymptotes</td>
<td>( y = \pm \frac{b}{a}x )</td>
<td>( y = \pm \frac{a}{b}x )</td>
</tr>
</tbody>
</table>

As with ellipses, there are several important relationships among the parts of hyperbolas.

- There are two axes of symmetry.
- The values of \( a, b, \) and \( c \) are related by the equation \( c^2 = a^2 + b^2 \).
Example 1  Write an Equation Given Vertices and Foci

Write an equation for the hyperbola shown in the graph.

Step 1  Find the center.

The vertices are equidistant from the center.
The center is at (0, 0).

Step 2  Find the values of \( a \), \( b \), and \( c \).

The value of \( a \) is the distance between a vertex and the center, or 4 units.
The value of \( c \) is the distance between a focus and the center, or 5 units.

\[
5^2 = a^2 + b^2 \quad c = 5 \text{ and } a = 3
\]

\[
9 = b^2 \quad \text{Subtract } 4^2 \text{ from each side.}
\]

Step 3  Write the equation.

The transverse axis is horizontal, so the equation is

\[
\frac{x^2}{16} - \frac{y^2}{9} = 1.
\]

Guided Practice

1. Write an equation for a hyperbola with vertices at (6, 0) and (−6, 0) and foci at (8, 0) and (−8, 0).

Hyperbolas can also be determined using the equations of their asymptotes.

Example 2  Write an Equation Given Asymptotes

The asymptotes for a vertical hyperbola are \( y = \frac{5}{3}x \) and \( y = -\frac{5}{3}x \) and the vertices are at (0, 5) and (0, −5). Write the equation for the hyperbola.

Step 1  Find the center.

The vertices are equidistant from the center.
The center of the hyperbola is at (0, 0).

Step 2  Find the values of \( a \) and \( b \).

The hyperbola is vertical, so \( a = 5 \).
From the asymptotes, \( b = 3 \).
The value of \( c \) is not needed.

Step 3  Write the equation.

The equation for the hyperbola is \( \frac{y^2}{25} - \frac{x^2}{9} = 1 \).

Guided Practice

2. The asymptotes for a horizontal hyperbola are \( y = \frac{2}{9}x \) and \( y = -\frac{2}{9}x \).
The vertices are (9, 0) and (−9, 0). Write an equation for the hyperbola.
Hyperbolas can be translated in the same manner as the other conic sections.

### Key Concept: Equations of Hyperbolas Centered at \((h, k)\)

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>(\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1)</th>
<th>(\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orientation</td>
<td>horizontal</td>
<td>vertical</td>
</tr>
<tr>
<td>Foci</td>
<td>((h \pm c, k))</td>
<td>((h, k \pm c))</td>
</tr>
<tr>
<td>Vertices</td>
<td>((h \pm a, k))</td>
<td>((h, k \pm a))</td>
</tr>
<tr>
<td>Co-vertices</td>
<td>((h, k \pm b))</td>
<td>((h \pm b, k))</td>
</tr>
<tr>
<td>Equations of Asymptotes</td>
<td>(y - k = \pm \frac{b}{a}(x - h))</td>
<td>(y - k = \pm \frac{a}{b}(x - h))</td>
</tr>
</tbody>
</table>

### Example 3: Graph a Hyperbola

Graph \(\frac{(x-3)^2}{4} - \frac{(y+2)^2}{16} = 1\). Identify the vertices, foci, and asymptotes.

**Step 1** Find the center. The center is at \((3, -2)\).

**Step 2** Find \(a, b,\) and \(c\). From the equation, \(a^2 = 4\) and \(b^2 = 16\), so \(a = 2\) and \(b = 4\).

- \(c^2 = a^2 + b^2\) \hspace{1cm} \text{Equation relating } a, b, \text{ and } c \text{ for a hyperbola}
- \(c^2 = 2^2 + 4^2\)
- \(c^2 = 20\) \hspace{1cm} \text{Simplify}
- \(c = \sqrt{20} \text{ or about } 4.47\) \hspace{1cm} \text{Take the square root of each side}

**Step 3** Identify the vertices and foci. The hyperbola is horizontal and the vertices are 2 units from the center, so the vertices are at \((1, -2)\) and \((5, -2)\).

The foci are about 4.47 units from the center.

The foci are at \((-1.47, -2)\) and \((7.47, -2)\).

**Step 4** Identify the asymptotes.

- \(y - k = \pm \frac{b}{a}(x - h)\) \hspace{1cm} \text{Equation for asymptotes of a horizontal hyperbola}
- \(y - (-2) = \pm \frac{4}{2}(x - 3)\) \hspace{1cm} \(a = 2, b = 4, h = 3,\) and \(k = -2\)

The equations for the asymptotes are \(y = 2x - 8\) and \(y = -2x + 4\).

**Step 5** Graph the hyperbola. The hyperbola is symmetric about the transverse and conjugate axes. Use this symmetry to plot additional points for the hyperbola.

Use the asymptotes as a guide to draw the hyperbola that passes through the vertices and the other points.

### Guided Practice

3. Graph \(\frac{(y-4)^2}{9} - \frac{(x+3)^2}{25} = 1\). Identify the vertices, foci, and asymptotes.
In the equation for any hyperbola, the value of $2a$ represents the constant difference. This is the absolute value of the difference between the distances from any point on the hyperbola to the foci of the hyperbola.

Any point on the hyperbola at the right will have the same constant difference, $|y - x|$ or $2a$.

Real-World Example 4 Write an Equation of a Hyperbola

**SPACE** Earth and the Sun are 146 million kilometers apart. A comet follows a path that is one branch of a hyperbola. Suppose the comet is 30 million miles farther from the Sun than from Earth. Determine the equation of the hyperbola centered at the origin for the path of the comet.

**Understand** We need to determine the equation for the hyperbola.

**Plan** Find the center and the values of $a$ and $b$. Once we have this information, we can determine the equation.

**Solve** The foci are Earth and the Sun, with the origin between them.

The value of $c$ is $146 \div 2$ or 73.

The difference of the distances from the comet to each body is 30. Therefore, $a$ is $30 \div 2$ or 15 million miles.

$$c^2 = a^2 + b^2$$  
Equation relating $a$, $b$, and $c$ for a hyperbola

$$73^2 = 15^2 + b^2$$  
$a = 15$ and $c = 73$

$$5104 = b^2$$  
Simplify.

The equation of the hyperbola is

$$\frac{x^2}{225} - \frac{y^2}{5104} = 1.$$  

Since the comet is farther from the Sun, it is located on the branch of the hyperbola near Earth.

**Check** $(21, 70)$ is a point that satisfies the equation.

The distance between this point and the Sun $(-73, 0)$ is

$$\sqrt{(21 - (-73))^2 + (70 - 0)^2}$$ or 117.2 million kilometers.

The distance between this point and Earth $(73, 0)$ is

$$\sqrt{(21 - 73)^2 + (70 - 0)^2}$$ or 87.2 million kilometers.

The difference between these distances is 30. ✓

Guided Practice

4. SEARCH AND RESCUE Two receiving stations that are 150 miles apart receive a signal from a downed airplane. They determine that the airplane is 80 miles farther from station $A$ than from station $B$. Determine the equation of the hyperbola centered at the origin on which the plane is located.
Check Your Understanding

Examples 1–2 Write an equation for each hyperbola.

1. \( \frac{x^2}{4} - \frac{y^2}{4} = 1 \)
2. \( \frac{y^2}{9} - \frac{x^2}{16} = 1 \)

Example 3 Graph each hyperbola. Identify the vertices, foci, and asymptotes.

5. \( \frac{x^2}{64} - \frac{y^2}{49} = 1 \)
6. \( \frac{y^2}{36} - \frac{x^2}{60} = 1 \)
7. \( 9y^2 + 18y - 16x^2 + 64x - 199 = 0 \)
8. \( 4x^2 + 24x - y^2 + 4y - 4 = 0 \)

Example 4 9. NAVIGATION A ship determines that the difference of its distances from two stations is 60 nautical miles. Write an equation for a hyperbola on which the ship lies if the stations are at \((-80, 0)\) and \((80, 0)\).

Practice and Problem Solving

Examples 1–2 Write an equation for each hyperbola.

10. \( \frac{x^2}{4} - \frac{y^2}{4} = 1 \)
11. \( \frac{y^2}{9} - \frac{x^2}{16} = 1 \)
12. \( y = \frac{2}{3}x - \frac{7}{3} \)
13. \( y = -\frac{2}{3}x - \frac{23}{3} \)
Example 3
Graph each hyperbola. Identify the vertices, foci, and asymptotes.

14. \( \frac{x^2}{36} - \frac{y^2}{4} = 1 \)  
15. \( \frac{y^2}{9} - \frac{x^2}{49} = 1 \)  
16. \( \frac{y^2}{36} - \frac{x^2}{25} = 1 \)  
17. \( \frac{x^2}{16} - \frac{y^2}{16} = 1 \)  
18. \( \frac{(x - 3)^2}{16} - \frac{(y + 1)^2}{4} = 1 \)  
19. \( \frac{(y + 5)^2}{16} - \frac{(x + 2)^2}{36} = 1 \)  
20. \( 9y^2 - 4x^2 - 54y + 32x - 19 = 0 \)  
21. \( 16x^2 - 9y^2 + 128x + 36y + 76 = 0 \)  
22. \( 25x^2 - 4y^2 - 100x + 48y - 144 = 0 \)  
23. \( 81y^2 - 16x^2 - 810y + 96x + 585 = 0 \)

Example 4
24. NAVIGATION A ship determines that the difference of its distances from two stations is 80 nautical miles. Write an equation for a hyperbola on which the ship lies if the stations are at \((-100, 0)\) and \((100, 0)\).

Determine whether the following equations represent ellipses or hyperbolas.

25. \( 4x^2 = 5y^2 + 6 \)  
26. \( 8x^2 - 2x = 8y - 3y^2 \)  
27. \( -5x^2 + 4x = 6y + 3y^2 \)  
28. \( 7y - 2x^2 = 6x - 2y^2 \)  
29. \( 6x - 7x^2 - 5y^2 = 2y \)  
30. \( 4x + 6y + 2x^2 = -3y^2 \)

31. SPACE Refer to the application at the beginning of the lesson. With the Sun as a focus and the center at the origin, a certain comet’s path follows a branch of a hyperbola. If two of the coordinates of the path are \((10, 0)\) and \((30, 100)\) where the units are in millions of miles, determine the equation of the path.

32. COOLING Natural draft cooling towers are shaped like hyperbolas for more efficient cooling of power plants. The hyperbola in the tower at the right can be modeled by \( \frac{x^2}{16} - \frac{y^2}{225} = 1 \), where the units are in meters. Find the width of the tower at the top and at its narrowest point in the middle.

33. MULTIPLE REPRESENTATIONS Consider \( xy = 16 \).
   a. Tabular Make a table of values for the equation for \(-12 \leq x \leq 12\).
   b. Graphical Graph the hyperbola represented by the equation.
   c. Logical Determine and graph the asymptotes for the hyperbola.
   d. Analytical What special property do you notice about the asymptotes? Hyperbolas that represent this property are called rectangular hyperbolas.
   e. Analytical Without any calculations, what do you think will be the coordinates of the vertices for \( xy = 25? \) for \( xy = 36? \)

34. SEARCH AND RESCUE Two receiving stations that are 250 miles apart receive a signal from a downed airplane. They determine that the airplane is 70 miles farther from station B than from station A. Determine the equation of the horizontal hyperbola centered at the origin on which the plane is located.

35. WEATHER Luisa and Karl live exactly 4000 feet apart. While on the phone at their homes, Luisa hears thunder out of her window and Karl hears it 3 seconds later out of his. If sound travels 1100 feet per second, determine the equation for the horizontal hyperbola on which the lightning is located.
36. **ARCHITECTURE** Large pillars with cross sections in the shape of hyperbolas were popular in ancient Greece. The curves can be modeled by the equation \( \frac{x^2}{0.16} - \frac{y^2}{4} = 1 \), where the units are in feet. If the pillars are 9 feet tall, find the width of the top of each pillar and the width of each pillar at the narrowest point in the middle. Round to the nearest hundredth of a foot.

Write an equation for the hyperbola that satisfies each set of conditions.

37. vertices \((-8, 0)\) and \((8, 0)\), conjugate axis of length 20 units

38. vertices \((0, -6)\) and \((0, 6)\), conjugate axis of length 24 units

39. vertices \((6, -2)\) and \((-2, -2)\), foci \((10, -2)\) and \((-6, -2)\)

40. vertices \((-3, 4)\) and \((-3, -8)\), foci \((-3, 9)\) and \((-3, -13)\)

41. centered at the origin with a horizontal transverse axis of length 10 units and a conjugate axis of length 4 units

42. centered at the origin with a vertical transverse axis of length 16 units and a conjugate axis of length 12 units

43. **TRIANGULATION** While looking for their lost dog in the woods, Lae, Meg, and Cesar hear a bark. Meg hears it 2 seconds after Lae and Cesar hears it 3 seconds after Lae. With Lae at the origin, determine the exact location of their dog if sound travels 1100 feet per second.

### H.O.T. Problems

**Use Higher-Order Thinking Skills**

44. **ERROR ANALYSIS** Simon and Gabriel are graphing \( \frac{y^2}{25} - \frac{x^2}{4} = 1 \). Is either of them correct? Explain your reasoning.

45. **CHALLENGE** The origin lies on a horizontal hyperbola. The asymptotes for the hyperbola are \( y = -x + 1 \) and \( y = x - 5 \). Find the equation for the hyperbola.

46. **REASONING** What happens to the location of the foci of a hyperbola as the value of \( a \) becomes increasingly smaller than the value of \( b \)? Explain your reasoning.

47. **REASONING** Consider \( \frac{y^2}{36} - \frac{x^2}{16} = 1 \). Describe the change in the shape of the hyperbola and the locations of the vertices and foci if 36 is changed to 9. Explain why this happens.

48. **OPEN ENDED** Write an equation for a hyperbola with a focus at the origin.

49. **WRITING IN MATH** Compare and contrast the characteristics of the equations and graphs of ellipses and hyperbolas.
50. You have 6 more dimes than quarters. You have a total of $5.15. How many dimes do you have?
   A 13       C 19
   B 16       D 25

51. How tall is a tree that is 15 feet shorter than a pole three times as tall as the tree?
   F 24.5 ft
   G 22.5 ft
   H 21.5 ft
   J 7.5 ft

52. SHORT RESPONSE A rectangle is 8 feet long and 6 feet wide. If each dimension is increased by the same number of feet, the area of the new rectangle formed is 32 square feet more than the area of the original rectangle. By how many feet was each dimension increased?

53. SAT/ACT When the equation \( y = 4x^2 - 5 \) is graphed in the coordinate plane, the graph is which of the following?
   A line       D hyperbola
   B circle     E parabola
   C ellipse

Spiral Review

Write an equation for an ellipse that satisfies each set of conditions. (Lesson 10-4)

54. endpoints of major axis at (2, 2) and (2, -10), endpoints of minor axis at (0, -4) and (4, -4)

55. endpoints of major axis at (0, 10) and (0, -10), foci at (0, 8) and (0, -8)

Find the center and radius of the circle with the given equation. Then graph the circle. (Lesson 10-3)

56. \((x - 3)^2 + y^2 = 16\)

57. \(x^2 + y^2 - 6y - 16 = 0\)

58. \(x^2 + y^2 + 9x - 8y + 4 = 0\)

59. BASKETBALL Zonta plays basketball for Centerville High School. So far this season, she has made 6 out of 10 free-throws. She is determined to improve her free-throw percentage. If she can make \(x\) consecutive free throws, her free-throw percentage can be determined using \(P(x) = \frac{6 + x}{10 + x}\). (Lesson 9-4)
   a. Graph the function.
   b. What part of the graph is meaningful in the context of the problem?
   c. Describe the meaning of the \(y\)-intercept.
   d. What is the equation of the horizontal asymptote? Explain its meaning with respect to Zonta’s shooting percentage.

Solve each equation. (Lesson 8-2)

60. \(\left(\frac{1}{2}\right)^{x - 3} = 343\)

61. \(10^{x - 1} = 100^{2x - 3}\)

62. \(36^{2p} = 216^{p - 1}\)

Graph each inequality. (Lesson 7-3)

63. \(y \geq \sqrt{5x - 8}\)

64. \(y \geq \sqrt{x - 3} + 4\)

65. \(y < \sqrt{6x - 2} + 1\)

Skills Review

66. Write an equation for a parabola with vertex at the origin that passes through (2, -8). (Lesson 5-7)

67. Write an equation for a parabola with vertex at (-3, -4) that opens up and has \(y\)-intercept 8. (Lesson 5-7)
1 Conics in Standard Form The equation for any conic section can be written in the form \(Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0\), where \(A, B,\) and \(C\) are not all zero. This general form can be converted to the standard forms below by completing the square.

<table>
<thead>
<tr>
<th>Conic Section</th>
<th>Standard Form of Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle</td>
<td>((x - h)^2 + (y - k)^2 = r^2)</td>
</tr>
<tr>
<td>Parabola</td>
<td>(y = a(x - h)^2 + k) or (x = a(y - k)^2 + h)</td>
</tr>
<tr>
<td>Ellipse</td>
<td>(\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1) or (\frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1)</td>
</tr>
<tr>
<td>Hyperbola</td>
<td>(\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1) or (\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1)</td>
</tr>
</tbody>
</table>

Example 1 Rewrite an Equation of a Conic Section

Write \(16x^2 - 25y^2 - 128x - 144 = 0\) in standard form. State whether the graph of the equation is a parabola, circle, ellipse, or hyperbola. Then graph the equation.

\[
16x^2 - 25y^2 - 128x - 144 = 0
\]

Original equation

\[
16(x^2 - 8x + 16) - 25y^2 = 144 + 16(16)
\]

Isolate terms.

\[
16(x - 4)^2 - 25y^2 = 400
\]

Complete the square.

\[
\frac{(x - 4)^2}{25} - \frac{y^2}{16} = 1
\]

Perfect square

Divide each side by 400.

The graph is a hyperbola with its center at \((4, 0)\).

Guided Practice

1. Write \(4x^2 + y^2 - 16x + 8y - 4 = 0\) in standard form. State whether the graph of the equation is a parabola, circle, ellipse, or hyperbola. Then graph the equation.
**Identify Conic Sections** You can determine the type of conic without having to write $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ in standard form. When there is an $xy$-term ($B \neq 0$), you can use the discriminant to identify the conic. $B^2 - 4AC$ is the discriminant of $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

**Concept Summary** Classify Conics with the Discriminant

<table>
<thead>
<tr>
<th>Discriminant</th>
<th>Conic Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^2 - 4AC &lt; 0$; $B = 0$ and $A = C$</td>
<td>circle</td>
</tr>
<tr>
<td>$B^2 - 4AC &lt; 0$; either $B \neq 0$ or $A \neq C$</td>
<td>ellipse</td>
</tr>
<tr>
<td>$B^2 - 4AC = 0$</td>
<td>parabola</td>
</tr>
<tr>
<td>$B^2 - 4AC &gt; 0$</td>
<td>hyperbola</td>
</tr>
</tbody>
</table>

When $B = 0$, the conic will be either vertical or horizontal. When $B \neq 0$, the conic will be neither vertical nor horizontal.

**Example 2** Analyze an Equation of a Conic Section

Without writing in standard form, state whether the graph of each equation is a parabola, circle, ellipse, or hyperbola.

**a.** $y^2 + 4x^2 - 3xy + 4x - 5y - 8 = 0$
   $A = 4$, $B = -3$, and $C = 1$
   The discriminant is $(-3)^2 - 4(4)(1) = -7$.
   Because the discriminant is less than 0 and $B \neq 0$, the conic is an ellipse.

**b.** $3x^2 - 6x + 4y - 5y^2 + 2xy - 4 = 0$
   $A = 3$, $B = 2$, and $C = -5$
   The discriminant is $2^2 - 4(3)(-5) = 64$.
   Because the discriminant is greater than 0, the conic is a hyperbola.

**c.** $4y^2 - 8x + 6y - 14 = 0$
   $A = 0$, $B = 0$, and $C = 4$
   The discriminant is $0^2 - 4(0)(4) = 0$.
   Because the discriminant equals 0, the conic is a parabola.

**Guided Practice**

2A. $8y^2 - 6x^2 + 4xy - 6x + 2y - 4 = 0$
2B. $3xy + 4x^2 - 2y + 9x - 3 = 0$
2C. $3x^2 + 16x - 12y + 2y^2 - 6 = 0$
Example 1  Write each equation in standard form. State whether the graph of the equation is a \textit{parabola, circle, ellipse, or hyperbola}. Then graph the equation.

1. \(x^2 + 4y^2 - 6x + 16y - 11 = 0\)
2. \(x^2 + y^2 + 12x - 8y + 36 = 0\)
3. \(9y^2 - 16x^2 - 18y - 64x - 199 = 0\)
4. \(6y^2 - 24y + 28 - x = 0\)

Example 2  Without writing in standard form, state whether the graph of each equation is a \textit{parabola, circle, ellipse, or hyperbola}.

5. \(4x^2 + 6y^2 - 3x - 2y = 12\)
6. \(5y^2 = 2x + 6y - 8 + 3x^2\)
7. \(8x^2 + 8y^2 + 16x + 24 = 0\)
8. \(4x^2 - 6y = 8x + 2\)
9. \(4x^2 - 3y^2 + 8xy - 12 = 2x + 4y\)
10. \(5xy - 3x^2 + 6y^2 + 12y = 18\)
11. \(8x^2 + 12xy + 16y^2 + 4y - 3x = 12\)
12. \(16xy + 8x^2 + 8y^2 - 18x + 8y = 13\)

13. \textbf{AVIATION}  A military jet performs for an air show. The path of the plane during one maneuver can be modeled by a conic section with equation \(24x^2 + 1000y - 31,680x - 45,600 = 0\), where distances are represented in feet.

   a. Identify the shape of the curved path of the jet. Write the equation in standard form.

   b. If the jet begins its path upward, or ascent, at \(x = 0\), what is the horizontal distance traveled by the jet from the beginning of the ascent to the end of the descent?

   c. What is the maximum height of the jet?

---

**Practice and Problem Solving**

Example 1  Write each equation in standard form. State whether the graph of the equation is a \textit{parabola, circle, ellipse, or hyperbola}. Then graph the equation.

14. \(3x^2 - 2y^2 + 18x + 8y - 35 = 0\)
15. \(3x^2 + 24x + 4y^2 - 40y + 52 = 0\)
16. \(x^2 + y^2 = 16 + 6y\)
17. \(32x + 28 = y - 8x^2\)
18. \(7x^2 - 8y = 84x - 2y^2 - 176\)
19. \(x^2 + 8y = 11 + 6x - y^2\)
20. \(4y^2 = 24y - x - 31\)
21. \(112y + 64x = 488 + 7y^2 - 8x^2\)
22. \(28x^2 + 9y^2 - 188 = 56x - 36y\)
23. \(25x^2 + 384y - 64y^2 + 200x = 1776\)

Example 2  Without writing in standard form, state whether the graph of each equation is a \textit{parabola, circle, ellipse, or hyperbola}.

24. \(4x^2 - 5y = 9x - 12\)
25. \(4x^2 - 12x = 18y - 4y^2\)
26. \(9x^2 + 12y = 9y^2 + 18y - 16\)
27. \(18x^2 - 16y = 12x - 4y^2 + 19\)
28. \(12y^2 - 4xy + 9x^2 = 18x - 124\)
29. \(5xy + 12x^2 - 16x = 5y + 3y^2 + 18\)
30. \(19x^2 + 14y = 6x - 19y^2 - 88\)
31. \(8x^2 + 20xy + 18 = 4y^2 - 12 + 9x\)
32. \(5x - 12xy + 6x^2 = 8y^2 - 24y - 9\)
33. \(18x - 24y + 324xy = 27x^2 + 3y^2 - 5\)

34. \textbf{LIGHT}  A lamp standing near a wall throws an arc of light in the shape of a conic section. Suppose the edge of the light can be represented by the equation \(3y^2 - 2y - 4x^2 + 2x - 8 = 0\). Identify the shape of the edge of the light and graph the equation.
Match each graph with its corresponding equation.

35. \[ y = x^2 + y^2 - 8x - 4y = -4 \]  
36. \[ 9x^2 - 16y^2 - 72x + 64y = 64 \]  
37. \[ 9x^2 + 16y^2 = 72x + 64y - 64 \]

For Exercises 38–41, match each situation with an equation that could be used to represent it.

a. \[ 47.25x^2 - 9y^2 + 18y + 33.525 = 0 \]  
b. \[ 25x^2 + 100y^2 - 1900x - 2200y + 45,700 = 0 \]  
c. \[ 16x^2 - 90x + y - 0.25 = 0 \]  
d. \[ x^2 + y^2 - 18x - 30y - 14,094 = 0 \]

38. **COMPUTERS** the boundary of a wireless network with a range of 120 feet
39. **FITNESS** the oval path of your foot on an exercise machine
40. **COMMUNICATIONS** the position of a cell phone between two cell towers
41. **SPORTS** the height of a football above the ground after being kicked

42. **ENGINEERING** The shape of the cables in a suspension bridge is approximately parabolic. If the towers for a planned bridge are 1000 meters apart and the lowest point of the suspension cables is 200 meters below the top of the towers, write the equation in standard form with the origin at the vertex.

43. **MULTIPLE REPRESENTATIONS** Consider an ellipse with center \((3, -2)\), vertex \(M(-1, -2)\), and co-vertex \(N(3, -4)\).
   a. **Analytical** Determine the standard form of the equation of the ellipse.
   b. **Algebraic** Convert part a to \(Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0\) form.
   c. **Graphical** Graph the ellipse.
   d. **Analytical** If the ellipse is rotated such that \(M\) is moved to \((3, -6)\), determine the location of \(N\) and the angle of rotation.

**H.O.T. Problems** Use Higher-Order Thinking Skills

44. **CHALLENGE** When a plane passes through the vertex of a cone, a degenerate conic is formed.
   a. Determine the type of conic represented by \(4x^2 + 8y^2 = 0\).
   b. Graph the conic.
   c. Describe the difference between this degenerate conic and a standard conic.

45. **REASONING** Determine whether the following statement is sometimes, always, or never true. Explain your reasoning.
   
   \(\text{When a conic is vertical and } A = C, \text{ it is a circle.}\)

46. **OPEN ENDED** Write an equation of the form \(Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0\), where \(A = 9C\), that represents a parabola.

47. **WRITING IN MATH** Compare and contrast the graphs of the four types of conics and their corresponding equations.
48. SAT/ACT  A class of 25 students took a science test. Ten students had a mean score of 80. The other students had an average score of 60. What is the average score of the whole class?
A 66  D 72
B 68  E 78
C 70

49. Six times a number minus 11 is 43. What is the number?
F 12
G 11
H 10
J 9

50. EXTENDED RESPONSE  The amount of water remaining in a storage tank as it is drained can be represented by the equation \( L = -4t^2 - 10t + 130 \), where \( L \) represents the number of liters of water remaining and \( t \) represents the number of minutes since the drain was opened. How many liters of water were in the tank initially? Determine to the nearest tenth of a minute how long it will take for the tank to drain completely.

51. Ruben has a square piece of paper with sides 4 inches long. He rolled up the paper to form a cylinder. What is the volume of the cylinder?
A \( \frac{4}{\pi} \)  C \( 4\pi \)
B \( \frac{16}{\pi} \)  D \( 16\pi \)

52. ASTRONOMY  Suppose a comet’s path can be modeled by a branch of the hyperbola with equation \( \frac{y^2}{225} - \frac{x^2}{400} = 1 \). Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola. Then graph the hyperbola. (Lesson 10-5)

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse. (Lesson 10-4)

53. \( \frac{y^2}{18} + \frac{x^2}{9} = 1 \)

54. \( 4x^2 + 8y^2 = 32 \)

55. \( x^2 + 25y^2 - 8x + 100y + 91 = 0 \)

Graph each function. (Lesson 9-3)

56. \( f(x) = \frac{3}{x} \)

57. \( f(x) = \frac{-2}{x + 5} \)

58. \( f(x) = \frac{6}{x - 2} - 4 \)

59. SPACE  A radioisotope is used as a power source for a satellite. The power output \( P \) (in watts) is given by \( P = 50e^{-\frac{t}{360}} \), where \( t \) is the time in days. (Lesson 8-8)

a. Is the formula for power output an example of exponential growth or decay? Explain your reasoning.

b. Find the power available after 100 days.

c. Ten watts of power are required to operate the equipment in the satellite. How long can the satellite continue to operate?

Skills Review

Solve each system of equations. (Lesson 3-2)

60. \( 6g - 8h = 50 \)
\( 6h = 22 - 4g \)

61. \( 3u + 5v = 6 \)
\( 2u - 4v = -7 \)

62. \( 10m - 9n = 15 \)
\( 5m - 4n = 10 \)
OBJECTIVE Solve linear-nonlinear systems using a graphing calculator.

You can use a TI-83/84 Plus application to solve linear-nonlinear systems by using the Y= menu to graph each equation on the same set of axes.

Example Linear-Quadratic System

Solve the system of equations.
\[ 3y - 4x = -7 \]
\[ 4x^2 + 3y^2 = 91 \]

**Step 1** Solve each equation for \( y \).
\[ 3y = 4x - 7 \]
\[ y = \frac{4}{3}x - \frac{7}{3} \]
\[ 4x^2 + 3y^2 = 91 \]
\[ y = \pm \sqrt{\frac{91 - 4x^2}{3}} \]

**Step 2** Enter \[ y = \frac{4}{3}x - \frac{7}{3} \] as \( Y1 \), \[ y = \sqrt{\frac{91 - 4x^2}{3}} \] as \( Y2 \), and \[ y = -\sqrt{\frac{91 - 4x^2}{3}} \] as \( Y3 \).

Then graph the equations.

**KEYSTROKES**:
\[ Y = 4 \div 3 X,T,\theta,n \] \( \pm 7 \) \( \div 3 \) \( \text{ENTER} \) \( 2\text{nd} \) \( x^2 \) \( ( \) \( 91 - 4 \) \( \text{ENTER} \)

**Step 3** Find the intersection of \( y = \frac{4}{3}x - \frac{7}{3} \) with \( y = \sqrt{\frac{91 - 4x^2}{3}} \).

**KEYSTROKES**: Press \( 2\text{nd} \) \( \text{TRACE} \) \( \text{5} \) \( \text{ENTER} \) \( \text{ENTER} \) \( \text{ENTER} \). The two graphs intersect at \((4,3)\).

**Step 4** Find the intersection of \( y = \frac{4}{3}x - \frac{7}{3} \) with \( y = -\sqrt{\frac{91 - 4x^2}{3}} \).

**KEYSTROKES**: Press \( 2\text{nd} \) \( \text{TRACE} \) \( \text{5} \) \( \text{ENTER} \) \( \text{ENTER} \) \( \text{ENTER} \). Then use \( \text{\textdownarrow} \) to move the cursor to the second intersection point.

Press \( \text{ENTER} \). The two graphs intersect at \((-2,-5)\).

The solutions of the system are \((4,3)\) and \((-2,-5)\).

**Exercises**

Use a graphing calculator to solve each system of equations.

1. \[ x^2 + y^2 = 100 \]
   \[ x + y = 2 \]
2. \[ 2y - x = 11 \]
   \[ 5x^2 + 2y^2 = 407 \]
3. \[ 21x + 9y = -36 \]
   \[ 7x^2 + 9y^2 = 1152 \]
Solving Linear-Nonlinear Systems

**Then**
- You solved systems of linear equations. (Lessons 3-1 and 3-2)

**Now**
1. Solve systems of linear and nonlinear equations algebraically and graphically.
2. Solve systems of linear and nonlinear inequalities graphically.

**Why?**
- Ever wonder how law enforcement agencies can track a cell phone user’s location? A person using a cell phone can be located in respect to three cellular towers. The respective coordinates and distances each tower is from the caller are used to pinpoint the caller’s location. This is accomplished using a system of quadratic equations.

### Systems of Equations
When a system of equations consists of a linear and a nonlinear equation, the system may have zero, one, or two solutions. Some of the possible solutions are shown below.

You can solve linear-quadratic systems by using graphical or algebraic methods.

#### Example 1 Linear-Quadratic System

Solve the system of equations.

\[
\begin{align*}
9x^2 + 25y^2 &= 225 \quad (1) \\
10y + 6x &= 6 \quad (2)
\end{align*}
\]

**Step 1** Solve the linear equation for \(y\).

\[
10y + 6x = 6 \quad \text{Equation (2)} \quad \Rightarrow \quad y = -0.6x + 0.6 \quad \text{Solve for } y.
\]

**Step 2** Substitute into the quadratic equation and solve for \(x\).

\[
\begin{align*}
9x^2 + 25y^2 &= 225 \quad \text{Quadratic equation} \\
9x^2 + 25(-0.6x + 0.6)^2 &= 225 \quad \text{Substitute } -0.6x + 0.6 \text{ for } y. \\
9x^2 + 25(0.36x^2 - 0.72x + 0.36) &= 225 \quad \text{Simplify.} \\
9x^2 + 9x^2 - 18x + 9 &= 225 \quad \text{Distribute.} \\
18x^2 - 18x - 216 &= 0 \quad \text{Simplify.} \\
6x^2 - x - 12 &= 0 \quad \text{Divide each side by 18.} \\
(x - 4)(x + 3) &= 0 \quad \text{Factor.} \\
x &= 4 \text{ or } -3 \quad \text{Zero Product Property}
\end{align*}
\]

**Step 3** Substitute \(x\)-values into the linear equation and solve for \(y\).

\[
\begin{align*}
y &= -0.6x + 0.6 \quad \text{Equation (2)} \\
y &= -0.6(4) + 0.6 \quad \text{Substitute the } x\text{-values} \\
y &= -2.4 + 0.6 \\
y &= -1.8 \quad \text{Simplify.} \\
y &= -1.8(2) + 0.6 \\
y &= 2.4 \\
\end{align*}
\]

The solutions of the system are \((4, -1.8)\) and \((-3, 2.4)\).

**Guided Practice** Solve each system of equations.

1A. \(3y + x^2 - 4x - 17 = 0\)
\(3y - 10x + 38 = 0\)

1B. \(3(y - 4) - 2(x - 3) = -6\)
\(5x^2 + 2y^2 - 53 = 0\)
If a quadratic system contains two conic sections, the system may have anywhere from zero to four solutions. Some graphical representations are shown below.

You can use elimination to solve quadratic-quadratic systems.

### Example 2 Quadratic-Quadratic System

Solve the system of equations.

\[
\begin{align*}
  x^2 + y^2 &= 45  \\
  y^2 - x^2 &= 27
\end{align*}
\]

1. \(x^2 + y^2 = 45\) \(\text{Equation (1), Commutative Property}\)
2. \(y^2 - x^2 = 27\) \(\text{Equation (2)}\)

\[
\begin{align*}
  2y^2 &= 72  \\
  y^2 &= 36  \\
  y &= \pm 6
\end{align*}
\]

Substitute 6 and \(-6\) into one of the original equations and solve for \(x\).

\[
\begin{align*}
  x^2 + y^2 &= 45  \\
  x^2 + 6^2 &= 45  \\
  x^2 &= 9  \\
  x &= \pm 3  \\
  x^2 - y^2 &= -1  \\
  2x^2 - y^2 &= -1
\end{align*}
\]

The solutions are \((-3, -6), (-3, 6), (3, -6), \text{and} (3, 6)\).

### Guided Practice

2A. \(x^2 + y^2 = 8\)  
2B. \(3x^2 + 4y^2 = 48\)  
   \(x^2 + 3y = 10\)  
   \(2x^2 - y^2 = -1\)

2 Systems of Inequalities

Systems of quadratic inequalities can be solved by graphing.

### Example 3 Quadratic Inequalities

Solve the system of inequalities by graphing.

\[
\begin{align*}
  x^2 + y^2 &\leq 49  \\
  x^2 - 4y^2 &> 16
\end{align*}
\]

The intersection of the graphs, shaded green, represents the solution of the system.

**CHECK** (6, 0) is in the shaded area. Use this point to check your solution.

\[
\begin{align*}
  x^2 + y^2 &\leq 49  \\
  6^2 + 0^2 &\leq 49  \\
  36 &\leq 49  \checkmark
\end{align*}
\]

\[
\begin{align*}
  x^2 - 4y^2 &> 16  \\
  6^2 - 4(0)^2 &> 16  \\
  36 &> 16 \checkmark
\end{align*}
\]

### Guided Practice

Solve each system of inequalities by graphing.

3A. \(5x^2 + 2y^2 \leq 10\)  
3B. \(x^2 - y^2 \leq 8\)  
   \(y \geq x^2 - 2x + 1\)  
   \(x^2 + y^2 \geq 120\)
Systems involving absolute value can also be solved by graphing.

Example 4 Quadratics with Absolute Value

Solve the system of inequalities by graphing.

\[ y \geq |2x - 4| \]
\[ y \leq -x^2 + 4x + 2 \]

Graph the boundary equations. Then shade appropriately.

The intersection of the graphs, shaded green, represents the solution to the system.

CHECK (2, 4) is in the shaded area. Use the point to check your solution.

\[ y \geq |2x - 4| \quad y \leq -x^2 + 4x + 2 \]
\[ 4 \geq |2(2) - 4| \quad 4 \leq -(2)^2 + 4(2) + 2 \]
\[ 4 \geq 0 \quad 4 \leq 6 \]

Guided Practice

Solve each system of inequalities by graphing.

4A. \( y > | -0.5x + 2 | \)
\[ \frac{x^2 + y^2}{16} \leq 1 \]
4B. \( x^2 + y^2 \leq 49 \)
\[ y \geq | x^2 + 1 | \]

Check Your Understanding

Examples 1–2 Solve each system of equations.

1. \( 8y = -10x \)
   \( y^2 = 2x^2 - 7 \)
2. \( x^2 + y^2 = 68 \)
   \( 5y = -3x + 34 \)
3. \( y = 12x - 30 \)
   \( 4x^2 - 3y = 18 \)
4. \( 6y^2 - 27 = 3x \)
   \( 6y - x = 13 \)
5. \( x^2 + y^2 = 16 \)
   \( x^2 - y^2 = 20 \)
6. \( y^2 - 2x^2 = 8 \)
   \( 3y^2 + x^2 = 52 \)
7. \( x^2 + 2y = 7 \)
   \( y^2 - x^2 = 8 \)
8. \( 4y^2 - 3x^2 = 11 \)
   \( 3y^2 + 2x^2 = 21 \)

9. CELL PHONES Refer to the beginning of the lesson. A person using a cell phone can be located in respect to three cellular towers. In a coordinate system where one unit represents one mile, the location of the caller is determined to be 50 miles from the tower at the origin. The person is also 40 miles from a tower at (0, 30) and 13 miles from a tower at (35, 18). Where is the caller?

Examples 3–4 Solve each system of inequalities by graphing.

10. \( 6x^2 + 9(y - 2)^2 \leq 36 \)
    \( x^2 + (y + 3)^2 \leq 25 \)
11. \( 16x^2 + 4y^2 \leq 64 \)
    \( y \geq -x^2 + 2 \)
12. \( 4x^2 - 8y^2 \geq 32 \)
    \( y \geq 1.5x \) – 8
13. \( x^2 + 8y^2 < 32 \)
    \( y < -|x - 2| + 2 \)
Examples 1–2 Solve each system of equations.

14. $3x^2 - 2y^2 = -24$
   \[ 2y = -3x \]

15. $5x^2 + 4y^2 = 20$
   \[ 5y = 7x + 35 \]

16. $x^2 + 3x = -4y - 2$
   \[ y = -2x + 1 \]

17. $y = 2x$
   \[ 4x^2 - 2y^2 = -36 \]

18. $2y = x + 10$
   \[ y^2 - 4y = 5x + 10 \]

19. $9y = 8x - 19$
   \[ 8x + 11 = 2y^2 + 5y \]

20. $2y^2 + 5x^2 = 26$
   \[ 2x^2 - y^2 = 5 \]

21. $x^2 + y^2 = 16$
   \[ x^2 - 4x + y^2 = 12 \]

22. $x^2 + y^2 = 8$
   \[ 5y^2 = 3x^2 \]

23. $y^2 - x^2 + 3y = 26$
   \[ x^2 + 2y^2 = 34 \]

24. $x^2 - y^2 = 25$
   \[ x^2 + y^2 + 7 = 0 \]

25. $x^2 - 10x + 2y^2 = 47$
   \[ y^2 - 2x^2 = -14 \]

26. **FIREWORKS** Two fireworks are set off simultaneously but from different altitudes. The height $y$ in feet of one is represented by $y = -16t^2 + 120t + 10$, where $t$ is the time in seconds. The height of the other is represented by $y = -16t^2 + 60t + 310$.

   a. After how many seconds are the fireworks the same height?

   b. What is that height?

Examples 3–4 Solve each system of inequalities by graphing.

27. $x^2 + y^2 \geq 36$
   \[ x^2 + 9(y + 6)^2 \leq 36 \]

28. $-x > y^2$
   \[ 4x^2 + 14y^2 \leq 56 \]

29. $12x^2 - 4y^2 \geq 48$
   \[ 16(x - 4)^2 + 25y^2 < 400 \]

30. $8y^2 - 3x^2 \leq 24$
   \[ 2y > x^2 - 8x + 14 \]

31. $y > x^2 - 6x + 8$
   \[ x^2 + 2x - 6y + 8 \]

32. $x^2 + y^2 \geq 9$
   \[ 25x^2 + 64y^2 \leq 1600 \]

33. $16(x - 3)^2 + 4y^2 \leq 64$
   \[ y \leq -x^2 - 2 + 2 \]

34. $x^2 - 4x + y^2 + 6y \leq 23$
   \[ y > |x - 2| - 6 \]

35. $2y - 4 \geq |x + 4|$
   \[ 12 - 2y > x^2 + 12x + 36 \]

36. $18y^2 - 3x^2 \leq 54$
   \[ y \geq 2x^2 - 6 \]

37. $x^2 + y^2 < 16$
   \[ y \geq |x - 2| + 6 \]

38. $x^2 < y - 2$
   \[ y \leq |x + 8| - 7 \]

39. **SPACE** Two satellites are placed in orbit about Earth. The equations of the two orbits are $\frac{x^2}{(300)^2} + \frac{y^2}{(900)^2} = 1$ and $\frac{x^2}{(600)^2} + \frac{y^2}{(690)^2} = 1$, where distances are in kilometers and Earth is the center of each curve.

   a. Solve each equation for $y$.

   b. Use a graphing calculator to estimate the intersection points of the two orbits.

   c. Compare the orbits of the two satellites.

40. **PETS** Taci’s dog was missing one day. Fortunately, he was wearing an electronic monitoring device. If the dog is 10 units from the tree, 13 units from the tower, and 20 units from the house, determine the coordinates of his location.

41. **BASEBALL** In 1997, after Mark McGuire hit a home run, the claim was made that the ball would have traveled 538 feet if it had not landed in the stands. The path of the baseball can be modeled by $y = -0.0037x^2 + 1.77x - 1.72$ and the stands can be modeled by $y = \frac{3}{2}x - 128.6$. How far vertically and horizontally from home plate did the ball land in the stands?
42. **ADVERTISING** The corporate logo for an automobile manufacturer is shown at the right. Write a system of three equations to model this logo.

Write a system of equations that satisfies each condition.

43. a circle and an ellipse that intersect at one point
44. a parabola and an ellipse that intersect at two points
45. a hyperbola and a circle that do not intersect
46. an ellipse and a parabola that intersect at three points
47. an ellipse and a hyperbola that intersect at four points

48. **FINANCIAL LITERACY** Prices are often set on an equilibrium curve, where the supply of a certain product equals its corresponding demand by consumers. An economist represents the supply of a product with \( y = p^2 + 10p \) and the corresponding demand with \( y = -p^2 + 40p \), where \( p \) is the price. Determine the equilibrium price.

49. **PAINTBALL** The shape of a paintball field is modeled by \( x^2 + 4y^2 = 10,000 \) in yards where the center is at the origin. The teams are provided with short-range walkie-talkies with a maximum range of 80 yards. Are the teams capable of hearing each other anywhere on the field? Explain your reasoning graphically.

50. **MOVING** Lena is moving to new city and needs for the location of her new home to satisfy the following conditions.

- It must be less than 10 miles from the office where she will work.
- Because of the terrible smell of the local paper mill, it must be at least 15 miles away from the mill.

If the paper mill is located 9.5 miles east and 6 miles north of Lena’s office, write and graph a system of inequalities to represent the area(s) were she should look for a home.

### H.O.T. Problems Use Higher-Order Thinking Skills

51. **CHALLENGE** Find all values of \( k \) for which the following system of equations has two solutions.

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad x^2 + y^2 = k^2
\]

52. **REASONING** When the vertex of a parabola lies on an ellipse, how many solutions can the quadratic system represented by the two graphs have? Explain your reasoning using graphs.

53. **OPEN ENDED** Write a system of equations, one a hyperbola and the other an ellipse, for which a solution is \((-4, 8)\).

54. **WRITING IN MATH** Explain how sketching the graph of a quadratic system can help you solve it.
55. SHORT RESPONSE  Solve.
\[ 4x - 3y = 0 \]
\[ x^2 + y^2 = 25 \]

56. You have 16 stamps. Some are postcard stamps that cost $0.23, and the rest cost $0.41. If you spent a total of $5.30 on the stamps, how many postcard stamps do you have?
A 7  B 8  C 9  D 10

57. Ms. Talbot received a promotion and a 7.2% raise. Her new salary is $53,600 a year. What was her salary before the raise?
F $50,000  G $53,600  H $55,000  J $57,500

58. SAT/ACT  When a number is multiplied by \( \frac{2}{3} \), the result is 188. Find the number.
A 292  B 282  C 272  D 262  E \( \frac{125}{3} \)

59. SPORTS  the flight of a baseball
60. PHOTOGRAPHY  the oval opening in a picture frame
61. GEOGRAPHY  the set of all points 20 miles from a landmark

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola. (Lesson 10-5)
62. \( \frac{y^2}{16} - \frac{x^2}{25} = 1 \)
63. \( \frac{(y - 3)^2}{25} - \frac{(x - 2)^2}{16} = 1 \)
64. \( 6y^2 = 2x^2 + 12 \)

Simplify each expression. (Lesson 9-1)
65. \( \frac{12p^2 + 6p - 6}{4(p + 1)^2} \div \frac{6p - 3}{2p + 10} \)
66. \( \frac{x^2 + 6x + 9}{x^2 + 7x + 6} \div \frac{4x + 12}{3x + 3} \)
67. \( \frac{r^2 + 2r - 8}{r^2 + 4r + 3} \div \frac{r - 2}{3r + 3} \)

Graph each function. State the domain and range. (Lesson 8-1)
68. \( f(x) = -\left(\frac{1}{5}\right)^x \)
69. \( y = -2.5(5)^x \)
70. \( f(x) = 2\left(\frac{1}{3}\right)^x \)

Skills Review

Solve each equation or formula for the specified variable. (Lesson 1-3)
71. \( d = rt \), for \( r \)
72. \( x = \frac{-b}{2a} \), for \( a \)
73. \( V = \frac{1}{3}\pi r^2 h \), for \( h \)
74. \( A = \frac{1}{2}h(a + b) \), for \( b \)
**Key Concepts**

**Midpoint and Distance Formulas** (Lesson 10-1)
- \[ M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]
- \[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

**Circles** (Lesson 10-2)
- The equation of a circle with center \((h, k)\) and radius \(r\) can be written in the form \((x - h)^2 + (y - k)^2 = r^2\).

**Parabolas** (Lesson 10-3)
- Standard Form: \(y = a(x - h)^2 + k\)
  \[ x = a(y - k)^2 + h \]

**Ellipses** (Lesson 10-4)
- Standard Form: horizontal \[ \frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1 \]
  vertical \[ \frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1 \]

**Hyperbolas** (Lesson 10-5)
- Standard Form: horizontal \[ \frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1 \]
  vertical \[ \frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1 \]

**Solving Quadratic Systems** (Lesson 10-7)
- Systems of quadratic equations can be solved using substitution and elimination.
- A system of quadratic equations can have zero, one, two, three, or four solutions.

**Study Organizer**

Be sure the Key Concepts are noted in your Foldable.

**Key Vocabulary**

- center (of a circle) (p. 631)
- center (of an ellipse) (p. 639)
- circle (p. 631)
- conjugate axis (p. 648)
- constant difference (p. 651)
- constant sum (p. 640)
- co-vertices (of a hyperbola) (p. 648)
- co-vertices (of an ellipse) (p. 639)
- directrix (p. 623)
- ellipse (p. 639)
- foci (of a hyperbola) (p. 648)
- foci (of an ellipse) (p. 639)
- focus (p. 623)
- hyperbola (p. 648)
- latus rectum (p. 623)
- major axis (p. 639)
- minor axis (p. 639)
- parabola (p. 623)
- radius (p. 631)
- transverse axis (p. 648)
- vertices (of a hyperbola) (p. 648)
- vertices (of an ellipse) (p. 639)

**Vocabulary Check**

State whether each sentence is true or false. If false, replace the underlined term to make a true sentence.

1. The set of all points in a plane that are equidistant from a given point in the plane, called the **focus**, forms a circle.
2. A(n) ellipse is the set of all points in a plane such that the sum of the distances from the two fixed points is constant.
3. The endpoints of the major axis of an ellipse are the **foci** of the ellipse.
4. The **radius** is the distance from the center of a circle to any point on the circle.
5. The line segment with endpoints on a parabola, through the focus of the parabola, and perpendicular to the axis of symmetry is called the **latus rectum**.
6. Every hyperbola has two axes of symmetry, the transverse axis and the **major axis**.
7. A **directrix** is the set of all points in a plane that are equidistant from a given point in the plane, called the center.
8. A hyperbola is the set of all points in a plane such that the absolute value of the sum of the distances from any point on the hyperbola to two given points is constant.
9. A parabola can be defined as the set of all points in a plane that are the same distance from the focus and a given line called the **directrix**.
10. The **major axis** is the longer of the two axes of symmetry of an ellipse.
### 10-1 Midpoint and Distance Formulas (pp. 617–622)

Find the midpoint of the line segment with endpoints at the given coordinates.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>11.</td>
<td>(−8, 6), (3, 4)</td>
<td>12.</td>
</tr>
<tr>
<td>13.</td>
<td>( \left( \frac{3}{4}, \frac{2}{3} \right) ) ( - \left( \frac{1}{3}, \frac{1}{4} \right) )</td>
<td>14.</td>
</tr>
</tbody>
</table>

Find the distance between each pair of points with the given coordinates.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>15.</td>
<td>(10, −3), (1, −5)</td>
<td>16.</td>
</tr>
<tr>
<td>17.</td>
<td>( \left( \frac{1}{4}, \frac{1}{2} \right) ) ( - \left( \frac{3}{2}, \frac{5}{4} \right) )</td>
<td>18.</td>
</tr>
</tbody>
</table>

#### Example 1

Find the midpoint of a line segment whose endpoints are at \((-4, 8)\) and \((10, −1)\).

Let \((x_1, y_1) = (-4, 8)\) and \((x_2, y_2) = (10, −1)\).

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-4 + 10}{2}, \frac{8 + (-1)}{2} \right) = \left( \frac{6}{2}, \frac{7}{2} \right) = (3, \frac{7}{2})
\]

#### Example 2

Find the distance between \(P(5, −3)\) and \(Q(−1, 5)\).

Let \((x_1, y_1) = (5, −3)\) and \((x_2, y_2) = (−1, 5)\).

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

Distance Formula

\[
= \sqrt{(-1 - 5)^2 + [5 - (-3)]^2}
\]

Substitute.

\[
= \sqrt{36 + 64}
\]

Subtract.

\[
= \sqrt{100} \text{ or } 10 \text{ units}
\]

Simplify.

---

### 10-2 Parabolas (pp. 623–629)

Graph each equation.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>20.</td>
<td>( y = 3x^2 + 24x - 10 )</td>
</tr>
<tr>
<td>22.</td>
<td>( x = \frac{1}{2}y^2 - 4y + 3 )</td>
</tr>
</tbody>
</table>

Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>24.</td>
<td>( y = -\frac{1}{2}x^2 )</td>
<td>25.</td>
</tr>
<tr>
<td>26.</td>
<td>( x - 6y = y^2 + 4 )</td>
<td>27.</td>
</tr>
</tbody>
</table>

#### Example 3

Write \(3y - x^2 = 4x + 7\) in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

Write the equation in the form \(y = a(x - h)^2 + k\) by completing the square.

\[
3y = x^2 + 4x + 7
\]

Isolate the terms with \(x\).

\[
3y = (x^2 + 4x + \boxed{4}) + 7 - 4
\]

Complete the square.

\[
3y = (x^2 + 4x + 4) + 7 - 4
\]

\[
\left(\frac{x}{2}\right)^2 = 4
\]

\[
y = \frac{1}{3}(x + 2)^2 + 3
\]

Divide each side by 3.

Vertex: \((-2, 1)\); axis of symmetry: \(x = -2\); direction of opening: upward since \(a > 0\).

---

**Hiking** Marc wants to hike from his camp to a waterfall. The waterfall is 5 miles south and 8 miles east of his campsite.

- **a.** How far away is the waterfall?
- **b.** Marc wants to stop for lunch halfway to the waterfall. Where should he stop?

**Sports** When a football is kicked, the path it travels is shaped like a parabola. Suppose a football is kicked from ground level, reaches a maximum height of 50 feet, and lands 200 feet away. Assuming the football was kicked at the origin, write an equation of the parabola that models the flight of the football.
### 10-3 Circles (pp. 631–637)

Write an equation for the circle that satisfies each set of conditions.

29. center \((-1, 6)\), radius 3 units
30. endpoints of a diameter \((2, 5)\) and \((0, 0)\)
31. endpoints of a diameter \((4, -2)\) and \((-2, -6)\)

Find the center and radius of each circle. Then graph the circle.

32. \((x + 5)^2 + y^2 = 9\)
33. \((x - 3)^2 + (y + 1)^2 = 25\)
34. \((x + 2)^2 + (y - 8)^2 = 1\)
35. \(x^2 + 4x + y^2 - 2y - 11 = 0\)

36. **SOUND** A loudspeaker in a school is located at the point \((65, 40)\). The speaker can be heard in a circle with a radius of 100 feet. Write an equation to represent the possible boundary of the loudspeaker sound.

**Example 4**

Find the center and radius of the circle with equation \(x^2 - 2x + y^2 + 6y + 6 = 0\). Then graph the circle.

Complete the squares.

\[
(x^2 - 2x + 1) + (y^2 + 6y + 9) = -6 + 1 + 9
\]

\[
(x - 1)^2 + (y + 3)^2 = 4
\]

The center of the circle is at \((1, -3)\) and the radius is 2.

---

### 10-4 Ellipses (pp. 639–646)

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

37. \(\frac{x^2}{9} + \frac{y^2}{36} = 1\)
38. \(\frac{y^2}{10} + \frac{x^2}{5} = 1\)
39. \(\frac{x^2}{36} + \frac{(y - 4)^2}{4} = 1\)
40. \(27x^2 + 9y^2 = 81\)
41. \(\frac{(x + 1)^2}{25} + \frac{(y - 2)^2}{16} = 1\)
42. \(9x^2 + 4y^2 + 54x - 8y + 49 = 0\)
43. \(9x^2 + 25y^2 - 18x + 50y - 191 = 0\)
44. \(7x^2 + 3y^2 - 28x - 12y = -19\)

45. **LANDSCAPING** The Martins have a garden in their front yard that is shaped like an ellipse. The major axis is 16 feet and the minor axis is 10 feet. Write an equation to model the garden. Assume the origin is at the center of the garden and the major axis is horizontal.

**Example 5**

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with equation \(9x^2 + 16y^2 - 54x + 32y - 47 = 0\). Then graph the ellipse.

First, convert to standard form.

\[
9(x^2 - 6x + \Box) + 16(y^2 + 2y + \Box) = 47 + 9(\Box) + 16(\Box)
\]

\[
9(x - 3)^2 + 16(y + 1)^2 = 144
\]

\[
\frac{(x - 3)^2}{16} + \frac{(y + 1)^2}{9} = 1
\]

The center of the ellipse is \((3, -1)\). The ellipse is horizontal. \(a^2 = 16\), so \(a = 4\). \(b^2 = 9\), so \(b = 3\).

The length of the major axis is \(2 \cdot 4 = 8\). The length of the minor axis is \(2 \cdot 3 = 6\). To find the foci: \(c^2 = 16 - 9 = 7\), so \(c = \sqrt{7}\). The foci are \((3 + \sqrt{7}, -1)\) and \((3 - \sqrt{7}, -1)\).
### 10-5 Hyperbolas (pp. 646–655)

Graph each hyperbola. Identify the vertices, foci, and asymptotes.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Vertices</th>
<th>Foci</th>
<th>Asymptotes</th>
</tr>
</thead>
<tbody>
<tr>
<td>46. ( \frac{y^2}{9} - \frac{x^2}{4} = 1 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>47. ( \frac{(x-3)^2}{1} - \frac{(y+2)^2}{4} = 1 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>48. ( \frac{(y+1)^2}{9} - \frac{(x-4)^2}{16} = 1 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>49. ( 4x^2 - 9y^2 = 36 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50. ( 9y^2 - x^2 - 4x + 18y + 4 = 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Example 6**

Graph \( 9x^2 - 4y^2 - 36x - 8y - 4 = 0 \). Identify the vertices, foci, and asymptotes.

Complete the square.

\[
9(x^2 - 4x + 4) - 4(y^2 + 2y + 1) = 4 + 9 - 4(1)
\]

\[
(x - 2)^2 - 4(y + 1)^2 = 36
\]

\[
\frac{(x - 2)^2}{36} - \frac{(y + 1)^2}{9} = 1
\]

The center is at \((2, -1)\). The vertices are at \((0, -1)\) and \((4, -1)\). The foci are at \((2 + \sqrt{13}, -1)\) and \((2 - \sqrt{13}, -1)\). The equations of the asymptotes are \(y + 1 = \pm \frac{3}{2}(x - 2)\).

### 10-6 Identifying Conic Sections (pp. 656–660)

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>52. ( 3x^2 + 12x - y + 8 = 0 )</td>
<td><em>Parabola</em></td>
</tr>
<tr>
<td>53. ( 9x^2 + 16y^2 = 144 )</td>
<td><em>Circle</em></td>
</tr>
<tr>
<td>54. ( x^2 + y^2 - 8x - 2y + 8 = 0 )</td>
<td><em>Ellipse</em></td>
</tr>
<tr>
<td>55. ( -9x^2 + y^2 + 36x - 45 = 0 )</td>
<td><em>Hyperbola</em></td>
</tr>
</tbody>
</table>

Without writing the equation in standard form, state whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>56. ( 7x^2 + 9y^2 = 63 )</td>
<td><em>Parabola</em></td>
</tr>
<tr>
<td>57. ( 5y^2 + 2y + 4x - 13x^2 = 81 )</td>
<td><em>Circle</em></td>
</tr>
<tr>
<td>58. ( x^2 - 8x + 16 = 6y )</td>
<td><em>Ellipse</em></td>
</tr>
<tr>
<td>59. ( x^2 + 4x + y^2 - 285 = 0 )</td>
<td><em>Hyperbola</em></td>
</tr>
</tbody>
</table>

**Example 7**

Write \( 3x^2 + 3y^2 - 12x + 30y + 39 = 0 \) in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

\[
3(x^2 - 4x + 4) + 3(y^2 + 10y + 25) = -39 + 3(4) + 3(25)
\]

\[
3(x - 2)^2 + 3(y + 5)^2 = 48
\]

\[
(x - 2)^2 + (y + 5)^2 = 16
\]

In this equation \( A = 3 \) and \( C = 3 \). Since \( A \) and \( C \) are both positive and \( A = C \), the graph is a circle. The center is at \((2, -5)\) and the radius is 4.

Light** Suppose the edge of a shadow can be represented by the equation \( 16x^2 + 25y^2 - 32x - 100y - 284 = 0 \).

a. What is the shape of the shadow?

b. Graph the equation.
10-7 Solving Quadratics Systems (pp. 662–667)

Solve each system of equations.

61. \( x^2 + y^2 = 8 \)
62. \( x - 2y = 2 \)
63. \( y + x^2 = 4x \)
64. \( 3x^2 - y^2 = 11 \)
65. \( 5x^2 + y^2 = 30 \)
66. \( \frac{x^2}{30} + \frac{y^2}{6} = 1 \)

67. PHYSICAL SCIENCE Two balls are launched into the air at the same time. The heights they are launched from are different. The height \( y \) in feet of one is represented by \( y = -16t^2 + 80t + 25 \) where \( t \) is the time in seconds. The height of the other ball is represented by \( y = -16t^2 + 30t + 100 \).

a. After how many seconds are the balls at the same height?
b. What is this height?

68. ARCHITECTURE An architect is building the front entrance of a building in the shape of a parabola with the equation \( y = -\frac{1}{10}(x - 10)^2 + 20 \). While the entrance is being built, the construction team puts in two support beams with equations \( y = -x + 10 \) and \( y = x - 10 \). Where do the support beams meet the parabola?

Solve each system of inequalities by graphing.

69. \( x^2 + y^2 < 64 \)
70. \( x^2 + y^2 < 49 \)
71. \( x + y < 4 \)
72. \( x^2 + y^2 < 25 \)
73. \( x^2 + y^2 < 36 \)
74. \( y^2 < x \)

Example 8

Solve the system of equations.
\( x^2 + y^2 = 100 \)
\( 3x - y = 10 \)

Use substitution to solve the system. First, rewrite \( 3x - y = 10 \) as \( y = 3x - 10 \).

\[
\begin{align*}
x^2 + y^2 &= 100 \\
x^2 + (3x - 10)^2 &= 100 \\
x^2 + 9x^2 - 60x + 100 &= 100 \\
10x^2 - 60x + 100 &= 0 \\
10x(x - 6) &= 0 \\
x &= 0 \quad \text{or} \quad x - 6 = 0 \\
x &= 0 \quad \text{or} \quad x = 6
\end{align*}
\]

Now solve for \( y \).
\[
\begin{align*}
y &= 3x - 10 \\
y &= 3x - 10 \\
= 3(0) - 10 &= 3(6) - 10 \\
= -10 &= 8
\end{align*}
\]

The solutions of the system are \((0, -10)\) and \((6, 8)\)

Example 9

Solve the system of inequalities by graphing.
\( x^2 + y^2 \leq 9 \)
\( 2y \geq x^2 + 4 \)

The solution is the green shaded region.
18. CARPENTRY Ellis built a window frame shaped like the top half of an ellipse. The window is 40 inches tall at its highest point and 160 inches wide at the bottom. What is the height of the window 20 inches from the center of the base?

Find the midpoint of the line segment with endpoints at the given coordinates.
1. (8, 3), (−4, 9)
2. \(\left(\frac{3}{4}, 0\right), \left(\frac{1}{2}, -1\right)\)
3. (−10, 0), (−2, 6)

Find the distance between each pair of points with the given coordinates.
4. (−5, 8), (4, 3)
5. \(\left(\frac{1}{3}, \frac{2}{3}\right), \left(-\frac{5}{6}, -\frac{11}{6}\right)\)
6. (4, −5), (4, 9)

State whether the graph of each equation is a parabola, circle, ellipse, or hyperbola. Then graph the equation.
7. \(y^2 = 64 - x^2\)
8. \(4x^2 + y^2 = 16\)
9. \(4x^2 - 9y^2 + 8x + 36y = 68\)
10. \(\frac{1}{2}x^2 - 3 = y\)
11. \(y = -2x^2 - 5\)
12. \(16x^2 + 25y^2 = 400\)
13. \(x^2 + 6x + y^2 = 16\)
14. \(\frac{y^2}{4} - \frac{x^2}{16} = 1\)
15. \((x + 2)^2 = 3(y - 1)\)
16. \(4x^2 + 16y^2 + 32x + 63 = 0\)

17. MULTIPLE CHOICE Which equation represents a hyperbola that has vertices at (−3, −3) and (5, −3) and a conjugate axis of length 6 units?
A. \(\frac{(y - 1)^2}{16} - \frac{(x + 3)^2}{9} = 1\)
B. \(\frac{(x - 1)^2}{16} - \frac{(y + 3)^2}{9} = 1\)
C. \(\frac{(y + 1)^2}{16} - \frac{(x - 3)^2}{9} = 1\)
D. \(\frac{(x + 1)^2}{16} - \frac{(y - 3)^2}{9} = 1\)

19. Solve each system of equations.
\[x^2 + y^2 = 100\]
\[y = -x - 2\]
20. Solve each system of inequalities.
\[x^2 + 2y^2 = 11\]
\[x + y = 2\]
21. Solve each system of inequalities.
\[y^2 - x^2 = 9\]
\[\frac{(x - 2)^2}{4} - \frac{(y - 4)^2}{9} \geq 1\]
\[y > -x^2 + 2\]
\[x - 4y < 8\]
22. Solve each system of inequalities.
\[\frac{(x - 2)^2}{4} - \frac{(y - 4)^2}{9} \geq 1\]
\[x - 4y < 8\]
24. MULTIPLE CHOICE Which is NOT the equation of a parabola?
F. \(y = 3x^2 + 5x - 3\)
G. \(2y + 3x^2 + x - 9 = 0\)
H. \(x = 3(y + 1)^2\)
J. \(x^2 + 2y^2 + 6x = 10\)

25. FORESTRY A forest ranger at an outpost in the Sam Houston National Forest and another ranger at the primary station both heard an explosion. The outpost and the primary station are 6 kilometers apart.
a. If one ranger heard the explosion 6 seconds before the other, write an equation that describes all the possible locations of the explosion. Place the two ranger stations on the x-axis with the midpoint between the stations at the origin. The transverse axis is horizontal. (Hint: The speed of sound is about 0.35 kilometer per second.)
b. Draw a sketch of the possible locations of the explosion. Include the ranger stations in the drawing.
Use a Formula

Sometimes it is necessary to use a formula to solve problems on standardized tests. In some cases you may even be given a sheet of formulas that you are permitted to reference while taking the test.

Strategies for Using a Formula

Step 1
Read the problem statement carefully.

Ask yourself:
- What am I being asked to solve?
- What information is given in the problem?
- Are there any formulas that I can use to help me solve the problem?

Step 2
Solve the problem and check your solution.
- Substitute the known quantities that are given in the problem statement into the formula.
- Simplify to solve for the unknown values in the formula.
- Check to make sure your answer makes sense. If time permits, check your answer.

Test Practice Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

What is the distance between points A and B on the coordinate plane? Round your answer to the nearest tenth if necessary.

<table>
<thead>
<tr>
<th>Scoring Rubric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criteria</td>
</tr>
<tr>
<td>Full Credit: The answer is correct and a full explanation is provided that shows each step.</td>
</tr>
<tr>
<td>Partial Credit:</td>
</tr>
<tr>
<td>- The answer is correct but the explanation is incomplete.</td>
</tr>
<tr>
<td>- The answer is incorrect but the explanation is correct.</td>
</tr>
<tr>
<td>No Credit: Either an answer is not provided or the answer does not make sense.</td>
</tr>
</tbody>
</table>
Read the problem statement carefully. You are given the coordinates of two points on a coordinate plane and asked to find the distance between them. To solve this problem, you must use the **Distance Formula**.

Example of a 2-point response:

Use the Distance Formula to find the distance between points \( A(-4, 3) \) and \( B(1, -5) \).

\[
\begin{align*}
\text{\( d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)} \\
= \sqrt{[1 - (-4)]^2 + [(-5) - 3]^2} \\
= \sqrt{5^2 + (-8)^2} \\
= \sqrt{25 + 64} \\
= \sqrt{89} \text{ or about 9.4} \\
\end{align*}
\]

The distance between points \( A \) and \( B \) is about 9.4 units.

The steps, calculations, and reasoning are clearly stated. The student also arrives at the correct answer. So, this response is worth the full 2 points.

**Exercises**

Read each problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

1. What is the midpoint of segment \( CD \) with endpoints \( C(5, -12) \) and \( D(-9, 4) \)?

2. Katrina is making a map of her hometown on a coordinate plane. She plots the school at \( S(7, 3) \) and the park at \( P(-4, 12) \). If the scale of the map is 1 unit = 250 yards, what is the actual distance between the school and the park? Round to the nearest yard.

3. Mr. Washington is making a concrete table for his backyard. The tabletop will be circular with a diameter of 6 feet and a depth of 6 inches. How much concrete will Mr. Washington need to make the top of the table? Round to the nearest cubic foot.

4. What is the equation, in standard form, of the hyperbola graphed below?

5. If the surface area of a cube is increased by a factor of 9, what is the change in the length of the sides of the cube?
   - A The length is 2 times the original length.
   - B The length is 3 times the original length.
   - C The length is 6 times the original length.
   - D The length is 9 times the original length.
Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which is the first incorrect step in simplifying \( \log_3 \frac{3}{48} \)?
   - A Step 1
   - B Step 2
   - C Step 3
   - D Each step is correct.

2. Which is the equation for the parabola that has vertex \((-3, -23)\) and passes through the point \((1, 9)\)?
   - A \( y = x^2 + 10x + 7 \)
   - B \( y = x^2 - 6x + 19 \)
   - C \( y = 2x^2 + 12x - 5 \)
   - D \( y = 2x^2 - 3x + 10 \)

3. What are the vertices of the ellipse with equation \( \frac{(x - 3)^2}{36} + \frac{(y - 2)^2}{144} = 1 \)?
   - A \((-3, 2)\) and \((9, 2)\)
   - B \((-2, 3)\) and \((10, 3)\)
   - C \((3, -10)\) and \((3, 14)\)
   - D \((2, -11)\) and \((4, 13)\)

4. Hooke’s Law states that the force needed to keep a spring stretched \(x\) units is directly proportional to \(x\). If a force of 40 N is required to maintain a spring stretched to 5 centimeters, what force is needed to keep the spring stretched 14 centimeters?
   - A 8 N
   - B 19 N
   - C 112 N
   - D 1600 N

5. Angela is making a map of her backyard on a coordinate grid. She plots point \(G(-4, -6)\) to represent her mom’s garden and point \(S(3, 7)\) to represent the rope swing hanging on an oak tree. If the scale of the map is 1 unit = 5 feet, what is the approximate distance between the garden and the rope swing?
   - A 74 feet
   - B 79 feet
   - C 82 feet
   - D 90 feet

6. If \( \sqrt{x + 5} + 1 = 4 \), what is the value of \(x\)?
   - A 4
   - B 10
   - C 11
   - D 20

7. The area of \( \text{the base of a rectangular suitcase measures}\ 3x^2 + 5x - 4 \text{ square units. The height of the suitcase measures}\ 2x \text{ units. Which polynomial expression represents the volume of the suitcase?}\)
   - A \( 3x^3 + 5x^2 - 4x \)
   - B \( 6x^2 + 10x - 8 \)
   - C \( 6x^3 + 10x^2 - 8x \)
   - D \( 3x^3 + 10x^2 - 4 \)

8. Malina was given this geoboard to model the slope \(\frac{5}{4}\).

   If the peg in the upper right-hand corner represents the origin on a coordinate plane, where could Malina place a rubber band to represent the given slope?
   - A from peg \(A\) to peg \(B\)
   - B from peg \(A\) to peg \(C\)
   - C from peg \(B\) to peg \(D\)
   - D from peg \(C\) to peg \(D\)
9. A placekicker kicks a ball upward with a velocity of 32 feet per second. Ignoring the height of the kicking tee, how long after the football is kicked does it hit the ground? Use the formula \( h(t) = v_0t - 16t^2 \), where \( h(t) \) is the height of an object in feet, \( v_0 \) is the object’s initial velocity in feet per second, and \( t \) is the time in seconds.

10. GRIDDED RESPONSE What is the maximum number of solutions of a system of equations that consists of a circle and a hyperbola?

11. Lupe is preparing boxes of assorted chocolates. Chocolate-covered peanuts cost \$7\) per pound. Chocolate-covered caramels cost \$6.50\) per pound. The boxes of assorted candies contain five more pounds of peanut candies than caramel candies. If the total amount sold was \$575\), how many pounds of each candy were needed to make the boxes?

12. GRIDDED RESPONSE What is the \( y \)-coordinate of the midpoint of segment \( AB \) with endpoints \( A(0.8, 5.32) \) and \( B(0.44, 2.2) \)?

13. Marc went shopping and bought two shirts, three pairs of pants, one belt, and two pairs of shoes. The following matrix shows the prices for each item respectively.

\[
\begin{bmatrix}
20.15 & 32 & 15 & 25.99
\end{bmatrix}
\]

Use matrix multiplication to find the total amount of money Marc spent while shopping.

14. Clarence graphed the quadratic equation \( h(t) = -16t^2 + 128t \) to model the flight of a firework. The parabola shows the height, in feet, of the firework \( t \) seconds after it was launched.

a. What is the vertex of the parabola?
b. What does the vertex of the parabola represent?
c. How long is the firework in the air before it lands?

15. The Colonial High School Yearbook Staff is selling yearbooks and chrome picture frames engraved with the year. The number of yearbooks and frames sold to members of each grade is shown in the table.

<table>
<thead>
<tr>
<th>Sales for Each Class</th>
<th>Yearbooks</th>
<th>Frames</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9th</td>
<td>423</td>
<td>256</td>
</tr>
<tr>
<td>10th</td>
<td>464</td>
<td>278</td>
</tr>
<tr>
<td>11th</td>
<td>546</td>
<td>344</td>
</tr>
<tr>
<td>12th</td>
<td>575</td>
<td>497</td>
</tr>
</tbody>
</table>

a. Find the difference in the sales of yearbooks and frames made to the 10th and 11th grade classes.
b. Find the total number of yearbooks and frames sold.
c. A yearbook costs \$48\) and a frame costs \$18. Find the sales of yearbooks and frames for each class.