## Chapter 12: Probability and Statistics

### Then
- In Chapter 9, you calculated weighted averages.

### Now
- In Chapter 12, you will:
  - Evaluate surveys, studies, and experiments.
  - Create and use graphs of probability distributions.
  - Use the Empirical Rule to find probabilities.
  - Compare sample statistics and population statistics.

### Why? ▲
- **EDUCATION** Probability and statistics are used in all facets of education. Surveys and experiments are done to find out which teaching methods promote the most learning. Statistics are used to determine grades when classes are curved, or when college professors weight their grades.

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- Graphing Calculator
- Audio
- Foldables
- Self-Check Practice
- Worksheets
Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

<table>
<thead>
<tr>
<th>QuickCheck</th>
<th>QuickReview</th>
</tr>
</thead>
<tbody>
<tr>
<td>State whether the events are independent or dependent. (Concepts and Skills Bank 4)</td>
<td>Example 1</td>
</tr>
<tr>
<td>1. selecting a fiction book and a nonfiction book at the library</td>
<td>Mary wants to take 7 different classes next year. Assuming that each class is offered each period, how many different schedules could she have?</td>
</tr>
<tr>
<td>2. choosing a president, vice-president, secretary, and treasurer for Key Club, assuming that a person can hold only one office</td>
<td>When Mary schedules a class for a given period, she cannot schedule that class for any other period. Therefore, the choices of which class to schedule each period are dependent events.</td>
</tr>
<tr>
<td>3. choosing a junior, a senior and a faculty member to coordinate a food drive</td>
<td>There are 7 · 6 · 5 · 4 · 3 · 2 · 1 or 5040 different schedules that Mary could have.</td>
</tr>
</tbody>
</table>

Determine whether each situation involves a permutation or a combination. (Lesson 0-5)

4. seven shoppers in line at a checkout counter
5. an arrangement of the letters in the word intercept
6. choosing 2 different pizza toppings from a list of 6

Expand each binomial. (Lesson 11-6)

7. \((a - 2)^4\) | Example 2 |
| 8. \((m - a)^5\) | Determine whether the situation involves a permutation or a combination. |
| 9. \((2b - x)^4\) | choosing 6 students from a class of 25 |
| 10. \((2a + b)^6\) | Because the order of the students that are chosen does not matter, this is a combination. |
| 11. \((3x - 2y)^5\) | Example 3 |
| 12. \((3x + 2y)^4\) | Expand \((a + b)^4\). |
| 13. \(\left(\frac{a}{2} + 2\right)^5\) | Replace \(n\) with 4 in the Binomial Theorem. |
| 14. \(\left(3 + \frac{m}{3}\right)^5\) | \((a + b)^4 = 4C_0a^4 + 4C_1a^3b + 4C^2a^2b^2 + 4C_3ab^3 + 4C_4b^4\) |

Example 2

<table>
<thead>
<tr>
<th>Expand ((a + b)^4).</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Replace (n) with 4 in the Binomial Theorem.</td>
<td></td>
</tr>
<tr>
<td>((a + b)^4 = 4C_0a^4 + 4C_1a^3b + 4C^2a^2b^2 + 4C_3ab^3 + 4C_4b^4)</td>
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<td></td>
</tr>
</tbody>
</table>

2 Online Option Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 12. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

New Vocabulary

<table>
<thead>
<tr>
<th>English</th>
<th>Español</th>
</tr>
</thead>
<tbody>
<tr>
<td>survey</td>
<td>exámenes</td>
</tr>
<tr>
<td>population</td>
<td>población</td>
</tr>
<tr>
<td>sample</td>
<td>muestra</td>
</tr>
<tr>
<td>biased</td>
<td>en polarización negativa</td>
</tr>
<tr>
<td>unbiased</td>
<td>imparcial</td>
</tr>
<tr>
<td>experiment</td>
<td>experimento</td>
</tr>
<tr>
<td>control group</td>
<td>grupo de control</td>
</tr>
<tr>
<td>parameter</td>
<td>parámetro</td>
</tr>
<tr>
<td>statistic</td>
<td>estadística</td>
</tr>
<tr>
<td>conditional probability</td>
<td>probabilidad condicional</td>
</tr>
<tr>
<td>probability</td>
<td>probabilidad</td>
</tr>
<tr>
<td>random variable</td>
<td>variable aleatoria</td>
</tr>
<tr>
<td>expected value</td>
<td>valor previsto</td>
</tr>
<tr>
<td>normal distribution</td>
<td>distribución normal</td>
</tr>
<tr>
<td>skewed distribution</td>
<td>distribución asimétrica</td>
</tr>
<tr>
<td>inferential statistics</td>
<td>estadística deductiva</td>
</tr>
<tr>
<td>binomial distribution</td>
<td>distribución binomial</td>
</tr>
<tr>
<td>binomial experiment</td>
<td>experimento binomio</td>
</tr>
</tbody>
</table>

Review Vocabulary

combination  p. P12  combinación  an arrangement or selection of objects in which order is not important
permutation  p. P12  permutación  a group of objects or people arranged in a certain order
random  arbitrario  Unpredictable, or not based on any predetermined characteristics of the population; when a die is tossed, a coin is flipped, or a spinner is spun, the outcome is a random event.
**Surveys, Studies, and Experiments** Surveys are used to collect information. If everyone involved with the school were surveyed, then the survey would involve the entire population. A survey in which every member of the population is polled is called a census. If only 100 people selected at random from the school were surveyed, then the survey would involve a sample.

A survey is biased if its design favors certain outcomes. If the students above only surveyed basketball players and their parents, then the survey would be biased toward accepting the team. A sample is unbiased if it is random, or not based on any predetermined characteristics of the population. If they sent surveys to 100 students selected at random, then the survey would be unbiased.

**Real-World Example 1** Biased and Unbiased Samples

**SURVEYS** State whether each survey would produce a random sample. Write yes or no. Explain.

a. asking every tenth person coming out of a theater how many times a week they go to the theater to determine how often city residents support the performing arts

No; the people surveyed probably go to the theater more often than the average person.

b. surveying people going into a pet store to find out if the city’s residents support the building and maintaining of a dog park

No; the people surveyed would probably be more likely than others to support pet activities.

c. A box contains the name of every student in the school. A hundred names are randomly pulled out of the box. Those students are asked their opinions on the new cafeteria rules.

Yes; everyone in the population has an equal chance to be part of the sample.

**Guided Practice**

1A. asking every player at a golf course what sport they prefer to watch on TV

1B. calling 100 randomly selected numbers and asking for their opinions on a local tax

1C. going to a football game and asking 100 random fans about their favorite sport

To avoid bias in a survey, two things are needed: a strong random sample and unbiased survey techniques. A strong random sample is an unbiased sample with a large number of participants.
Experiments, Surveys, and Observational Studies

**Real-World Example 2  Survey Design**

**SCHOOL SURVEYS** Christopher wants to determine the most desired location for the senior class trip. Which questions will get him the answer he is seeking?

a. Do you like Disneyland?
   This question is biased in favor of Disneyland.

b. Which is better, King’s Island or Cedar Point?
   This question is biased because it only gives two options.

c. Where would you most like to go on the senior trip?
   This is an unbiased question that will produce the answer he is seeking.

**Guided Practice**

Which question will determine the most popular horror movie at school?

2A. Did you enjoy the last horror movie you saw?

2B. Which is better, romance or comedy?

2C. What is your favorite horror movie?

In an **observational study**, individuals are observed and no attempt is made to influence the results. In an **experiment**, something is intentionally done to people, animals, or objects, and then the response is observed.

<table>
<thead>
<tr>
<th>Observational Study</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Find 100 people, 50 of whom have been taking a treatment.</td>
<td></td>
</tr>
<tr>
<td>• Collect the data.</td>
<td></td>
</tr>
<tr>
<td>• Analyze and interpret the data.</td>
<td></td>
</tr>
<tr>
<td>• Find 100 people. Randomly select 50 people for treatment. Give the other 50 a placebo.</td>
<td></td>
</tr>
<tr>
<td>• Collect and analyze the data.</td>
<td></td>
</tr>
</tbody>
</table>

In an experiment, the people, animals, or objects given the treatment are called the **treatment group**. Those given the **placebo**, or false treatment, are the **control group**. The placebo is given so none of the participants will know which group he or she is in, and the experiment will be unbiased.

**Real-World Example 3  Experiments and Observational Studies**

**EXPERIMENTS** State whether each situation represents an **experiment** or an **observational study**. If it is an experiment, identify the **control** group and the **treatment** group. Then determine whether there is bias.

a. Find 200 students, half of whom participated in extracurricular activities, and compare their grade-point averages.
   This is an observational study.

b. Find 200 people and randomly split them into two groups. One group jogs 2 miles per day and the other group does not jog at all.
   This is an experiment because the people are put into groups at random. The treatment group is the joggers, and the control is the other group. This is a biased experiment because the participants all know which group they are in.

**Guided Practice**

3. Find 80 college students, half of whom took a statistics course in high school, and compare their grades in a college statistics course.
How do you know when to use a survey, an observational study, or an experiment? A survey involves the random sampling of subjects from a population, while experiments involve the random assignment of treatments to subjects. In an experiment, you have control. In an observational study, you do not.

**Example 4  Experiments and Observational Studies**

Determine whether each situation calls for a survey, an observational study, or an experiment. Explain the process.

**a. You want to test a treatment for a disease.**
This calls for an experiment. The test subjects are people with the disease. The treatment group receives the treatment while the control group gets a placebo.

**b. You want to find opinions on a presidential election.**
This calls for a survey. It is best to call random numbers throughout the country in order to get an unbiased sample.

**c. You want to find out if 10 years of smoking affects lung capacity.**
This calls for an observational study. The lung capacity of people who have smoked for 10 years is compared to the lung capacity of an equal number of nonsmokers.

**Guided Practice**

4. Two hundred randomly selected high school students rate their opinions regarding the new lunch rules from 1 (Totally Disagree) to 5 (Totally Agree).

**2 Distinguish Between Correlation and Causation**  An observed association between the results of an experiment and the treatment does not necessarily imply that the treatment caused the results.

When there is a correlation between two events, the two events are related. When there is a causation, one event is shown to be the direct cause of another event. While a correlation between two events can be shown, causation is much more difficult to prove.

**Example 5  Correlation Versus Causation**

Determine whether the following statements show correlation or causation. Explain your reasoning.

**a. Studies have shown that students are less energetic after they eat lunch.**
Correlation; the statement ignores crucial factors that might have a causal influence on both.

**b. If I lift weights, I can make the football team.**
Correlation; there are more factors involved.

**c. When the Sun is visible, we have daylight.**
A good way to determine causation is to look for other alternatives that could cause daylight. Since there are none, it shows causation.

**Guided Practice**

5. When I study, I will get an A.
Check Your Understanding

Example 1 State whether each survey would produce a random sample. Write yes or no. Explain.
1. Survey every third person coming out of an ice cream shop to find people’s favorite type of dessert.
2. A teacher sends every student whose last name ends with a chosen letter to the blackboard.

Example 2 Determine the survey question that will best obtain the desired answer.
3. Taylor wants to determine the most popular football team at the school.
   a. What is your favorite college football team?
   b. What is your favorite football team?
   c. Do you like the Dallas Cowboys or the Pittsburgh Steelers?

Example 3 State whether each situation represents an experiment or an observational study. If it is an experiment, identify the control group and the treatment group. Then determine whether there is bias.
4. A teacher has his first class complete review activities the day before the test. His second class does no review activities. He compares their test results.
5. Jaime finds 100 people, half of whom volunteer at a homeless shelter, and compares their average annual incomes.

Example 4 Determine whether the situation calls for a survey, an observational study, or an experiment. Explain the process.
6. You want to test a drug that reverses male pattern baldness.
7. You want to find voters’ opinions on recent legislation.

Example 5 Determine whether the following statements show correlation or causation. Explain.
8. When I exercise, I am in a better mood.
9. If we have a Level 2 snow emergency, we do not have school.

Practice and Problem Solving

Example 1 State whether each survey would produce a random sample. Write yes or no. Explain.
10. A sporting goods store owner sends a survey to everyone whose address ends in a particular digit.
11. Students in an honors science class are asked what their favorite subject is.
12. Every other shopper coming out of a mall is surveyed to determine how much people spend during the holidays.
13. Every twentieth person coming out of your high school is asked for whom they will vote in the upcoming student council race.

Example 2 Determine the survey question that will best obtain the desired answer.
14. Sabrina wants to determine interest in starting a chess club at her school.
   a. What day do you have free to stay after school?
   b. Do you like chess?
   c. Would you be willing to join a chess club at school?
15. Lauren wants to determine the most popular presidential candidate.
   a. For whom would you vote in the upcoming election?
   b. Do you prefer a particular political party?
   c. If you could vote, would you?
Example 3  State whether each situation represents an experiment or an observational study. If it is an experiment, identify the control group and the treatment group. Then determine whether there is bias.

16. Find 300 people and randomly split them into two groups. One group listens to Mozart for an hour every night before bed, and the other group does not listen to anything. Then compare how well they slept.

17. Find 250 students, half of whom are in the marching band, and compare the amounts of time spent on homework.

18. Find 100 students, half of whom are in the French Club, and compare their grades in French class.

Example 4  Determine whether each situation calls for a survey, an observational study, or an experiment. Explain the process.

19. You want to find out if years of running affect knee movement.

20. You want to find out if drinking soda affects stomach linings.

21. You want to test a treatment that keeps deer out of your garden.

Example 5  Determine whether the following statements show correlation or causation. Explain.

22. When it is very hot in the summer, there are ice cream vendors outside in New York.

23. Reading more will enable you to become more intelligent.

24. Researchers have concluded that Americans who speak more than one language are less likely to become ill.

25. Sleeping with your shoes on will cause you to have a headache.

26. **SELECTION BIAS** In a call-in poll, 81% of the more than 6000 respondents said that a certain businessman “symbolizes what makes the U.S.A. a great country.” How is this an example of sampling bias?

27. **QUESTIONNAIRES** A company gives an exit questionnaire to employees who are leaving the company. One of the questions asks how the employee felt about his or her experience with the company. Is this survey biased? Explain why or why not.

**H.O.T. Problems**  Use Higher-Order Thinking Skills

28. **ERROR ANALYSIS** Jordan and Kyle were asked to design an unbiased experiment. Is either of them correct? Explain your reasoning.

<table>
<thead>
<tr>
<th>Jordan</th>
<th>Kyle</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Get a group of 20 random people.</td>
<td>• Get a group of 20 football players.</td>
</tr>
<tr>
<td>• Randomly put half of them on an all-fruit diet for 3 weeks.</td>
<td>• Make half of them do 500 push-ups per day.</td>
</tr>
<tr>
<td>• Compare their weight gain/loss at the end of the 3 weeks.</td>
<td>• Compare the number of push-ups each group can do after 3 weeks.</td>
</tr>
</tbody>
</table>

29. **CHALLENGE** How could a telephone survey introduce sampling bias into the results?

30. **WRITING IN MATH** Compare and contrast the random sampling of units from a population and the random assignment of treatments to experimental units.

31. **OPEN ENDED** Design one of each of the following.
    a. survey     b. observational study     c. experiment

32. **REASONING** How can bias occur in an experiment, and how does it affect the results? Provide an example to explain your reasoning.
33. GEOMETRY  In \(\triangle ABC\), \(BC > AB\).
Which of the following must be true?
A  \(AB = BC\)
B  \(AC < AB\)
C  \(a > 60\)
D  \(a = b\)

34. SHORT RESPONSE  What is the solution set of
\[4x^2 - 2x - 4 = 4 - 2?\]

35. SAT/ACT  A pie is divided evenly between 3 boys and a girl. If one boy gives one half of his share to the girl and a second boy keeps two thirds of his share and gives the rest to the girl, what portion will the girl have in all?
F  \(\frac{5}{24}\)
H  \(\frac{1}{2}\)
K  \(\frac{13}{12}\)
G  \(\frac{11}{24}\)
J  \(\frac{13}{24}\)

36. Which equation represents a hyperbola?
A  \(y^2 = 49 - x^2\)
C  \(y = 49x^2\)
B  \(y = 49 - x^2\)
D  \(y = \frac{49}{x}\)

Spiral Review

37. Prove that the statement \(9^n - 1\) is divisible by 8 is true for all natural numbers.  (Lesson 11-7)

38. INTRAMURALS  Ofelia is taking ten shots in the intramural free-throw shooting competition. How many sequences of hits and misses are there that result in her making eight shots and missing two?  (Lesson 11-6)

Solve each system of equations.  (Lesson 10-7)

39. \(y = x + 3\)
\(y = 2x^2\)
40. \(x^2 + y^2 = 36\)
\(y = x + 2\)

41. \(y^2 + x^2 = 9\)
\(y = 7 - x\)

42. \(y + x^2 = 3\)
\(x^2 + 4y^2 = 36\)
43. \(x^2 + y^2 = 64\)
\(x^2 + 64y^2 = 64\)

44. \(y^2 = x^2 - 25\)
\(x^2 - y^2 = 7\)

Find the distance between each pair of points with the given coordinates.  (Lesson 10-1)

45. \((9, -2), (12, -14)\)

46. \((-4, -10), (-3, -11)\)

47. \((1, -14), (-6, 10)\)

48. \((-4, 9), (1, -3)\)

49. \((2.3, -1.2), (-4.5, 3.7)\)

50. \((0.23, 0.4), (0.68, -0.2)\)

Simplify. Assume that no variable equals 0.  (Lesson 6-1)

51. \((5cd^2)(-c^4d)\)

52. \((7x^3y^{-5})(4xy^3)\)

53. \(\frac{a^2n^6}{mn^5}\)

54. \((n^4)^4\)

55. \(-\frac{y^5z^7}{y^2z^5}\)

56. \((-2r^2t^4)^3(3rt^2)\)

Write a quadratic equation with the given root(s). Write the equation in the form \(ax^2 + bx + c = 0\), where \(a\), \(b\), and \(c\) are integers.  (Lesson 5-3)

57. \(-3, 9\)

58. \(-\frac{1}{3}, -\frac{3}{4}\)

59. \(4, -5\)

Skills Review

60. TESTS  Ms. Bonilla’s class of 30 students took a biology test. If 20 of her students had an average of 83 on the test and the other students had an average score of 74, what was the average score of the whole class?  (Lesson 9-6)

61. DRIVING  During a 10-hour trip, Kwan drove 4 hours at 60 miles per hour and 6 hours at 65 miles per hour. What was her average rate, in miles per hour, for the entire trip?  (Lesson 9-6)
You can use a TI-83/84 Plus graphing calculator with the CelSheet application to evaluate data found in the media.

A newspaper ran a series of articles about high school students who study abroad for at least one semester. To support the claim that international study was gaining in popularity, the reporter presented the graph at the right. It includes information from a state university about the number of students earning credit through the university while studying abroad in the International Academic Programs.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Students in IAP</td>
<td>316</td>
<td>451</td>
<td>561</td>
<td>704</td>
<td>823</td>
</tr>
</tbody>
</table>

**Tennessee Curriculum Standards**
CLE 3103.5.2 Evaluate and critique various ways of collecting data and using information based on data published in the media. Also addresses SPI 3103.1.2, CLE 3103.5.3, and ✓ 3103.5.11.

**Activity**

Evaluate the graph of the data.

**Step 1** Enter data in the CelSheet application.
- Press [APPs] then press [▼] until CelSheet is highlighted. Press [ENTER]. Then press any key to exit the title and help pages.
- Press [ALPHA] [“] 85 [ENTER] 89 [ENTER] to enter the first range of years into cell A1. Repeat for the remaining years.
- Use the arrow keys to highlight cell B1. Enter the data for each range of years.

**Step 2** Make a bar graph of the data.
- Enter the Category range: press [ALPHA] [A] 1 [ALPHA] [;] [ALPHA] [A] 5 [ENTER].
- For Ser1Name enter STDNTS. At this prompt, the alpha is assumed.
- Use the arrow keys to scroll past Series 2 and Series 3 information.
  At Title, enter IAP. Press [ENTER] three times to display graph.
- Press [TRACE] then [►] and [◄] to see information about each bar.

**Analyze the Results**

Compare your graph to the newspaper’s graph.

1. Do the graphs display the same information?
2. Which graph seems to show a more dramatic increase? Why?
3. Why would the reporter choose to display the graph in this way? Is it acceptable? Why or why not?
Statistical Analysis

1 Measures of Central Tendency

Data in one variable, or data type, like the finishing times are called univariate data. These data can be described by a measure of central tendency because it represents the center or middle of the data. The most commonly used measures of central tendency are the mean, median, and mode.

When deciding which measure of central tendency to use to represent a set of data, look closely at the data values.

<table>
<thead>
<tr>
<th>Key Concept</th>
<th>Measures of Central Tendency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use</td>
<td>Which Is…</td>
</tr>
<tr>
<td>mean</td>
<td>the sum of the data divided by the number of items in the data set</td>
</tr>
<tr>
<td>median</td>
<td>the middle number of the ordered data, or the mean of the middle two numbers</td>
</tr>
<tr>
<td>mode</td>
<td>the number or numbers that occur most often</td>
</tr>
</tbody>
</table>

Real-World Example 1: Measures of Central Tendency

a. RACING TIMES Refer to the information above. Which measure of central tendency best represents the data, and why?

Since the data are spread out and there do not appear to be any outliers, the mean best represents the data.

b. Which measure of central tendency best represents the data at the right, and why?

Since there are outliers and no big gaps in the middle, the median best represents the data.

Guided Practice

1. RAFFLE A raffle is offering a grand prize worth $1000 and thirty other prizes worth $5 each. Which measure of central tendency best represents the data, and why?

Two types of measures can be applied to sets of data. A parameter is a measure that describes a characteristic of a population. One example of a parameter is the mean income of the United States. A statistic is a measure that describes a characteristic of a sample. An example of a statistic is the mean income of the people who live on your street.
**Example 2 Samples Versus Populations**

Determine whether each of the following represents a population or a sample.

a. The Nielsen Poll estimates the average number of hours of television watched per week for U.S. households.
   This represents a sample because only a fraction of U.S. residents are polled.

b. A mathematics exam is given to every graduating senior in the country to analyze certain mathematics skills.
   This represents a population because the exam tests every graduating senior.

**Guided Practice**

2A. A teacher compares the scores on a test in her class.

2B. A teacher compares her class with the rest of the country on a national test.

When a single sample is drawn from a population, there is a risk of incurring a sampling error. As the size of the sample increases, the margin of error decreases. The margin of sampling error provides the interval that shows how much the responses from the sample would differ from the population.

**Key Concept Margin of Sampling Error**

When a random sample \( n \) is taken from a population, the margin of sampling error can be approximated by \( ± \frac{1}{\sqrt{n}} \).

**Example 3 Margin of Sampling Error**

In a random survey of 2148 people, 58% said that football is their favorite sport.

a. What is the margin of sampling error?
   \[
   \text{Margin of sampling error} = ± \frac{1}{\sqrt{n}} \quad \text{Margin of Sampling Error Formula}
   \]
   \[
   = ± \frac{1}{\sqrt{2148}} \quad n = 2148
   \]
   \[
   ≈ ±0.0216 \quad \text{Simplify.}
   \]
   The margin of sampling error is about ±2.16%.

b. What is the likely interval that contains the percentage of the population that claims football is their favorite sport?
   \[
   0.58 + 0.0216 = 0.6016 \quad 0.58 - 0.0216 = 0.5584
   \]
   The likely interval that contains the percentage of the population that claims football is their favorite sport is between 55.84% and 60.16%.

**Guided Practice**

In a random survey of 3247 people, 41% said that they are satisfied with the government’s performance.

3A. What is the margin of sampling error?

3B. What is the likely interval that contains the percentage of the population that is satisfied with the government?
2 Measures of Variation  Measures of variation describe the dispersion or spread of a set of data. Two common measures of variation are the variance and standard deviation. These measures describe how closely a set of data clusters about the mean.

The sample mean \( \bar{x} \), read \( x \) bar, and the population mean \( \mu \), or mu, are calculated the same way. The formulas for calculating the sample standard deviation \( s \) and the population standard deviation \( \sigma \), or sigma, are given below.

### KeyConcept Standard Deviation Formulas

<table>
<thead>
<tr>
<th>Sample</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ s = \sqrt{\frac{\sum_{k=1}^{n} (x_k - \bar{x})^2}{n-1}} } ]</td>
<td>[ \sigma = \sqrt{\frac{\sum_{k=1}^{n} (x_k - \mu)^2}{n}} } ]</td>
</tr>
</tbody>
</table>

### Real-World Example 4  Standard Deviation

**TEST SCORES** The Chapter 3 and Chapter 4 scores from Mr. Hoff’s class both have a mean of 75. Find and compare their standard deviations.

#### a. Find the standard deviation for the Chapter 3 scores.

**Step 1** This is a population. Since the mean of each set was 75, \( \mu = 75 \).

**Step 2** Find the standard deviation.

\[
\sigma = \sqrt{\frac{\sum_{k=1}^{n} (x_k - \mu)^2}{n}}
\]

Standard Deviation Formula

\[
\approx \sqrt{\frac{(85 - 75)^2 + (80 - 75)^2 + \cdots + (75 - 75)^2}{23}} \approx 3.9
\]

The class mean of the Chapter 3 test is 75 with a standard deviation of about 3.9.

#### b. Use a calculator to find the standard deviation of the Chapter 4 scores.

Clear all lists. Then press [STAT] ENTER and enter each data value, pressing ENTER after each value.

To view the statistics, press [STAT] [1] ENTER.

The class mean of the Chapter 4 test is 75 with a standard deviation of about 36.

#### c. Compare the standard deviations of the two tests.

The standard deviation of the Chapter 4 test is far greater than for Chapter 3. Therefore, the scores are more dispersed in the Chapter 4 test, and they are much closer to the mean in the Chapter 3 test. Mr. Hoff can conclude that 75 is a stronger mean for Chapter 3, meaning that the majority of his students scored very close to 75.
Guided Practice

4A. Calculate the mean and standard deviation of the population of data.

4B. Change 30 to 70. What should happen to the mean and standard deviation? Recalculate to confirm your results.

Study Tip
Standard Deviation
The greater the standard deviation, the more the data deviate from the mean.

In a given set of data, the majority of the values fall within one standard deviation of the mean. Almost all of the data will fall within 2 standard deviations. Mr. Hoff’s Chapter 3 scores had a mean of 75 and a standard deviation \( \sigma \) of 3.9. We can illustrate this graphically on a number line.

If Mr. Hoff were to compare his students’ scores with other students throughout the country on a national test, the class would be considered a sample of all of the students who took the test. He would then need to calculate a sample mean \( \bar{x} \) and a sample standard deviation \( s \).

Check Your Understanding

Example 1
Which measure of central tendency best represents the data, and why?

1. \{833, 796, 781, 776, 758\}
2. \{27.2, 36.8, 50.4, 71.6, 194.7\}
3. \{65, 21, 17, 52, 25, 17, 11, 22, 60, 44\}
4. \{53, 61, 46, 59, 61, 55, 49\}

Example 2
Determine whether each of the following represents a population or a sample.

5. Jerry’s math club wants to compare their SAT scores to the scores of all students who took the SAT.
6. The tennis team wants to compare their first-serve percentages with each other.
7. Jennifer conducts an online survey on political opinions.
8. Veronica compares the student-teacher ratios of all of the schools in her county.

Example 3
OLYMPICS In a random survey of 5824 people, 29% said they will watch some of the Summer Olympics on television.

a. What is the margin of sampling error?

b. What is the likely interval that contains the percentage of the population that will watch the Summer Olympics on television?

Example 4
DRIVING The maximum speed limits in miles per hour for interstate highways are given.

a. Is this a sample or a population?

b. Find the standard deviation of the speeds.

<table>
<thead>
<tr>
<th>Maximum Speed Limits Per State</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
</tr>
<tr>
<td>55</td>
</tr>
<tr>
<td>65</td>
</tr>
<tr>
<td>75</td>
</tr>
<tr>
<td>70</td>
</tr>
</tbody>
</table>

Source: National Motorists Association
Example 1  Which measure of central tendency best represents the data, and why?

11. **NUTRITION**  The table shows the number of Calories per serving of each vegetable.

<table>
<thead>
<tr>
<th>Vegetable</th>
<th>Calories</th>
<th>Vegetable</th>
<th>Calories</th>
<th>Vegetable</th>
<th>Calories</th>
<th>Vegetable</th>
<th>Calories</th>
</tr>
</thead>
<tbody>
<tr>
<td>asparagus</td>
<td>14</td>
<td>broccoli</td>
<td>25</td>
<td>cauliflower</td>
<td>10</td>
<td>lettuce</td>
<td>9</td>
</tr>
<tr>
<td>beans</td>
<td>30</td>
<td>cabbage</td>
<td>17</td>
<td>celery</td>
<td>17</td>
<td>spinach</td>
<td>9</td>
</tr>
<tr>
<td>bell pepper</td>
<td>20</td>
<td>carrots</td>
<td>28</td>
<td>corn</td>
<td>66</td>
<td>zucchini</td>
<td>17</td>
</tr>
</tbody>
</table>

12. **WEATHER**  The table below shows daytime high temperatures for a week.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>64°F</td>
<td>73°F</td>
<td>69°F</td>
<td>70°F</td>
<td>71°F</td>
<td>75°F</td>
<td>74°F</td>
</tr>
</tbody>
</table>

Example 2  Determine whether each of the following represents a population or a sample.

13. Carissa calculates the average number of pineapples in 25 cans of pineapple.

14. The IRS calculates the mean income per household.

15. Middleburg Elementary School calculates the average height of all of its students.

16. Members of the football team want to compare their times in the 40 meter dash to those of the rest of the conference.

17. Jermaine asks 100 random people at the mall for their opinions on education.

18. The NFL compares the yards per game allowed by each team’s defense.

19. Tomás compares the populations of every state.

20. Dona asks 400 random people what their favorite season is.

Example 3  MOVIES  A survey of 5669 random people found that 31% go to the movies at least once a month.

- a. What is the margin of sampling error?
- b. What is the likely interval that contains the percentage of the population that goes to the movies at least once a month?

Example 4  DOGSLED  The Iditarod is a 1150-mile dogsled race across Alaska. At the right are the winning times, in days, for recent years.

- a. Is this a sample or a population?
- b. Find the standard deviation of the winning times.

Example 4  TRAINING  While training, Aiden recorded his times in the 40-meter dash. Find the standard deviation of the data.

<table>
<thead>
<tr>
<th>Iditarod Winning Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 9 10 9 9 8 9 9 9 9</td>
</tr>
<tr>
<td>17 15 15 14 12 16 13 13 18 12</td>
</tr>
<tr>
<td>11 11 11 11 13 11 11 11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>40-Meter Dash Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.8 4.9 4.8 4.7 5.0 4.9 4.8 4.9 4.8 5.0</td>
</tr>
<tr>
<td>5.0 5.1 4.8 4.9 4.6 4.8 4.7 4.9 4.8 4.8</td>
</tr>
<tr>
<td>5.0 4.9 4.9 5.0 4.9 5.0 4.8 4.8 4.7 4.6</td>
</tr>
</tbody>
</table>
25. **EDUCATION**  Below are ACT scores for a recent year.

<table>
<thead>
<tr>
<th>Mean ACT Scores by State</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.2</td>
</tr>
<tr>
<td>20.0</td>
</tr>
<tr>
<td>20.8</td>
</tr>
<tr>
<td>21.2</td>
</tr>
<tr>
<td>19.3</td>
</tr>
</tbody>
</table>

Source: ACT, Inc.

a. Compare the mean and median of the data.
b. Is this a sample or a population?
c. Find the standard deviation of the data. Round to the nearest hundredth.
d. Suppose the state with a mean score of 20.0 incorrectly reported the results. The score for the state is actually 22.5. How are the mean and median of the data affected by this change?

26. **STUDENT-TEACHER RATIOS**  The table at the right shows the number of students in every math class at Principal Johnson’s high school.

<table>
<thead>
<tr>
<th>Students Per Math Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
</tr>
<tr>
<td>24</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>28</td>
</tr>
<tr>
<td>23</td>
</tr>
</tbody>
</table>

a. Which measure of central tendency best represents the data? Why?
b. Is this a sample or a population?
c. Find the standard deviation of the data. Round to the nearest hundredth.

27. **VACATIONS**  The table shows the number of annual vacation days for nine countries. Which measure of central tendency best represents the data? Justify your selection, and then find the measure of central tendency.

<table>
<thead>
<tr>
<th>Annual Vacation Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Country</td>
</tr>
<tr>
<td>Brazil</td>
</tr>
<tr>
<td>Canada</td>
</tr>
<tr>
<td>France</td>
</tr>
<tr>
<td>Germany</td>
</tr>
<tr>
<td>Italy</td>
</tr>
</tbody>
</table>

Source: USA TODAY

**H.O.T. Problems**  Use Higher-Order Thinking Skills

28. **OPEN ENDED**  Find and analyze a set of univariate real-world data of interest to you. Describe its measures of central tendency and variation.

29. **CHALLENGE**  If 67% of the people surveyed responded positively and the likely interval that contains the percentage of the population is 64.8%–69.2%, how many people were surveyed?

30. **REASONING**  A large outlier is eliminated from a set of data. How does this affect the mean and the standard deviation of the data? Explain.

31. **REASONING**  With a linear transformation of data, all of the values are increased or decreased by the same value. If all of the values of the data are increased by 10, how does this affect the median, mean, and standard deviation? Explain.

32. **Writing in Math**  Compare and contrast the mean and median as measures of central tendency for a univariate data set.

33. **REASONING**  The East basketball team has an average height of 6 feet with a standard deviation of 1.1 inches. The West basketball team has an average height of 6 feet with a standard deviation of 4.1 inches. Compare and contrast the heights of the players on the two teams.
34. **STATISTICS** In a set of nine different numbers, which of the following cannot affect the value of the median?
   - A doubling each number
   - B increasing each number by 10
   - C increasing the smallest number only
   - D increasing the largest number only

35. **SHORT RESPONSE** The average of the test scores of a class of \(c\) students is 80, and the average test scores of a class of \(d\) students is 85. When the scores of both classes are combined, the average score is 82. What is the value of \(\frac{c}{d}\)?

36. **SAT/ACT** What is the multiplicative inverse of \(2i\)?
   - F \(-2i\)
   - G \(-2\)
   - H \(-\frac{i}{2}\)
   - J \(\frac{1}{2}\)
   - K \(\frac{i}{2}\)

37. Which equation best represents the graph?
   - A \(y = 4x\)
   - B \(y = x^2 + 4\)
   - C \(y = 4^{-x}\)
   - D \(y = -4^x\)

---

**Spiral Review**

State whether each survey would produce a random sample. Explain. (Lesson 12-1)

38. the government sending a tax survey to everyone whose social security number ends in a particular digit

39. finding the heights of all the boys on the varsity basketball team to determine the average height of all the boys in your school

40. **PARTIES** Suppose each time a new guest arrives at a party, he or she shakes hands with each person already at the party. Prove that after \(n\) guests have arrived, a total of \(\frac{n(n - 1)}{2}\) handshakes have taken place. (Lesson 11-7)

41. **ASTRONOMY** The orbit of Pluto can be modeled by the equation \(\frac{x^2}{39.5^2} + \frac{y^2}{38.3^2} = 1\), where the units are astronomical units. Suppose a comet is following a path modeled by the equation \(x = y^2 + 20\). (Lesson 10-7)
   a. Find the point(s) of intersection of the orbits of Pluto and the comet.
   b. Will the comet necessarily hit Pluto? Explain.
   c. Where do the graphs of \(y = 2x + 1\) and \(2x^2 + y^2 = 11\) intersect?
   d. What are the coordinates of the points that lie on the graphs of both \(x^2 + y^2 = 25\) and \(2x^2 + 3y^2 = 66\)?

---

**Skills Review**

Determine whether each situation involves a permutation or a combination. Then find the number of possibilities. (Lesson 0-5)

42. the winner of the first, second, and third prizes in a contest with 8 finalists

43. selecting two of eight employees to attend a business seminar

44. an arrangement of the letters in the word algebra

Conditional Probability

The probability of an event given that another event has already occurred is called **conditional probability**. The conditional probability that event \( B \) occurs given that event \( A \) has already occurred can be represented by \( P(B \mid A) \). This is read **the probability of B given A**.

**Key Concept**  Conditional Probability

Given that \( A \) and \( B \) are dependent events, the conditional probability of an event \( B \), given that an event \( A \) has already occurred, is defined as

\[
P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}, \text{ where } P(A) \neq 0.
\]

**Example 1  Conditional Probability**

Carolina rolls a six-sided die. What is the probability that she has rolled a 3 given that she has rolled an odd number?

There are 6 possible results of rolling a six-sided die.

Let event \( A \) be that she rolled an odd number.

Let event \( B \) be that she rolled a 3.

\[
P(A) = \frac{1}{2} \quad \text{Three of the six outcomes are odd numbers.}
\]

\[
P(A \text{ and } B) = \frac{1}{6} \quad \text{One of the six outcomes is 3 and odd.}
\]

\[
P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)} \quad \text{Probability of } B \text{ given } A
\]

\[
= \frac{1}{6} \div \frac{1}{2} \quad \text{or} \quad \frac{1}{3}
\]

\[
P(A) = \frac{1}{2} \quad \text{and} \quad P(A \text{ and } B) = \frac{1}{6}
\]

The probability of rolling a 3 given that the roll is odd is \( \frac{1}{3} \).

**Guided Practice**

1. Chen draws a card from a standard deck of 52 cards. Find the probability that he drew a king given that he drew a king, a queen, or a jack.
2 Contingency Tables  A contingency table or two-way table records data in which different possible situations result in different possible outcomes. Each value represents the relative frequency of an outcome. These tables can be used to find conditional probabilities.

**Real-World Example 2** Contingency Tables

**MEDICINE** Find the probability that a test subject stayed healthy, given that he or she used an experimental drug.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Number of Subjects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using Drug (D)</td>
<td></td>
</tr>
<tr>
<td>sick (S)</td>
<td>1600</td>
</tr>
<tr>
<td>healthy (H)</td>
<td>800</td>
</tr>
<tr>
<td>Using Placebo (P)</td>
<td></td>
</tr>
<tr>
<td>sick (S)</td>
<td>1200</td>
</tr>
<tr>
<td>healthy (H)</td>
<td>400</td>
</tr>
</tbody>
</table>

There is a total of 1600 + 800 + 1200 + 400 or 4000 people in the study. We need to find the probability of H given that D occurs.

$$P(H \mid D) = \frac{P(H \text{ and } D)}{P(D)}$$

$$= \frac{800 + 400}{4000} = \frac{1200}{4000} = \frac{800}{2400} = \frac{1}{3}$$

The probability that a subject stayed healthy given that he or she used the drug is $\frac{1}{3}$.

**Guided Practice**

2. Find the probability that a test subject remained healthy, if the placebo was used.

Contingency tables can be used to represent any number of possible situations.

**Test Example 3**

The table below shows the number of students who are varsity athletes. Find the probability that a student is a varsity athlete given he or she is a junior.

<table>
<thead>
<tr>
<th>Class</th>
<th>Freshman</th>
<th>Sophomore</th>
<th>Junior</th>
<th>Senior</th>
</tr>
</thead>
<tbody>
<tr>
<td>varsity</td>
<td>7</td>
<td>22</td>
<td>36</td>
<td>51</td>
</tr>
<tr>
<td>nonvarsity</td>
<td>269</td>
<td>262</td>
<td>276</td>
<td>257</td>
</tr>
</tbody>
</table>

**Math History Link**

Christian Huygens (1629–1695) This Dutchman was the first to discuss games of chance. “Although in a pure game of chance the results are uncertain, the chance that one player has to win or to lose depends on a determined value.” This became known as the expected value.
Check Your Understanding

Example 1  A bag contains 8 blue marbles, 6 red marbles, 10 yellow marbles, 6 white marbles, and 5 green marbles. A marble is chosen at random. Find each probability.

1. The marble is green, given that it is not blue.
2. The marble is red, given that it is not green.
3. The marble is yellow, given that it is not red or blue.
4. The marble is green or white, given that it is not red.

Example 2  5. DRIVING TESTS  The table shows how students in Mr. Diaz’s class fared on their first driving test. Some took a class to prepare, while others did not.

<table>
<thead>
<tr>
<th></th>
<th>Class</th>
<th>No Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passed</td>
<td>64</td>
<td>48</td>
</tr>
<tr>
<td>Failed</td>
<td>18</td>
<td>32</td>
</tr>
</tbody>
</table>

a. Find the probability that Paige passed, given that she took the class.
b. Find the probability that Elizabeth failed, given that she did not take the class.
c. Find the probability that Terrence did not take the class, given that he passed.

Example 3  6. MULTIPLE CHOICE  The number of students who have attended a football game at North Coast High School is listed below. Find the probability that a student who has attended a game is a junior or a senior.

<table>
<thead>
<tr>
<th>Class</th>
<th>Freshman</th>
<th>Sophomore</th>
<th>Junior</th>
<th>Senior</th>
</tr>
</thead>
<tbody>
<tr>
<td>attended</td>
<td>48</td>
<td>90</td>
<td>224</td>
<td>254</td>
</tr>
<tr>
<td>not attended</td>
<td>182</td>
<td>141</td>
<td>36</td>
<td>8</td>
</tr>
</tbody>
</table>

A 48.6%  B 77.6%  C 86.2%  D 91.6%

Practice and Problem Solving

Example 1  A tip jar contains 7 pennies, 15 nickels, 25 dimes, and 32 quarters. A coin is chosen at random. Find each probability.

7. The coin is a nickel, given that it is silver.
8. The coin is a quarter, given that it is not a dime.
9. The coin is a penny, given that it is not a quarter.
10. The coin is a dime or a quarter, given that it is silver.

Example 2  11. SCHOOL CLUBS  King High School tallied the number of males and females that were members of at least one after school club. Find each probability.

<table>
<thead>
<tr>
<th></th>
<th>Clubs</th>
<th>No Clubs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>156</td>
<td>242</td>
</tr>
<tr>
<td>Female</td>
<td>312</td>
<td>108</td>
</tr>
</tbody>
</table>

a. A student is a member of a club given that he is male.
b. A student is not a member of a club given that she is female.
c. A student is a male given that he is not a member of a club.

Example 3  12. MULTIPLE CHOICE  Naoko, Keisha, and Joshua compared the music on their MP3 players. Find the probability that a selected song is country given that it is not on Naoko’s player.

<table>
<thead>
<tr>
<th>Person</th>
<th>Rock</th>
<th>Country</th>
<th>R &amp; B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Naoko</td>
<td>521</td>
<td>316</td>
<td>44</td>
</tr>
<tr>
<td>Keisha</td>
<td>119</td>
<td>145</td>
<td>302</td>
</tr>
<tr>
<td>Joshua</td>
<td>244</td>
<td>4</td>
<td>182</td>
</tr>
</tbody>
</table>

F 17.2%  G 24.8%  H 35.9%  J 15.0%
A card is chosen at random from a standard deck. Find each probability. Assume that an ace represents a 1.

13. The card is a ten, given that it is red.
14. The card is a five, given that it is not a face card.
15. The card is a queen, given that it is a face card.
16. The card is greater than 7, given that it is not a face card.

**SOFTBALL** On average, Paloma gets a single 14% of the time, a double 6% of the time, a triple 1% of the time, a home run 13% of the time, and is walked 3% of the time.

17. What is the probability that Paloma gets a double, given that she is not out?
18. What is the probability that Paloma gets a triple or a home run, given that she is not walked?

**COMPUTER GAMES** The table shows a distribution of computer games sold by a company.

<table>
<thead>
<tr>
<th>Type</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>strategy</td>
<td>0.19</td>
</tr>
<tr>
<td>children’s</td>
<td>0.12</td>
</tr>
<tr>
<td>family</td>
<td>0.08</td>
</tr>
<tr>
<td>action</td>
<td>0.25</td>
</tr>
<tr>
<td>role playing</td>
<td>0.17</td>
</tr>
<tr>
<td>sports</td>
<td>0.16</td>
</tr>
<tr>
<td>other</td>
<td>0.03</td>
</tr>
</tbody>
</table>

19. a. Find the probability that a game is an action game, given that it is not a sports or role playing game.
19. b. Find the probability that a game is a family game, given that it is not a strategy or action game.

20. **FIRE DRILLS** Marburn High School will have a fire drill at a randomly-chosen time between 8:30 and 3:00. Mr. Woodruff has a test planned for 1:20 to 1:55. If the fire drill is in the afternoon, what is the probability that it will start during the test?

21. **FUNDRAISING** Mercedes and Victoria are trying to raise funds for their charity by calling numbers in the local phone book and asking for donations. They only reach 40% of the people they call. Of the people they reach, 20% promise to donate funds. Of the people who promise to donate, only 25% actually send money. What is the probability that a person who is called will actually contribute?

22. **HONORS CLASS** The probability that a student is in honors, given that he or she is in Mrs. Rollins’ class, is \( \frac{28}{51} \). The probability that a student is not in Mrs. Rollins’ class, given that he or she is not in honors, is \( \frac{33}{56} \). If there are 165 students that are neither in Mrs. Rollins’ class nor in honors, how many students are in Mrs. Rollins’ class and in honors?

**H.O.T. Problems** Use Higher-Order Thinking Skills

23. **CHALLENGE** The probability that a student has a MyRoom page, given that he or she is a freshman, is \( \frac{43}{55} \). The probability that a student does not have a MyRoom page, given that he or she is a sophomore, is \( \frac{4}{27} \). If there are 82 students, determine the probability that a student is a freshman, given that he or she does have a MyRoom page.

24. **WRITING IN MATH** Explain the difference between conditional probability for dependent events and conditional probability for independent events. Provide examples of each type.

25. **REASONING** Which branches of a tree diagram represent conditional probability? Provide a sample tree diagram and explain your reasoning.

26. **REASONING** If a fair coin is flipped 20 times in a row and comes up heads every single time, what is the probability that it comes up heads on the 21st flip? Explain your reasoning.

27. **OPEN ENDED** Create a contingency table and calculate a conditional probability using the students in your class.
28. GEOMETRY If the perimeter of an equilateral triangle is 45, then what is the length of the altitude of the triangle?
   A 9  C 7.5
   B $9\sqrt{3}$  D $7.5\sqrt{3}$

29. Which expression is equivalent to $\frac{1}{4} + \frac{1}{x} + \frac{1}{x}$?
   F $\frac{x + 1}{x + 4}$  H $\frac{4x + 4}{x + 4}$
   G 4  J $\frac{1}{4}$

30. SAT/ACT Aisha is late to practice 30% of the time each Wednesday because of French Club. What is the probability that she will be late on at least 3 of the next 5 Wednesdays?
   A 3%  D 18%
   B 13%  E 84%
   C 16%

31. SHORT RESPONSE If $\frac{12}{7} + \frac{15}{x} = 1$, what is the value of $x$?

32. FINANCIAL LITERACY The list shows the median income per capita in a recent year for 12 states in a region of the country. (Lesson 12-2)
   a. Compare the mean and median for the region.
   b. Find the standard deviation of the data. Round to the nearest hundredth.
   c. Suppose the state’s reported per capita income of $22,861 is incorrect, and the actual value is $24,861. How are the mean and median for the region affected?

Determine whether each situation would produce a random sample. Write yes or no and explain your answer. (Lesson 12-1)

33. surveying band members to find the most popular type of music at your school
34. surveying people coming into a post office to find out what color cars are most popular

35. ENTERTAINMENT A basketball team has a halftime promotion in which a fan gets to shoot a 3-pointer to try to win a jackpot. The jackpot starts at $5000 for the first game and increases $500 each time there is no winner. Ellis has tickets to the fifteenth game of the season. How much will the jackpot be for that game if no one wins by then? (Lesson 11-2)

36. Find $x$. Round to the nearest tenth if necessary. (Lesson 0-7)

37.

38.

39.

40.

41.
Probability

The probability of an event is a ratio that measures the chances of the event occurring. A desired outcome is called a **success**. Any other outcome is called a **failure**. The set of all possible outcomes is called the **sample space**. The closer the probability of an event is to 1, the more likely the event is to occur.

### Key Concept
**Probability of Success and Failure**

**Words**

If an event can succeed in \(s\) ways and fail in \(f\) ways, then the probabilities of success \(P(S)\) and of failure \(P(F)\) are as follows.

**Symbols**

\[
P(S) = \frac{s}{s + f} \quad P(F) = \frac{f}{s + f}
\]

### Example 1
**Probability with Combinations**

Twelve male and 16 female students have been selected as equal qualifiers for 6 college scholarships. If the qualifiers interviewed on the first day are to be chosen at random, what is the probability that 3 will be male and 3 will be female?

**Step 1**

Determine the number of successes.

\[12C_3 \quad \text{3 males chosen from 12 males}\]

\[16C_3 \quad \text{3 females chosen from 16 females}\]

Use combinations and the Fundamental Counting Principle to find \(s\).

\[12C_3 \cdot 16C_3 = \frac{12!}{9!3!} \cdot \frac{16!}{13!3!} = 123,200 \text{ possible groups}\]

**Step 2**

Determine the number of possibilities, \(s + f\).

\[28C_6 = \frac{28!}{22!6!} = 376,740 \text{ total possible groups}\]

**Step 3**

Find the probability.

\[P(\text{3 males and 3 females}) = \frac{s}{s + f} \quad \text{Probability of success}\]

\[
= \frac{123,200}{376,740} = 0.327016 \quad s = 123,200 \text{ and } s + f = 376,740
\]

Use a calculator.

The probability of selecting 3 males and 3 females is about 0.327016 or 33%.

### Guided Practice

1. Three juniors and eleven seniors have been nominated for 4 spots to represent the school at a city-wide charity event. If the winners are drawn at random, what is the probability that 2 juniors and 2 seniors are selected?
When a group of objects or people is arranged in a certain order, the arrangement is called a permutation. An arrangement or selection of objects in which order is not important is called a combination. (Lesson 0-5)

**Real-World Example 2** Probability with Permutations

**MUSIC** Courtney has a playlist of 6 songs on her MP3 player. What is the probability that the player will randomly play her favorite song first, then her second favorite song, and the three least favorite songs last?

**Step 1** Determine the number of successes.

\[
P_1 \quad \text{Play two favorite songs first and in order.}
\]
\[
P_3 \quad \text{Play the least favorite songs last, but in any order.}
\]

Use permutations and the Fundamental Counting Principle to find \( s \).

\[
P_1 \cdot P_3 = 1! \cdot 3! = 6
\]

**Step 2** Determine the number of possibilities, \( s + f \).

\[
P_6 = 6! = 720 \text{ possible orders of 6 songs}
\]

**Step 3** Find the probability.

\[
P(\text{Courtney’s desired order}) = \frac{s}{s + f} = \frac{6}{720} = 0.0083 \quad \text{Use a calculator.}
\]

The probability of the songs playing in Courtney’s desired order is about 0.8%.

**Guided Practice**

2. **RACING** Taryn, Stephanie, and Julie are in the 400-meter race with 5 other athletes. What is the probability that they all finish in the top three?

Sometimes, permutations and combinations are both used in determining a probability.

**Example 3** Probability with Combinations and Permutations

Suppose Hernanda pulls 5 marbles without replacement from a bag of 28 marbles in which 7 are red, 7 are black, 7 are blue, and 7 are white. What is the probability that 2 are of one color and 3 are of another color?

**Step 1** Determine the number of successes.

\[
P_2 \quad 2 \text{ colors chosen from 4 if order matters}
\]
\[
C_2 \quad 2 \text{ marbles of one color chosen from a group of 7}
\]
\[
C_3 \quad 3 \text{ marbles of another color chosen from a group of 7}
\]

Use permutations and combinations, along with the Fundamental Counting Principle, to find \( s \).

\[
P_2 \cdot C_2 \cdot C_3 = 12 \cdot 21 \cdot 35 = 8820
\]

**Step 2** Determine the number of possibilities, \( s + f \).

\[
C_5 = 98,280 \text{ ways to pull 5 marbles from a bag of 28}
\]

**Step 3** Find the probability.

\[
P(2 \text{ of one color, } 3 \text{ of another}) = \frac{s}{s + f} = \frac{8820}{98,280} = 0.0897 \quad \text{Use a calculator.}
\]

The probability of pulling 2 of one color and 3 of another is about 9%.
**Guided Practice**

3. If 7 green marbles are added to the bag, what is the probability that Hernanda pulls out 4 of one color and 3 of another?

**Probability Distributions**  The value of a random variable is the numerical outcome of a random event. A probability distribution for a particular random variable is a function that maps the sample space to its probabilities of the outcomes in the sample space.

A variable is said to be random if the sum of its probabilities is 1. The table below illustrates the probability distribution for rolling a die.

<table>
<thead>
<tr>
<th>Sample space: $R = {1, 2, 3, 4, 5, 6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
</tr>
<tr>
<td>Probability</td>
</tr>
</tbody>
</table>

$P(R = 4) = \frac{1}{6}$

A distribution in which all of the probabilities are equal is called a uniform distribution. For example, the distribution for rolling a die is uniform.

To help visualize a probability distribution, you can use a table or a bar graph or histogram of probabilities called a relative-frequency graph.

**Example 4  Probability Distribution**

The spinner shows the probability distribution of the spinner landing on each color.

a. Create a relative-frequency bar graph.

b. Use the graph to determine which outcome is most likely.
   The most likely outcome is purple, and its probability is $\frac{1}{4}$.

c. Find $P$(blue or green).
   The probability of spinning blue or green is $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$.

**Guided Practice**

Two six-sided dice are rolled, and their sum is recorded.

4A. Create a frequency table and a relative-frequency graph of the data.
4B. Which outcomes are the least likely to occur? What is their probability?
4C. Find $P$(5 or 11).
Probability distributions like the one in Example 4 are called **discrete probability distributions** because there are only a finite number of possible outcomes.

The probabilities discussed here are **theoretical probabilities** because they are based on assumptions of what is expected to happen. The **expected value** $E(x)$ is the weighted average of the values in a probability distribution if the weight applied to each value is its theoretical probability. It tells you what you could expect in the “long run”—that is, after many trials. This is not to be confused with **expected number**. When there are multiple trials, the expected number of times an event occurs is the probability of success in one trial multiplied by the total number of trials.

### Example 5  Expected Value

**a.** A die is rolled. Find the expected value of one roll of the die.

$$E(x) = (1 \cdot \frac{1}{6}) + (2 \cdot \frac{1}{6}) + (3 \cdot \frac{1}{6}) + (4 \cdot \frac{1}{6}) + (5 \cdot \frac{1}{6}) + (6 \cdot \frac{1}{6})$$  

**Weighted Average Formula**

$$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6}$$  

$$= \frac{21}{6} = 3.5$$  

**Multiply.**

**b.** If the die is rolled 4 times, find the expected number of even rolls.

$$E(n) = P(S) \cdot n$$  

**Expected number**

$$= \frac{1}{2} \cdot 4 = 2$$  

**Multiply.**

### Guided Practice

5. Find the expected value of the sum of two dice. What is the expected number of 7s in 100 rolls?

### Check Your Understanding  

**Example 1** **ART** A museum curator at the Art Institute of Chicago is randomly selecting 4 paintings out of the 20 on display to showcase the work in a special exhibit. What is the probability that 3 of the 8 Paul Gauguin paintings are selected?

**Example 2** **TOURNAMENTS** Eight players entered a tournament. If the names are drawn randomly, what is the probability that the first four players selected are, in order, Alicia, Andrew, Marco, and Zack?

**Example 3** **CARDS** Suppose Justin draws 5 cards from a standard deck of 52 cards. What is the probability that those 5 cards contain 3 of one suit and two of another suit?

**Example 4** **FLOWERS** The relative-frequency graph shows the distribution of the number of red flowers if 4 seeds are planted.

- **a.** Find $P(R = 0)$.
- **b.** What is the probability that at least 2 are red?
- **c.** If ten pots are planted with 4 seeds each, how many would you expect to have 1 red flower?

**Example 5** **RAFFLES** The French Club sold 500 raffle tickets for $1 each. The first prize ticket will win $100, 2 second prize tickets will each win $10, and 5 third prize tickets each win $5. What is the expected value of a single ticket?
6. **DRAWINGS** Twenty-four students entered a random drawing for 10 new calculators. What is the probability that 3 of the 5 students who entered from Mr. Kline’s class won a calculator?

7. **RAFFLES** Fifty kids, including Lorena, Rebecca, and Melia, entered a raffle for 4 game consoles. What is the probability that two of these girls won?

8. **PERFORMANCES** During a magic show, the magician selects at random five members of the audience to assist in his performance. If there are 124 people in the audience, what is the probability that at least one of ten friends is selected?

9. **SEATING CHARTS** The new seating chart in Mr. Lian’s class of 26 students was randomly generated. What is the probability that Jamila, Candace, and Haley are in the first, second, and third seats, respectively?

10. **LOTTERIES** In a lottery, 3 numbers from 1 through 10 are drawn without replacement, and the person who selects the correct numbers in the order in which they are drawn wins the prize. If Eva buys 5 different tickets, what is the probability that she will win?

11. **BALLOONS** A package of 48 balloons contains an equal number of red, white, blue, and purple balloons. If Shelby is given a handful of them to blow up, what is the probability that she gets 3 balloons of one color and 4 of another color?

12. **TRIVIA CONTESTS** Ten students from every grade level at West High were invited to a district-wide trivia contest. At the contest, 6 students are randomly selected to be alternates. What is the probability that 4 of these students are seniors and 2 are sophomores?

13. **RAFFLES** The table and relative-frequency graph show the distribution of winning a raffle if 100 tickets are sold. There is 1 prize for first, 10 prizes for second, and 25 prizes for third. Find $P(Z > 0)$.

<table>
<thead>
<tr>
<th>Prize</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>no prize</td>
<td>0.64</td>
</tr>
<tr>
<td>1st</td>
<td>0.01</td>
</tr>
<tr>
<td>2nd</td>
<td>0.1</td>
</tr>
<tr>
<td>3rd</td>
<td>0.25</td>
</tr>
</tbody>
</table>

14. **MARBLES** Tai has a sack of 35 marbles. Eight are black, 12 are red, 9 are green, and the rest are white. Brianna pulls 2 marbles out of the bag.

   a. Create a frequency table and a relative-frequency graph of the data.
   b. Which outcome is the most likely to occur?
   c. Find $P$(black and green).

15. **CARDS** In a standard deck of 52 cards, there are 4 different suits.

   a. If jacks = 11, queens = 12, kings = 13, and aces = 1, what is the expected value of a card that is drawn from a standard deck?
   b. If you are dealt 7 cards, what is the expected number of spades?
16. **BILLIARD BALLS** In a rack of 16 billiard balls, there are 9 different colors, including the black eight ball and the white cue ball. Of the remaining 14 balls, 7 are striped and 7 are solid.

a. If 5 balls are randomly selected, what is the expected number of stripes?

b. If 4 balls are randomly selected, what is the expected number of white balls?

c. If the value of the cue ball is 0 and the other balls are numbered 1–15, what is the expected value of a randomly-selected ball?

17. **SNOW DAYS** The following probability distribution lists the probable number of snow days per school year at North High School. Use this information to determine the expected number of snow days per year.

<table>
<thead>
<tr>
<th>Number of Snow Days Per Year</th>
<th>Days</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0.02</td>
</tr>
</tbody>
</table>

18. **BASKETBALL** The distribution lists the probability of the number of upsets in the first round of a basketball tournament. Determine the expected number of upsets.

<table>
<thead>
<tr>
<th>Number of Upsets Per Year</th>
<th>Upsets</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1/32</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1/16</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3/32</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1/8</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1/8</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5/16</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>1/8</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>3/32</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>1/32</td>
</tr>
</tbody>
</table>

19. **STUDENT GOVERNMENT** Based on previous data, the probability distribution of the number of students running for class president per year is listed at the right. Determine the expected number of students who will run.

<table>
<thead>
<tr>
<th>Number of Students Running</th>
<th>Students</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.2</td>
</tr>
</tbody>
</table>

20. **MARBLES** In a bag of 25 marbles with an equal number of red, blue, green, black, and clear marbles, what is the probability of pulling out 4 of one color and 2 of another?

21. **CONTESTS** The bar graph at the right shows the probability of each student winning a prize.

<table>
<thead>
<tr>
<th>Probability</th>
<th>( \text{Aaron} )</th>
<th>( \text{Brett} )</th>
<th>( \text{Pablo} )</th>
<th>( \text{Damon} )</th>
<th>( \text{Brian} )</th>
<th>( \text{Damon} )</th>
<th>( \text{Tora} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>( \text{Damon} )</td>
<td>( \text{Brett} )</td>
<td>( \text{Aaron} )</td>
<td>( \text{Brett} )</td>
<td>( \text{Brian} )</td>
<td>( \text{Tora} )</td>
<td>( \text{Damon} )</td>
</tr>
<tr>
<td>0.25</td>
<td>( \text{Brett} )</td>
<td>( \text{Aaron} )</td>
<td>( \text{Damon} )</td>
<td>( \text{Brett} )</td>
<td>( \text{Brian} )</td>
<td>( \text{Tora} )</td>
<td>( \text{Damon} )</td>
</tr>
<tr>
<td>0.2</td>
<td>( \text{Brett} )</td>
<td>( \text{Aaron} )</td>
<td>( \text{Damon} )</td>
<td>( \text{Brett} )</td>
<td>( \text{Brian} )</td>
<td>( \text{Tora} )</td>
<td>( \text{Damon} )</td>
</tr>
<tr>
<td>0.15</td>
<td>( \text{Brett} )</td>
<td>( \text{Aaron} )</td>
<td>( \text{Damon} )</td>
<td>( \text{Brett} )</td>
<td>( \text{Brian} )</td>
<td>( \text{Tora} )</td>
<td>( \text{Damon} )</td>
</tr>
<tr>
<td>0.1</td>
<td>( \text{Brett} )</td>
<td>( \text{Aaron} )</td>
<td>( \text{Damon} )</td>
<td>( \text{Brett} )</td>
<td>( \text{Brian} )</td>
<td>( \text{Tora} )</td>
<td>( \text{Damon} )</td>
</tr>
<tr>
<td>0.05</td>
<td>( \text{Brett} )</td>
<td>( \text{Aaron} )</td>
<td>( \text{Damon} )</td>
<td>( \text{Brett} )</td>
<td>( \text{Brian} )</td>
<td>( \text{Tora} )</td>
<td>( \text{Damon} )</td>
</tr>
</tbody>
</table>

a. Who has the best chance to win, and what is the probability?

b. Which two players combined have the same chance of winning as Brian?

c. Who has a better chance of winning, Damon or Brett?

d. Find \( P(\text{Aaron or Pablo}) \).

e. Find \( P(\text{neither Damon nor Tora}) \).

22. **CARDS** Three eights, 2 tens, 4 sixes, 3 fives, 2 twos, and a three are drawn from a deck of cards. If one card is drawn from these cards, what is its expected value?
23. **VOLUNTEERING** Twenty girls and 25 boys sign up to volunteer at a shelter. If only eight students are allowed to go, what is the probability that 3 will be boys?

24. **HORSE RACING** In a race involving seven horses, Delsin randomly chose three horses to place first through third. What is the probability that he wins?

25. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate geometric probability.

   a. **Tabular** The spinner shown has a radius of 2.5 inches. Copy and complete the table below.

<table>
<thead>
<tr>
<th>Color</th>
<th>Probability</th>
<th>Sector Area</th>
<th>Total Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>orange</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>yellow</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>green</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>blue</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. **Verbal** Make a conjecture about the relationship between the ratio of the area of the sector to the total area and the probability of the spinner landing on each color.

   c. **Analytical** Consider the dartboard shown. Predict the probability of a dart landing in each area of the board. Assume that any dart thrown will land on the board and is equally likely to land at any point on the board.

**H.O.T. Problems** Use Higher-Order Thinking Skills

26. **ERROR ANALYSIS** Liana and Shannon created probability distributions for the sum of two spins on the spinner at the right. Is either of them correct? Explain your reasoning.

27. **REASONING** Determine whether the following statement is **true** or **false**. Explain your reasoning.

   *Theoretical probabilities are based on results of experiments.*

28. **OPEN ENDED** Create a discrete probability distribution that shows five different outcomes and their associated probabilities.
29. **GRIDDED RESPONSE**  The height \( f(x) \) of a bouncing ball after \( x \) bounces is represented by \( f(x) = 140(0.8)^x \). How many times higher is the first bounce than the fifth bounce?

30. **PROBABILITY**  Andres has a bag that contains 4 red, 6 yellow, 2 blue, and 4 green marbles. If he reaches into the bag and removes a marble without looking, what is the probability that it will not be yellow?

\[
\begin{align*}
A & \quad \frac{1}{8} \quad C \quad \frac{3}{8} \\
B & \quad \frac{1}{4} \quad D \quad \frac{5}{8}
\end{align*}
\]

31. **GEOMETRY**  Find the area of the shaded portion of the figure to the nearest square inch.

\[
\begin{align*}
F & \quad 79 \quad H \quad 589 \\
G & \quad 94 \quad J \quad 707
\end{align*}
\]

32. **SAT/ACT**  If \( x \) and \( y \) are positive integers, which of the following expressions is equivalent to \( \frac{(5^y)^y}{5^x} \)?

\[
\begin{align*}
A & \quad 1^y \\
B & \quad \pm 1 \\
C & \quad 5^y \\
D & \quad 5^{xy - 1} \\
E & \quad 5^{xy - x}
\end{align*}
\]

### Spiral Review

33. **JOBS**  A family’s job jar contains 8 cleaning jobs, 2 painting jobs, 10 yard jobs, and 6 repair jobs. Find the probability that a randomly-chosen job is a cleaning or painting job, given that it is not a yard job.  *(Lesson 12-3)*

Determine whether each of the following represents a population or a sample.  *(Lesson 12-2)*

34. Shenae calculates the average number of people on 50 bus rides.

35. The U.S. Census Bureau conducts a demographics survey every 10 years.

36. East State College calculates the average cost of tuition of the entire student body.

37. Jared conducts a survey in his department to determine what time employees usually arrive in the morning.

Find the first five terms of each geometric sequence described.  *(Lesson 11-3)*

\[
\begin{align*}
38. & \quad a_1 = 0.125, \quad r = 1.5 \\
39. & \quad a_1 = 0.5, \quad r = 2.5 \\
40. & \quad a_1 = 4, \quad r = 0.5 \\
41. & \quad a_1 = 12, \quad r = \frac{1}{3} \\
42. & \quad a_1 = 21, \quad r = \frac{2}{3} \\
43. & \quad a_1 = 80, \quad r = \frac{5}{4}
\end{align*}
\]

44. **COMMUNICATION**  A microphone is placed at the focus of a parabolic reflector to collect sound for the television broadcast of a football game. Write an equation for the cross section, assuming that the focus is at the origin, the focus is 6 inches from the vertex, and the parabola opens to the right. *(Lesson 10-2)*

Solve each equation. Check your solutions.  *(Lesson 8-4)*

\[
\begin{align*}
45. & \quad \log_9 x = \frac{3}{2} \\
46. & \quad \log_{\frac{1}{10}} x = -3 \\
47. & \quad \log_b 9 = 2
\end{align*}
\]

### Skills Review

Find each percent. Round to the nearest tenth.

\[
\begin{align*}
48. & \quad 65\% \text{ of } 27 \\
49. & \quad 89\% \text{ of } 120 \\
50. & \quad 11\% \text{ of } 30 \\
51. & \quad 25\% \text{ of } 373 \\
52. & \quad 77\% \text{ of } 200 \\
53. & \quad 30\% \text{ of } 48
\end{align*}
\]
State whether each survey would produce a random sample. Write yes or no. Explain. (Lesson 12-1)

1. Every other shopper coming out of a mall is surveyed to determine how many children they have.
2. Every tenth person in an office is surveyed to determine their feelings about their jobs.
3. Every other student in a high school is asked who their vote for Teacher of the Year is.
4. Henry surveys thirty random friends to determine who should be Homecoming Queen.

5. MULTIPLE CHOICE Determine which of the following statements show a causation. (Lesson 12-1)
   A. If you practice every day, you can become a professional basketball player.
   B. If you read your textbook, you will pass the test.
   C. If you apply for ten different jobs, you will get an offer from at least one.
   D. If you stand outside in the rain with no shelter, you will get wet.

State whether each situation represents an experiment or an observational study. If it is an experiment, identify the control group and the treatment group. Then determine whether there is bias. (Lesson 12-1)

6. Find 250 students, half of whom are on the honor roll, and compare their study habits.
7. Give a random half of the employees an extra hour lunch break every day and compare their attitudes toward work with their coworkers.

8. MULTIPLE CHOICE Determine which of the following represents a population. (Lesson 12-2)
   F. Mr. Noble compares 100 random times in the 400 meter run in his gym classes.
   G. Heather compares the ratings of every quarterback in the NFL.
   H. Omar completes an online survey regarding the state of education in the United States.
   J. A national newspaper sends out a survey with every paper asking for public opinion on the upcoming presidential election.

9. Which measure of central tendency best represents the data, and why? (Lesson 12-2)

   | Number of Years Playing an Instrument |
   |------------------|---|---|---|---|---|---|
   | 2                | 2 | 3 | 2 | 4 | 1 | 2 |
   | 2                | 3 | 1 | 3 | 4 | 2 | 1 |
   | 3                | 2 | 3 | 2 | 3 | 1 | 4 |
   | 2                | 3 | 4 | 1 | 1 | 1 | 0 |
   | 1                | 2 | 1 | 2 | 2 | 2 | 3 |

10. SCHOOL CLUBS The table below shows the number of students who took algebra in eighth grade and the number who took calculus in high school. Use this information to determine the probability of each of the following. (Lesson 12-3)

<table>
<thead>
<tr>
<th>Did take calculus</th>
<th>Did not take calculus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Did take algebra</td>
<td>48</td>
</tr>
<tr>
<td>Did not take algebra</td>
<td>42</td>
</tr>
<tr>
<td>6</td>
<td>144</td>
</tr>
</tbody>
</table>

   a. Lori took calculus given that she took algebra in eighth grade.
   b. Kenny did not take algebra in eighth grade given that he did not take calculus.

A spinner has 16 equal-sized spaces numbered 1 to 16. Determine each probability for a single spin. (Lesson 12-3)

11. The number is odd, given that it is greater than 3.
12. The number is greater than 7, given that it is even.
13. The number is divisible by 3, given that it is even.

14. FOOTBALL The number of freshmen that make the varsity roster for Eddie’s school each year is listed in the table below. Find the number of freshmen expected to make the next varsity roster. (Lesson 12-4)

<table>
<thead>
<tr>
<th>Year</th>
<th>Freshmen</th>
<th>Year</th>
<th>Freshmen</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>4</td>
<td>2004</td>
<td>4</td>
</tr>
<tr>
<td>2001</td>
<td>2</td>
<td>2005</td>
<td>2</td>
</tr>
<tr>
<td>2002</td>
<td>1</td>
<td>2006</td>
<td>3</td>
</tr>
<tr>
<td>2003</td>
<td>5</td>
<td>2007</td>
<td>2</td>
</tr>
</tbody>
</table>

15. RAFFLES Sixty students, including Michelle and her 8 friends, entered a raffle for 5 gift certificates. What is the probability that only Michelle or one of her friends wins a gift certificate? (Lesson 12-4)
Normal and Skewed Distributions  In a **continuous probability distribution**, the outcome can be any value in an interval of real numbers and is best represented by a curve. The most common example is the **normal distribution**.

**Key Concept  Characteristics of the Normal Distribution**

- The maximum occurs at the mean. The mean, median, and mode are equal.
- The distribution can extend from negative infinity to positive infinity, but never touches the x-axis.
- The population mean μ and standard deviation σ are used to determine probabilities. Probabilities are cumulative and are expressed as inequalities.
- Because the area under the normal curve represents probabilities, this area is 1.

While the normal distribution is continuous, discrete distributions like the one above can have a normal shape. Distributions with other shapes are called **skewed distributions**.

**Example 1  Classify a Data Distribution**

Determine whether the following data appear to be positively skewed, negatively skewed, or normally distributed.

**a.**

<table>
<thead>
<tr>
<th>10</th>
<th>12</th>
<th>13</th>
<th>15</th>
<th>13</th>
<th>15</th>
<th>14</th>
<th>16</th>
<th>15</th>
<th>18</th>
<th>16</th>
<th>18</th>
<th>19</th>
<th>16</th>
<th>14</th>
<th>13</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>16</td>
<td>15</td>
<td>14</td>
<td>18</td>
<td>17</td>
<td>11</td>
<td>19</td>
<td>17</td>
<td>18</td>
<td>13</td>
<td>15</td>
<td>14</td>
<td>21</td>
<td>14</td>
<td>15</td>
<td>17</td>
</tr>
</tbody>
</table>

Use the frequency table to make a graph. Since the graph is high in the middle and appears to be somewhat symmetric, the data are normally distributed.
The Normal Distribution

Study Tip
Discrete vs Continuous
A discrete probability distribution is defined for a discrete number of values that are usually integers. A continuous probability distribution is defined for an infinite number of points over a continuous interval. The probability at a single point is always zero.

Study Tip
Normal Distributions
In all of these cases, the number of data values must be large for the distribution to be approximately normal.

Guided Practice
1. Determine whether the data at the right appear to be positively skewed, negatively skewed, or normally distributed.

ACT Scores

Guided Practice
2. Find the probability that a randomly selected value in the distribution above is less than 49.

Example 2 Normal Distribution

A normal distribution of data has a mean of 34 and standard deviation of 5.

Find the probability that random value $x$ is greater than 24, that is, $P(x > 24)$.

$\mu = 34$ and $\sigma = 5$

The probability that a randomly selected value in the distribution is greater than $\mu - 2\sigma$, that is, $34 - 2(5)$ or 24, is the shaded area under the normal curve.

$$P(x > 24) = 13.5 + 34 + 34 + 13.5 + 2 + 0.5$$

$= 97.5\%$

Guided Practice
2. Find the probability that a randomly selected value in the distribution above is less than 49.
A sample that is normally distributed can be represented by the normal curve as if it were a population.

### Real-World Example 3  Normally Distributed Sample

**HEIGHTS** The heights of 1800 teenagers are normally distributed with a mean of 66 inches and a standard deviation of 2 inches.

**a.** About how many teens are between 62 and 70 inches?

Draw a normal curve.

62 and 70 are 2σ away from the mean. Therefore, about 95% of the data are between 62 and 70.

Since $1800 \times 95\% = 1710$, we know that about 1710 of the teenagers are between 62 and 70 inches tall.

**b.** What is the probability that a teenager selected at random has a height greater than 68 inches?

From the curve, values greater than 68 are more than 1σ from the mean. 13.5% are between 1σ and 2σ, 2% are between 2σ and 3σ, and 0.5% are greater than 3σ.

So, the probability that a teenager selected at random has a height greater than 68 inches is $13.5 + 2 + 0.5$ or 16%.

### Guided Practice

**GRADES** The grade-point averages of 1200 students at East High School are normally distributed with a mean of 2.6 and a standard deviation of 0.6.

3A. About how many students have a grade-point average between 2.0 and 3.2?

3B. What is the probability that a randomly selected student has an average less than 3.8?

---

**Check Your Understanding**

1. **ACT** The table at the right shows recent composite ACT scores. Determine whether the data appear to be positively skewed, negatively skewed, or normally distributed.

<table>
<thead>
<tr>
<th>Score</th>
<th>% of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>33–36</td>
<td>1</td>
</tr>
<tr>
<td>28–32</td>
<td>9</td>
</tr>
<tr>
<td>24–27</td>
<td>19</td>
</tr>
<tr>
<td>20–23</td>
<td>29</td>
</tr>
<tr>
<td>16–19</td>
<td>27</td>
</tr>
<tr>
<td>13–15</td>
<td>12</td>
</tr>
</tbody>
</table>

Source: ACT, Inc.

2. A normal distribution of data has a mean of 161 and standard deviation of 12. Find the probability that random value $x$ is less than 149, that is $P(x < 149)$.

3. **SCHOOL** Mr. Bash gave a quiz in his social studies class. The scores were normally distributed with a mean of 21 and a standard deviation of 2.

   **a.** What percent would you expect to score between 19 and 23?
   
   **b.** What percent would you expect to score between 23 and 25?
   
   **c.** What is the probability that a student scored between 17 and 25?
Example 1
Determine whether the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

4. Additionally, provide the table for 20 Most Visited National Parks:

<table>
<thead>
<tr>
<th>Visitors (millions)</th>
<th>Number of Parks</th>
</tr>
</thead>
<tbody>
<tr>
<td>3–4</td>
<td>10</td>
</tr>
<tr>
<td>4–5</td>
<td>2</td>
</tr>
<tr>
<td>5–6</td>
<td>2</td>
</tr>
<tr>
<td>6–7</td>
<td>1</td>
</tr>
<tr>
<td>7–8</td>
<td>1</td>
</tr>
<tr>
<td>8+</td>
<td>4</td>
</tr>
</tbody>
</table>

5. Additionally, provide the table for Tallest Buildings in the World:

<table>
<thead>
<tr>
<th>Stories</th>
<th>Number of Buildings</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–39</td>
<td>1</td>
</tr>
<tr>
<td>40–59</td>
<td>11</td>
</tr>
<tr>
<td>60–79</td>
<td>35</td>
</tr>
<tr>
<td>80–99</td>
<td>9</td>
</tr>
<tr>
<td>100+</td>
<td>6</td>
</tr>
</tbody>
</table>

Example 2
A normal distribution of data has each mean and standard deviation. Find each probability.

6. $\mu = 74, \sigma = 6, P(x > 86)$

7. $\mu = 13, \sigma = 0.4, P(x < 12.6)$

8. $\mu = 63, \sigma = 4, P(59 < x < 71)$

9. $\mu = 91, \sigma = 6, P(73 < x < 103)$

Example 3
10. **CAR BATTERIES** The useful life of a certain car battery is normally distributed with a mean of 100,000 miles and a standard deviation of 10,000 miles. The company makes 20,000 batteries a month.

   a. About how many batteries will last between 90,000 and 110,000 miles?

   b. About how many batteries will last more than 120,000 miles?

   c. About how many batteries will last less than 90,000 miles?

   d. What is the probability that if you buy a car battery at random, it will last between 80,000 and 110,000 miles?

11. **HEALTH** The cholesterol level for adult males of a specific racial group is normally distributed with a mean of 158.3 and a standard deviation of 6.6.

   a. About what percent of the males have cholesterol below 151.7?

   b. How many of the 900 men in a study have cholesterol between 145.1 and 171.5?

12. **FOOD** The shelf life of a particular snack chip is normally distributed with a mean of 180 days and a standard deviation of 30 days.

   a. About what percent of the product lasts between 150 and 210 days?

   b. About what percent of the product lasts between 180 and 210 days?

   c. About what percent of the product lasts less than 90 days?

   d. About what percent of the product lasts more than 210 days?

13. **VENDING** A vending machine dispenses about 8 ounces of coffee. The amount varies and is normally distributed with a standard deviation of 0.3 ounce.

   a. What percent of the time will you get more than 8 ounces of coffee?

   b. What percent of the time will you get less than 8 ounces of coffee?

   c. What percent of the time will you get between 7.4 and 8.6 ounces of coffee?
14. **FINANCIAL LITERACY**  The insurance industry uses various factors including age, type of car driven, and driving record to determine an individual’s insurance rate. Suppose insurance rates for a sample population are normally distributed.

a. If the mean annual cost per person is $829 and the standard deviation is $115, what is the range of rates you would expect 68% of the population to pay annually?

b. If 900 people were sampled, how many would you expect to pay more than $1059 annually?

c. Where on the distribution would you expect a person with several traffic citations to lie? Explain your reasoning.

d. How do you think auto insurance companies use each factor to calculate risk?

15. **RAINFALL**  Use the table at the right.

<table>
<thead>
<tr>
<th>City</th>
<th>Precipitation (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albuquerque</td>
<td>9</td>
</tr>
<tr>
<td>Boise</td>
<td>12</td>
</tr>
<tr>
<td>Phoenix</td>
<td>8</td>
</tr>
<tr>
<td>Reno</td>
<td>7</td>
</tr>
<tr>
<td>Salt Lake City</td>
<td>17</td>
</tr>
<tr>
<td>San Francisco</td>
<td>20</td>
</tr>
</tbody>
</table>

a. Find the mean.

b. Find the standard deviation.

c. If the data are normally distributed, what percent of the time will annual precipitation in these cities be between 7.36 and 16.98 inches?

---

**H.O.T. Problems**  Use Higher-Order Thinking Skills

16. **ERROR ANALYSIS**  A set of normally distributed tree diameters have mean 11.5 cm, standard deviation 2.5, and range 3.6 to 19.8. Monica and Hiroko are to find the range that represents the middle 68% of the data. Is either of them correct? Explain.

- **Monica**
  - The data span 16.2 cm. 68% of 16.2 is about 11 cm. Center this 11-cm range around the mean of 11.5 cm. This 68% group will range from about 6 cm to about 17 cm.

- **Hiroko**
  - The middle 68% span from \( \mu + \sigma \) to \( \mu - \sigma \). So we move 2.5 cm below 11.5 and then 2.5 cm above 11.5.
  - The 68% group will range from 9 cm to 14 cm.

17. **CHALLENGE**  A case of digital audio players has an average battery life of 8.0 hours with a standard deviation of 0.7 hour. Eight of the players have a battery life greater than 10.1 hours. If the sample is normally distributed, how many players are in the case?

18. **WRITING IN MATH**  Explain the difference between *positively skewed*, *negatively skewed*, and *normally distributed* sets of data and describe an example of each.

19. **REASONING**  True or false: According to the Empirical Rule, in a normal distribution, most of the data will fall within one standard deviation of the mean. Explain.

20. **OPEN ENDED**  Find a real-world data set that appears to represent a normal distribution and one that does not. Describe the characteristics of each distribution. Create a visual representation of each set of data.

21. **OPEN ENDED**  Provide examples of a discrete probability distribution and a continuous probability distribution. Describe the differences between them.

22. **REASONING**  The term *six sigma process* comes from the notion that if one has six standard deviations between the mean of a process and the nearest specification limit, there will be practically no items that fail to meet the specifications. Is this a true assumption? Explain.
23. The lifetimes of 10,000 light bulbs are normally distributed. The mean lifetime is 300 days, and the standard deviation is 40 days. How many light bulbs will last between 260 and 340 days?

A 2500  C 5000
B 3400  D 6800

24. Which description best represents the graph?

F negatively skewed  H normal distribution
G no correlation  J positively skewed

25. SHORT RESPONSE In the figure below, RT = TS and QR = QT. What is the value of x?

26. SAT/ACT The integer 99 can be expressed as a sum of \( n \) consecutive positive integers. The value of \( n \) could be which of the following?

I. 2
II. 3
III. 6
A I only  D I and II only
B II only  E I, II, and III
C III only

Spiral Review

27. DOGS Three spaniels and eleven retrievers have been nominated for 4 spots to visit people at a hospital. If the winners are drawn at random, what is the probability that 2 spaniels and 2 retrievers are selected? (Lesson 12-4)

28. GAMES To begin a game, Kaylee, Abbey, and Brent agree to roll two dice and the player who rolls the greatest sum will go first. Kaylee rolled a sum of 5 and Abbey rolled a sum of 7. Assuming there is no tie, what is the probability that Brent will go first? (Lesson 12-3)

29. BRIDGES The Sydney Harbour Bridge connects the Sydney central business district to northern metropolitan Sydney. It has an arch in the shape of a parabola that opens downward. Write an equation of a parabola to model the arch, assuming that the origin is at the surface of the water, beneath the vertex of the arch. (Lesson 10-2)

Identify the type of function represented by each graph. (Lesson 2-7)

30. 

31. 

32. 

Skills Review

Find the standard deviation for each sample set of data. (Lesson 12-2)

33. \[\{3, 11, 27, 14, 19, 19, 2, 33, 16, 12\}\]
34. \[\{45, 47, 49, 49, 51, 53, 46, 47, 50, 48\}\]
35. \[\{320, 400, 350, 410, 380, 390, 330, 400, 370, 360\}\]
36. \[\{505, 527, 512, 517, 509, 513, 522, 520, 516, 511\}\]
If you know the mean and standard deviation of a normal distribution, you know that about 68%, 95%, and 99% of the data are within 1, 2, and 3 standard deviations of the mean, respectively. This is called the **Empirical Rule**. You can use the Empirical Rule to report percentiles. A **percentile** describes what percent of the data were at or below a given level.

Here is some additional information about percentiles.

- Percentiles measure rank from the bottom.
- There is no 0 percentile rank. The lowest score is at the 1st percentile.
- There is no 100th percentile rank. The highest score is at the 99th percentile.

**Activity**

A county-wide math contest was held for students in grades 9–12. Participants took at least three different tests during the competition. For the Problem-Solving Test, the scores were normally distributed with a mean of 30 and a standard deviation of 5.

**Step 1** Draw a normal curve for the Problem-Solving Test scores, similar to the one shown at the right. Label the mean and the mean plus or minus multiples of the standard deviation. Label the percent as shown.

**Step 2** The score of 30 is the mean. Looking at the diagram you can see that 50% of the scores are at or below the score of 30. You can say that a score of 30 is at the 50th percentile. What percent of the total scores was at or below a score of 25?

**Step 3** What percent of the total scores was at or below a score of 40?

**Step 4** What score is at the 99th percentile?
What score is at the 0th percentile?

**Exercises**

Make a drawing similar to the drawing in Step 1. Then find the percentiles or scores.

1. For the geometry test, the scores were normally distributed with a mean of 15 and a standard deviation of 2. Find the percentiles for the following scores: 21, 15, 13, 9.

2. For the chemistry test, the scores were normally distributed with a mean of 40 and a standard deviation of 4. Find the scores for the following percentiles: 99th, 2nd, 50th, 84th.
Hypothesis Testing

While a distribution can provide general data about populations, it cannot give you any specifics. You can use inferential statistics to draw conclusions about a population by using a sample. When you use information from a sample to draw conclusions about the entire population, you are making a statistical inference.

To account for differences between sample statistics and population parameters, you can use an estimate. A confidence interval is an estimate of a population parameter stated as a range with a specific degree of certainty. Typically, statisticians use 90%, 95%, and 99% confidence intervals, but any other percentage can be considered. The most often used interval is 95%.

Key Concept 95% Confidence Interval Formula

A 95% confidence interval estimate can be found by using the formula $CI = \bar{x} \pm 1.96 \cdot \frac{s}{\sqrt{n}}$, where $\bar{x}$ is the mean of the sample, $s$ is the standard deviation of the sample, and $n$ is the size of the sample.

These intervals are an estimation of the mean of the population, $\mu$.

Real-World Example 1 Find Confidence Intervals

SCHOOL WORK A sample of 200 students was asked for the average amount of time they spend on their homework during a week night. The mean time was 52.5 minutes with a standard deviation of 5.1 minutes. Determine a 95% confidence interval.

$$CI = \bar{x} \pm 1.96 \cdot \frac{s}{\sqrt{n}}$$

$$= 52.5 \pm 1.96 \cdot \frac{5.1}{\sqrt{200}}$$

$$\approx 52.5 \pm 0.71$$

The 95% confidence interval is $51.79 \leq \mu \leq 53.21$.

Guided Practice

1. Find a 95% confidence interval for $\bar{x} = 48.3$, $s = 6.4$, and $n = 80$. 

New Vocabulary

- inferential statistics
- statistical inference
- confidence interval
- hypothesis
- null hypothesis
- alternative hypothesis
2 **Hypothesis Testing** A hypothesis is an assumption that can be verified by testing. A specific hypothesis to be tested is called the **null hypothesis** \( H_0 \) (read \( H \) null). It is expressed as an equality and is considered true until evidence indicates otherwise.

If you conclude that the null hypothesis is false, then the alternative hypothesis must be true. The **alternative hypothesis** \( H_1 \) is mutually exclusive to the null hypothesis. It is stated as an inequality using \(<, \leq, \neq, \geq, \text{ or } >\). The alternative hypothesis represents the conclusion reached by rejecting the null hypothesis. Use these steps to test a hypothesis.

### Key Concept: Hypothesis Testing

<table>
<thead>
<tr>
<th>Step</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>State the null hypothesis ( H_0 ) and the alternative hypothesis ( H_1 ).</td>
</tr>
<tr>
<td>2</td>
<td>Design the experiment.</td>
</tr>
<tr>
<td>3</td>
<td>Conduct the experiment and collect the data.</td>
</tr>
<tr>
<td>4</td>
<td>Find the confidence interval.</td>
</tr>
<tr>
<td>5</td>
<td>Make the correct statistical inference. Accept the null hypothesis if the population parameter falls into the confidence interval.</td>
</tr>
</tbody>
</table>

### Real-World Example 2 Hypothesis Test

**STUDENT COUNCIL** Lindsey, the president of the junior class, has heard complaints that the cafeteria lunch line moves too slowly. The dining services coordinator assures her that the average wait time is 6 minutes. The students think it is much longer. Test the hypothesis that the average wait time is 6 minutes.

**Step 1** State the hypotheses: \( H_0: \mu = 6 \) and \( H_1: \mu > 6 \).

**Step 2** Design the experiment.
Lindsey will collect data and decide whether the data provide evidence for or against the null hypothesis. The results must differ enough from the null hypothesis to reject it, so Libby will use a 95% confidence level.

**Step 3** Conduct the experiment and collect the data.
Lindsey selected a random sample of 40 students and measured their wait times. She found that \( \bar{x} = 7.3 \) and \( s = 2.821 \).

**Step 4** Find the confidence interval.

\[
CI = \bar{x} \pm 1.96 \cdot \frac{s}{\sqrt{n}}
\]

Confidence Interval Formula

\[
= 7.3 \pm 1.96 \cdot \frac{2.821}{\sqrt{40}} = 6.426 \text{ or } 8.174
\]

Use a calculator.

This means that 95% of the time, the experiment will produce a mean between 6.426 and 8.174.

**Step 5** Make the correct statistical inference.
The 95% confidence interval does not include \( H_0 \), so Lindsey can reject the null hypothesis. She can assume that the average wait time is longer than 6 minutes.

### Guided Practice

2. **FIRE DRILLS** Ms. Guzman was told that it takes an average of 2 minutes to evacuate the building during a fire drill. She believes it takes longer. After collecting data for 20 fire drills, she arrives at a mean of 3.2 minutes and a standard deviation of 0.9 minute. Test the hypothesis that the average length of time is greater than 2 minutes.
Check Your Understanding

Example 1  
Find a 95% confidence interval for each of the following.
1. \( \bar{x} = 90, s = 5.6, \text{ and } n = 50 \)  
2. \( \bar{x} = 72, s = 4.6, \text{ and } n = 100 \)  
3. \( \bar{x} = 84, s = 3.5, \text{ and } n = 120 \)  
4. \( \bar{x} = 62.5, s = 2.3, \text{ and } n = 150 \)  
5. VIDEO GAMES  
A sample of 100 students was asked for the average amount of time they spend playing video games each day. The mean time was 75 minutes with a standard deviation of 5.1 minutes. Determine a 95% confidence interval.

Example 2  
Test each null hypothesis. Write accept or reject.
6. \( H_0 = 10, H_1 < 10, n = 50, \bar{x} = 8.75, \text{ and } s = 0.9 \)  
7. \( H_0 = 48.8, H_1 > 48.8, n = 100, \bar{x} = 49, \text{ and } s = 1.5 \)  
8. \( H_0 = 75, H_1 > 75, n = 150, \bar{x} = 77, \text{ and } s = 2 \)  
9. \( H_0 = 90, H_1 > 90, n = 200, \bar{x} = 93, \text{ and } s = 3.5 \)  
10. SWIMMING  
Yolanda’s average time for the 400-meter butterfly was 8 minutes. She wants to test to see if that time is still accurate. After timing herself for 25 drills, she came to a mean of 8 minutes 10 seconds and a standard deviation of 30 seconds. Test the hypothesis that the average time is 8 minutes.

Practice and Problem Solving
Extra Practice begins on page 947.

Example 1  
Find a 95% confidence interval for each of the following.
11. \( \bar{x} = 26, s = 3.7, \text{ and } n = 180 \)  
12. \( \bar{x} = 47, s = 5.9, \text{ and } n = 200 \)  
13. \( \bar{x} = 58, s = 7.1, \text{ and } n = 225 \)  
14. \( \bar{x} = 66, s = 6.3, \text{ and } n = 250 \)  
15. \( \bar{x} = 92, s = 8.4, \text{ and } n = 300 \)  
16. \( \bar{x} = 74, s = 6.8, \text{ and } n = 350 \)  
17. FINANCIAL LITERACY  
A sample of 500 students was asked for the average amount of money they spend in a day. The mean amount was $6.55 with a standard deviation of $2.75.
   a. Determine a 95% confidence interval.
   b. Suppose the sample is expanded to 750 students, but the mean and standard deviation remain the same. Determine a new 95% confidence interval.
   c. How does a larger sample affect the confidence interval?

Example 2  
Test each null hypothesis. Write accept or reject.
18. \( H_0 = 14, H_1 < 14, n = 80, \bar{x} = 12.75, \text{ and } s = 0.8 \)  
19. \( H_0 = 64.2, H_1 > 64.2, n = 200, \bar{x} = 64, \text{ and } s = 2.5 \)  
20. \( H_0 = 95, H_1 > 95, n = 150, \bar{x} = 97, \text{ and } s = 1.5 \)  
21. \( H_0 = 50, H_1 < 50, n = 400, \bar{x} = 49.5, \text{ and } s = 0.9 \)  
22. \( H_0 = 81, H_1 > 81, n = 300, \bar{x} = 81.5, \text{ and } s = 3.4 \)  
23. \( H_0 = 72, H_1 < 72, n = 350, \bar{x} = 71.7, \text{ and } s = 4.1 \)  
24. WALKING  
Evan thought it took about 5 minutes to walk to school, while his sister Angela thought it took longer. They timed themselves for 40 days and calculated a mean of 5.8 minutes with a standard deviation of 0.6 minutes. Test the hypothesis.
**GAS MILEAGE** The manufacturer of Diana’s car claimed that the car averages 28 miles per gallon in the city, but Diana believes it is less than that. The following data represent her calculations for the last 30 tanks of gas for her car. Conduct a hypothesis test to see if she is correct.

<table>
<thead>
<tr>
<th>28.2</th>
<th>25.3</th>
<th>24.6</th>
<th>27.2</th>
<th>29.3</th>
<th>27.1</th>
<th>26.4</th>
<th>29.1</th>
<th>26.2</th>
<th>25.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>26.6</td>
<td>25.8</td>
<td>24.9</td>
<td>27.3</td>
<td>28.6</td>
<td>28.4</td>
<td>28.3</td>
<td>25.8</td>
<td>25.8</td>
<td>28.2</td>
</tr>
<tr>
<td>27.2</td>
<td>28.1</td>
<td>29.3</td>
<td>26.3</td>
<td>25.9</td>
<td>28.0</td>
<td>27.2</td>
<td>26.1</td>
<td>27.4</td>
<td>26.4</td>
</tr>
</tbody>
</table>

**QUALITY CONTROL** Grace is a quality tester for a manufacturing company. The company wants to claim that their new rechargeable battery lasts 8 hours. Grace tests 50 different batteries to see if they actually last fewer than 8 hours. Use the data below to conduct a hypothesis test.

<table>
<thead>
<tr>
<th>8.1</th>
<th>7.9</th>
<th>7.8</th>
<th>8.0</th>
<th>8.2</th>
<th>7.8</th>
<th>7.7</th>
<th>8.1</th>
<th>7.8</th>
<th>7.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.2</td>
<td>8.4</td>
<td>8.2</td>
<td>7.8</td>
<td>7.7</td>
<td>7.7</td>
<td>7.9</td>
<td>8.3</td>
<td>8.1</td>
<td>8.0</td>
</tr>
<tr>
<td>8.0</td>
<td>7.9</td>
<td>7.9</td>
<td>8.4</td>
<td>8.1</td>
<td>8.2</td>
<td>8.0</td>
<td>7.6</td>
<td>7.7</td>
<td>7.9</td>
</tr>
<tr>
<td>7.8</td>
<td>7.9</td>
<td>8.0</td>
<td>8.0</td>
<td>8.1</td>
<td>8.2</td>
<td>7.6</td>
<td>7.8</td>
<td>7.8</td>
<td>8.0</td>
</tr>
<tr>
<td>8.0</td>
<td>8.1</td>
<td>8.1</td>
<td>7.9</td>
<td>7.9</td>
<td>7.9</td>
<td>8.1</td>
<td>7.8</td>
<td>7.8</td>
<td>8.0</td>
</tr>
</tbody>
</table>

**COOKIES** A cookie manufacturer stated that there were 20 chocolate chips in every cookie. Lamar thought there were fewer than 20, so he tested 40 random cookies. Use the data below to conduct a hypothesis test.

<table>
<thead>
<tr>
<th>21</th>
<th>19</th>
<th>20</th>
<th>20</th>
<th>19</th>
<th>18</th>
<th>21</th>
<th>19</th>
<th>17</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>18</td>
<td>18</td>
<td>20</td>
<td>20</td>
<td>19</td>
<td>18</td>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td>21</td>
<td>21</td>
<td>19</td>
<td>17</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>22</td>
<td>21</td>
<td>21</td>
<td>20</td>
<td>20</td>
<td>19</td>
<td>17</td>
<td>17</td>
<td>20</td>
</tr>
</tbody>
</table>

**CANNED FOOD** The label on Leah’s can of sliced peaches promises 12 slices in every can. Leah decides to test her hypothesis that there are more than 12 slices in every can by finding the number in 40 random cans. Test her hypothesis.

<table>
<thead>
<tr>
<th>13</th>
<th>14</th>
<th>13</th>
<th>14</th>
<th>12</th>
<th>12</th>
<th>11</th>
<th>15</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>13</td>
<td>14</td>
<td>13</td>
<td>14</td>
<td>12</td>
<td>15</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>13</td>
<td>14</td>
<td>14</td>
<td>13</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>11</td>
<td>14</td>
<td>14</td>
<td>13</td>
<td>14</td>
<td>13</td>
<td>14</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

**H.O.T. Problems** Use Higher-Order Thinking Skills

29. **CHALLENGE** A 95% confidence interval for the mean weight of a 20-ounce box of cereal was $19.996 \leq \bar{x} \leq 20.004$ with a sample standard deviation of 0.128 ounces. Determine the sample size that led to this interval.

30. **WRITING IN MATH** Describe how the size of the sample affects hypothesis testing.

31. **REASONING** Determine whether the following statement is *sometimes, always, or never* true. Explain your reasoning.

   If a confidence interval contains $H_0$, then it is not rejected.

32. **OPEN ENDED** Conduct your own research study, and draw conclusions based on the results of a hypothesis test. Write a brief summary of your findings.

33. **CHALLENGE** If $H_0 = 85, H_1 > 85$, $\bar{x} = 85.5$, and $n = 300$, what is the minimum sample standard deviation for which the null hypothesis will be accepted with 95% confidence?
34. GEOMETRY  In the graph below, line \( \ell \) passes through the origin. What is the value of \( \frac{a}{b} \)?

- **A** 4
- **B** 1/4
- **C** -4
- **D** -1/4

![Graph with line passing through the origin](image)

35. SAT/ACT  If \( 5 + i \) and \( 5 - i \) are the roots of \( x^2 - 10x + c = 0 \), what is the value of \( c \)?

- **A** -26
- **B** 24
- **C** 26
- **D** -25

36. The Service Club at Corey’s school was founded 8 years ago. The number of members of the club by year is shown in the table. Which linear equation best models the data?

<table>
<thead>
<tr>
<th>Year</th>
<th>Participation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>8</td>
<td>22</td>
</tr>
</tbody>
</table>

- **A** \( y = 1.4x \)
- **B** \( y = 1.4x + 10.4 \)
- **C** \( y = 1.6x \)
- **D** \( y = 1.6x + 11.1 \)

37. GRIDDED RESPONSE  Solve for \( x \): \( \log_2 (x - 6) = 3 \).

Spiral Review

38. HEALTH  The heights of students at Madison High School are normally distributed with a mean of 66 inches and a standard deviation of 2 inches. Of the 1080 students in the school, how many would you expect to be less than 62 inches tall?  (Lesson 12-5)

39. RETAIL  The posters for 8 newly released DVDs can be displayed in a store window. If there are 6 new comedies, 9 new family movies, and 4 new dramas this week, what is the probability that 4 posters will be for comedies and 4 will be for family movies if the posters are chosen at random?  (Lesson 12-4)

Find \( a_n \) for each geometric sequence.  (Lesson 11-3)

40. \( a_1 = \frac{1}{3}, r = 3, n = 8 \)

41. \( a_1 = \frac{1}{64}, r = 4, n = 9 \)

42. \( a_1 = 16, r = 0.5, n = 8 \)

Write each equation in standard form. State whether the graph of the equation is a parabola, circle, ellipse, or hyperbola. Then graph the equation.  (Lesson 10-6)

43. \( 4x^2 + 2y^2 = 8 \)

44. \( x^2 = 8y \)

45. \( (x - 1)^2 - 9(y - 4)^2 = 36 \)

Write an equation in slope-intercept form for each graph.  (Lesson 2-4)

46. \( y = \frac{1}{2}x + 1 \)

47. \( y = -2x + 6 \)

48. \( y = \frac{1}{3}x - 2 \)

Skills Review

Expand each power.  (Lesson 11-7)

49. \( (a - b)^3 \)

50. \( (m + n)^4 \)

51. \( (r + n)^8 \)
A simulation uses a probability experiment to mimic a real-life situation. You can use a simulation to solve the following problem.

A fast food restaurant is offering one of six different food prize tickets on every soft drink cup. If the prizes are equally and randomly distributed, how many drinks, on average, would you have to buy in order to get at least one of each prize?

**Activity**

Work in pairs or small groups to complete Steps 1 through 4.

**Step 1** Use the six numbers on a die to represent the six different food prizes.

**Step 2** Roll the die and record which food prize was on the first soft drink cup. Use a tally sheet like the one shown at the right.

**Step 3** Continue to roll the die and record the prize number until you have a complete set of food prizes. Stop as soon as you have a complete set. This is the end of one trial in your simulation. Record the number of drinks required for this trial.

**Step 4** Repeat Steps 1, 2, and 3 until your group has carried out 25 trials. Use a new tally sheet for each trial.

<table>
<thead>
<tr>
<th>Simulation Tally Sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prize Number</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>Total Needed</td>
</tr>
</tbody>
</table>

**Analyze the Data**

1. Create two different statistical graphs of the data collected for 25 trials.

2. Determine the mean, median, maximum, minimum, and standard deviation of the total number of drinks needed in the 25 trials.

3. Combine the small-group results and determine the mean, median, maximum, minimum, and standard deviation of the number of drinks required for all the trials conducted by the class.

**Make a Conjecture**

4. If you carry out 25 additional trials, will your results be the same as in the first 25 trials? Explain.

5. Should the small-group results or the class results give a better idea of the average number of drinks required to get a complete set of food prizes? Explain.

6. If there were 8 prizes instead of 6, would you need to buy more drinks or fewer drinks on average?

7. **DESIGN A SIMULATION** What if one of the 6 prizes was more common than the other 5? For instance, suppose that one prize, a free ice cream sundae, appears on 25% of all the drinks and the other 5 prizes are equally and randomly distributed among the remaining 75% of the drinks. Design and carry out a new simulation to predict the average number of drinks you would need to buy to get a complete set. Include some measures of central tendency and dispersion with your data.
Binomial Distributions

Another type of discrete probability distribution is the binomial distribution. A binomial distribution shows the probabilities of the outcomes of a binomial experiment. A binomial experiment is a random experiment with an outcome that is one of two simple events. In a binomial experiment, the following are true.

- There are only two possible outcomes, success or failure.
- There is a fixed number of trials, \( n \).
- The probability of success is the same in every trial.
- The trials are independent.
- The random variable is the number of successes in \( n \) trials.

The experimental probability is what is estimated from observed simulations or experiments. When a simulation is conducted, the observed data are analyzed and the experimental probability is determined from these results.

### Example 1 Design a Binomial Experiment

In a certain dice game, a player tries to roll a total of 7 or 11 with two dice. Design and conduct a binomial experiment for 10 rolls of the dice.

**Step 1** Describe the trial for the situation.
Each roll is a trial. There will be 10 trials.

**Step 2** Describe a success. What is the probability of a success?
A success is rolling 7 or 11. The probability of success is \( \frac{6}{36} + \frac{2}{36} = \frac{2}{9} \).

**Step 3** Design and conduct a simulation to determine the experimental probability of rolling a 7 or 11 at least two out of ten times. Let \( s \) represent success, and let \( f \) represent failure.

<table>
<thead>
<tr>
<th>Roll</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Total Successes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation 1</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>s</td>
<td>f</td>
<td>f</td>
<td>s</td>
<td>f</td>
<td>2</td>
</tr>
<tr>
<td>Simulation 2</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>s</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>1</td>
</tr>
<tr>
<td>Simulation 3</td>
<td>s</td>
<td>s</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>2</td>
</tr>
<tr>
<td>Simulation 4</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>s</td>
<td>s</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>2</td>
</tr>
</tbody>
</table>

Three of the four simulations produced at least 2 successes, so the experimental probability of rolling a 7 or 11 twice in 10 rolls is 75%.
Guided Practice

1. Design and conduct an experiment, and then find the experimental probability of a fair coin landing on heads 6 out of 10 tosses.

Binominal distributions are often represented graphically, usually with a tree diagram.

Real-World Example 2  Find a Probability

STATE FAIR  Antonia earned 3 prize tokens for shooting baskets at the state fair. According to the advertisement, 30% of the tokens win prizes. Find the probability that exactly 2 of Antonia’s tokens win a prize.

Each token has a probability of success of 0.3. The probability of failure is 1 – 0.3 or 0.7. The tree diagram shows all of the possibilities and the probability of each.

Three distinct branches of the tree indicate exactly two successes. The sum of the probabilities of these branches will produce the overall probability.

\[
0.3 \cdot 0.3 \cdot 0.7 + 0.3 \cdot 0.7 \cdot 0.3 + 0.7 \cdot 0.3 \cdot 0.3 = 0.189 \text{ or } 18.9% 
\]

Guided Practice

2. Becky bought 5 game cards at the store. Each game card has a 10% chance of winning. Find the probability that at least 2 of her game cards are winners.

2 Binominal Distribution  The tree diagram in Example 2 is an example of a binominal distribution. This distribution can be simplified by the following formula.

Key Concept  Binominal Distribution Functions

The probability of \( x \) successes in \( n \) independent trials is

\[
P(x) = \binom{n}{x} s^x f^{n-x},
\]

where \( s \) is the probability of success of an individual trial and \( f \) is the probability of failure on that same individual trial \((s + f = 1)\).

The expected value of a binominal distribution can also be determined.

Key Concept  Expected Value of a Binominal Distribution

The expected value for a binominal distribution is \( E(X) = ns \), where \( n \) is the total number of trials and \( s \) is the probability of success.
StudyTip

Selecting Probabilities
Sometimes it is easier to find the probability of failure and subtract from 1 to get the probability of success. These probabilities are complements of each other.

StudyTip

Expected Values and Probability
A probability is not a guarantee that something will occur. For example, if the probability shows that a ball team is expected to win 3 of the next 5 games, it does not mean they will.

Example 3 Binomial Probability

A chocolate company makes boxes of assorted chocolates, 40% of which are dark chocolate on average. The production line mixes the chocolates randomly and packages 10 per box.

a. What is the probability that at least 3 chocolates in a box are dark?

A success is a dark chocolate, so \( s = 0.4 \) and \( f = 1 - 0.4 \) or 0.6.

Calculate the probability of the box having exactly 0, 1, or 2 dark chocolates, and then subtract that sum from 1.

\[
P(\geq 3 \text{ dark chocolates}) = 1 - P(< 3 \text{ dark chocolates})
\]

\[
= 1 - [P(0) + P(1) + P(2)]
\]

\[
= 1 - [C(10, 0)(0.4)^0(0.6)^{10} + C(10, 1)(0.4)^1(0.6)^9 + C(10, 2)(0.4)^2(0.6)^8]
\]

\[
= 1 - 0.1673 \text{ or } 0.8327
\]

The probability of at least 3 chocolates being dark is 0.8327 or 83.27%.

b. What is the expected number of dark chocolates in a box?

\[
E(X) = np
\]

\[
= 10(0.4) \text{ or } 4
\]

The expected number of dark chocolates in a box is 4.

Guided Practice

3. If 20% of the chocolates are white chocolates, what is the probability that at least one chocolate in a given box of 10 is a white chocolate?

You can find the full probability distribution for a binomial experiment by expanding the binomial.

Example 4 Full Probability Distribution

Autumn ran out of time when she took her multiple-choice test so she randomly circled answers for the last 5 questions. Each question had 5 possible choices. Determine the probabilities associated with the number of answers she got correct on the last 5 questions.

We are asked to find the probability for each possible number of correct answers on the 5 she guessed on.

Expand the binomial \((s + f)^n\) with \(n = 5\) questions.

There are five equal possibilities for each question, so \( s = \frac{1}{5} \) or 0.2 and \( f = \frac{4}{5} \) or 0.8.

\[
(s + f)^n = 1s^5 + 5s^4f + 10s^3f^2 + 10s^2f^3 + 5sf^4 + 1f^5
\]

\[
= (0.2)^5 + 5(0.2)^4(0.8) + 10(0.2)^3(0.8)^2 + 10(0.2)^2(0.8)^3 + 5(0.2)(0.8)^4 + (0.8)^5
\]

\[
= 0.032\% + 0.64\% + 5.12\% + 20.48\% + 40.96\% + 32.768\%
\]

Guided Practice

4. Ricky guessed on the last 6 questions of his test. Each question had 4 options. Determine the probabilities associated with the number of answers he got correct on the last 6 questions.
The graph of a binomial probability distribution can be drawn with the possible outcomes on the $x$-axis and their probabilities of success on the $y$-axis.

**Example 5** Graphing a Binomial Distribution

Graph the binomial probability distribution in Example 4. Describe the shape of the distribution.

List the number of correct answers along the $x$-axis.

The maximum probability is 40.96%, so the $y$-axis should range from 0 to 0.5.

The graph is positively skewed.

**Guided Practice**

5. Graph the binomial probability distribution in Guided Practice 4. Describe the shape of the distribution.

When the number of trials increases, a normal distribution can be used to approximate a binomial distribution.

**Key Concept** Normal Approximation of a Binomial Distribution

In a binomial distribution with $n$ trials, a probability of success $s$, and a probability of failure $f$, such that $ns \geq 5$ and $nf \geq 5$, the binomial distribution can be approximated by a normal distribution with $\bar{x} = ns$ and $\sigma = \sqrt{nsf}$.

**Example 6** Normal Approximation of a Binomial Distribution

According to an online poll, 64% of middle-aged college graduates feel that their college years were the most exciting. Bernardo conducts a survey of 300 random middle-aged adults with college degrees. What is the probability that at least 200 of the responses will agree?

The number of people surveyed who say that their college years were the most exciting has a binomial distribution with $n = 300$, $s = 0.64$, and $f = 0.36$. Use a normal distribution to approximate the probability.

$$\bar{x} = ns$$

$$= 300(0.64) \text{ or } 192$$

$$\sigma = \sqrt{nsf}$$

$$= \sqrt{300(0.64)(0.36)}$$

$$\approx 8.31$$

200 is about 1 standard deviation greater than the mean, so the probability that at least 200 responses agree is 16%.

**Guided Practice**

6. According to an online poll, 32% of adults feel that school should be in session year-round. Suki thinks the number should be lower, so she conducts a survey of 250 random adults. What is the probability that no more than 65 of the surveyed adults feel that school should be in session year-round?
Check Your Understanding

Example 1 1. CARDS Design and conduct an experiment, and use a table like the one at the right to find the experimental probability of drawing an ace or a king 1 out of 10 times when drawing from a deck of 52 cards with replacement.

<table>
<thead>
<tr>
<th>Draw</th>
<th>Ace/King</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>No</td>
</tr>
<tr>
<td>3</td>
<td>No</td>
</tr>
<tr>
<td>4</td>
<td>No</td>
</tr>
<tr>
<td>5</td>
<td>No</td>
</tr>
</tbody>
</table>

Example 2 2. PETS Chloe’s cat is having kittens. The probability of a kitten being male is 0.5.
   a. If Chloe’s cat has 4 kittens, what is the probability that at least 3 will be male?
   b. What is the expected number of males in a litter of 6?

Example 3 3. BASEBALL What is the probability that at least 5 of the 25 students trying out for the baseball team are left-handed if 11% of the population is left-handed?

Example 4 4. GUESSING Loranzo had to guess on the last 10 questions of his test. Luckily, they were true and false questions. Determine the probabilities associated with the number of answers he guessed correctly and make a table listing the probability for 0 correct, 1 correct, 2 correct, etc.

Example 5 5. PARKING A poll at Steve’s high school showed that 85% of the students were in favor of expanding the junior-senior parking lot. Steve asked 6 random students who participated in the poll if they were in favor of expanding the parking lot.
   a. Graph the binomial probability distribution.
   b. What is the probability that all 6 of the students were in favor of expansion?

Example 6 6. SUMMER JOBS According to an online poll, 85% of high school upperclassmen have summer jobs. Tadeo thinks the number should be lower so he conducts a survey of 200 random upperclassmen. What is the probability that no more than 160 of the surveyed upperclassmen have summer jobs?

Practice and Problem Solving

Example 1 7. DICE Design and conduct an experiment, and then find the experimental probability of rolling a 7 with 2 six-sided dice 2 out of 10 times.

8. CARDS Design and conduct an experiment, and then find the experimental probability of drawing a face card out of a standard deck of cards 4 out of 10 times with replacement.

9. MARBLES Design and conduct an experiment using a bag of 4 blue, 3 green, and 5 red marbles. Then find the experimental probability of pulling out a red marble 6 out of 10 times with replacement.

Examples 2–3 10. MP3 PLAYERS According to a recent survey, 85% of high school students own an MP3 player. What is the probability that at least 6 of 10 random high school students own an MP3 player?

11. CARS According to a recent survey, 92% of high school seniors own their own car. What is the probability that fewer than 8 out of 10 random high school students own their own car?
12. **SENIOR PROM** According to a recent survey, 25% of high school upperclassmen think that the junior-senior prom is the most important event of the school year. What is the probability that no more than 3 out of 10 random high school upperclassmen think this way?

13. **FOOTBALL** A certain football team has won 75.7% of their games. Find the probability that they win at least 7 of their next 10 games.

14. **GARDENING** Peter is planting 24 irises in his front yard. The flowers he bought were a combination of two varieties, blue and white. The flowers are not blooming yet, but Peter knows that the probability of having a blue flower is 75%. What is the probability that at least 20 of the flowers will be blue?

15. **FOOTBALL** What is the probability that a field goal kicker makes at least 7 of his next 10 kicks from within 34 yards?

<table>
<thead>
<tr>
<th>Range (yd)</th>
<th>Accuracy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–34</td>
<td>75</td>
</tr>
<tr>
<td>35–44</td>
<td>62</td>
</tr>
<tr>
<td>45+</td>
<td>20</td>
</tr>
</tbody>
</table>

16. **BABIES** Mr. and Mrs. Davis are planning to have 3 children and the probability of each child being a boy is 50%. What is the probability that they will have at least 2 boys?

**Example 4**

17. **LAPTOPS** According to a recent survey, 95% of high school students own a laptop. Ten random students are chosen.
   a. Determine the probabilities associated with the number of students who own a laptop.
   b. What is the probability that at least 8 of the 10 students own a laptop?

18. **ATHLETICS** According to a recent survey, 80% of high school students have participated in at least one sport for their school. Six random students are chosen.
   a. Determine the probabilities associated with the number of students playing in at least one sport.
   b. What is the probability that no more than 2 of the students participated in a sport?

**Example 5**

19. **CAR WASH** Some students are doing a car wash to raise money for the Spanish Club. They have determined that 65% of the time the customers donate more than the minimum amount for the car wash. What is the probability that at least 4 of the next 5 customers will donate more than the minimum?

20. **DRAWINGS** One in five students will win a prize in the class drawing. If there are 25 students in the drawing, including Jake, Leslie, Roberto, Ika, and Nicholas, what is the probability that at least one of them wins a prize?

**Example 6**

21. **MUSIC** An online poll showed that 5% of adults still play vinyl records. Moe surveyed 8 random people from the population.
   a. Graph the binomial probability distribution.
   b. What is the probability that no more than 2 of the people surveyed still play vinyl records?

22. **PROMOTIONS** A beverage company has a promotion in which 30% of the bottles purchased during the promotion have bottle caps that win a free beverage. Melanie bought 10 bottles.
   a. Graph the binomial probability distribution.
   b. What is the probability that Melanie won at least 4 free beverages?

23. **REALITY SHOWS** According to an online poll, 70% of teens watch at least one reality show. Dillon surveyed 200 random teens. What is the probability that at least 146 of the teens surveyed watch at least one reality show?
24. **COLLEGE** A poll of students at Jacqui’s school determined that 88% of the students wanted to go to college. Jacqui surveyed 150 random students from the school. What is the probability that at least 10 of the polled students did not want to go to college?

A binomial distribution has a 60% rate of success. There are 18 trials.

25. What is the probability that there will be at least 12 successes?

26. What is the probability that there will be 12 failures?

27. What is the expected number of successes?

28. **TENNIS** A player has won 85% of his matches over his career. Find each probability.
   a. He wins 3 of the next 5 matches.
   b. He wins at least 2 of his next 5 matches.
   c. He loses at least 1 of his next 5 matches.

Each binomial distribution has \( n \) trials and \( p \) probability of success. Determine the most likely number of successes.

29. \( n = 8, p = 0.6 \)  
30. \( n = 10, p = 0.4 \)  
31. \( n = 6, p = 0.8 \)

32. \( n = 12, p = 0.55 \)  
33. \( n = 9, p = 0.75 \)  
34. \( n = 11, p = 0.35 \)

35. **SWEETSTAKES** A beverage company is having a sweepstakes. The probability of winning selected prizes are shown at the right. If Ernesto purchases 8 beverages, what is the probability that he wins at least one prize?

<table>
<thead>
<tr>
<th>Probability of Winning</th>
</tr>
</thead>
<tbody>
<tr>
<td>beverage 1 in 10</td>
</tr>
<tr>
<td>CD 1 in 200</td>
</tr>
<tr>
<td>hat 1 in 250</td>
</tr>
<tr>
<td>MP3 player 1 in 20,000</td>
</tr>
<tr>
<td>car 1 in 25,000,000</td>
</tr>
</tbody>
</table>

Each binomial distribution has \( n \) trials and \( p \) probability of success. Determine the probability of \( s \) successes.

36. \( n = 8, p = 0.3, s \geq 2 \)  
37. \( n = 10, p = 0.2, s > 2 \)  
38. \( n = 6, p = 0.6, s \leq 4 \)

39. \( n = 9, p = 0.25, s \leq 5 \)  
40. \( n = 10, p = 0.75, s \geq 8 \)  
41. \( n = 12, p = 0.1, s < 3 \)

**H.O.T. Problems** Use Higher-Order Thinking Skills

42. **CHALLENGE** In a normal approximation of a binomial distribution, there is a 34% probability of there being between 60 and 66 successes. If \( \bar{x} = 60 \) and the probability of success is 36%, how many trials are there?

43. **WRITING IN MATH** You poll a sample of your classmates to find out if they support using school funds for the science wing project. How could you use a binomial distribution to predict the number of people in the school who support the project?

44. **REASONING** Determine whether the following statement is sometimes, always, or never true. Explain your reasoning.
   
   It is more beneficial to find the probability of failure and subtract it from 1 in order to determine the probability of success.

45. **OPEN ENDED** Describe a real-world setting within your school or community activities that seems to fit a binomial distribution. Identify the key components of your setting that connect to binomial distributions.

46. **WRITING IN MATH** Describe how binomial distributions are connected to Pascal’s triangle.

47. **WRITING IN MATH** Explain the relationship between a binomial experiment and a binomial distribution.
48. **EXTENDED RESPONSE** Carly is taking a 10-question multiple-choice test in which each question has four choices. If she guesses on each question, what is the probability that she will get

- a. 7 questions correct?
- b. 9 questions correct?
- c. 0 questions correct?
- d. 3 questions correct?

49. What is the maximum point of the graph of the equation \( y = -2x^2 + 16x + 5? \)

- A \((-4, -59)\)
- B \((-4, -91)\)
- C \((4, 37)\)
- D \((4, 101)\)

50. **GEOMETRY** On a number line, point \( X \) has coordinate \(-8\) and point \( Y \) has coordinate \( 4. \) Point \( P \) is \( \frac{2}{3} \) of the way from \( X \) to \( Y \). What is the coordinate of \( P? \)

- F \(-4\)
- G \(-2\)
- H \( 0 \)
- J \( 2 \)

51. **SAT/ACT** The cost of 4 CDs is \( d \) dollars. At this rate, what is the cost, in dollars, of 36 CDs?

- A \(9d\)
- B \(144d\)
- C \(9d\)
- D \(\frac{d}{36}\)
- E \(\frac{36}{d}\)

52. \( H_0 = 33, H_1 > 33, n = 100, \bar{x} = 32.1, \) and \( s = 1.2 \)

53. \( H_0 = 5, H_1 < 5, n = 50, \bar{x} = 5.2, \) and \( s = 0.8 \)

54. \( H_0 = 0.04, H_1 > 0.04, n = 100, \bar{x} = 0.042, \) and \( s = 0.1 \)

55. \( H_0 = 300, H_1 < 300, n = 25, \bar{x} = 301, \) and \( s = 1.5 \)

56. **SPEED** A system collected and recorded the speed of drivers on a road near a school. The speeds were normally distributed with a mean of 37 miles per hour and a standard deviation of 4 miles per hour. Of the 425 cars sampled, how many would you expect were driving less than 33 miles per hour? (Lesson 12-5)

**Test each null hypothesis. Write accept or reject.** (Lesson 12-6)

- 52. \( H_0 = 33, H_1 > 33, n = 100, \bar{x} = 32.1, \) and \( s = 1.2 \)
- 53. \( H_0 = 5, H_1 < 5, n = 50, \bar{x} = 5.2, \) and \( s = 0.8 \)
- 54. \( H_0 = 0.04, H_1 > 0.04, n = 100, \bar{x} = 0.042, \) and \( s = 0.1 \)
- 55. \( H_0 = 300, H_1 < 300, n = 25, \bar{x} = 301, \) and \( s = 1.5 \)

57. \( a_5 = 12, a_{16} = 133, d = ? \)

58. \( a_9 = -34, a_{22} = 44, d = ? \)

59. \( a_4 = 18, a_n = 95, d = 7, n = ? \)

60. \( a_8 = ?, a_{19} = 31, d = 8 \)

61. \( a_6 = ?, a_{20} = 64, d = 7 \)

62. \( a_7 = -28, a_n = 76, d = 8, n = ? \)

57. \( a_5 = 12, a_{16} = 133, d = ? \)

58. \( a_9 = -34, a_{22} = 44, d = ? \)

60. \( a_8 = ?, a_{19} = 31, d = 8 \)

61. \( a_6 = ?, a_{20} = 64, d = 7 \)

63. **ASTRONOMY** The table at the right shows the closest and farthest distances of Venus and Jupiter from the center of the Sun in millions of miles. (Lesson 10-4)

<table>
<thead>
<tr>
<th>Planet</th>
<th>Closest</th>
<th>Farthest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Venus</td>
<td>66.8</td>
<td>67.7</td>
</tr>
<tr>
<td>Jupiter</td>
<td>460.1</td>
<td>507.4</td>
</tr>
</tbody>
</table>

- a. Write an equation for the orbit of each planet. Assume that the center of the orbit is the origin and the center of the Sun is a focus that lies on the x-axis.

- b. Which planet has an orbit that is closer to a circle?

Write an equivalent exponential or logarithmic function. (Lesson 8-7)

- 64. \( e^{-x} = 5 \)
- 65. \( e^2 = 6x \)
- 66. \( \ln e = 1 \)
- 67. \( \ln 5.2 = x \)
- 68. \( e^{x + 1} = 9 \)
- 69. \( e^{-1} = x^2 \)
- 70. \( \ln \frac{7}{3} = 2x \)
- 71. \( \ln e^x = 3 \)

**Skills Review**

72. **MUSIC** Tina owns 11 pop, 6 country, 16 rock, and 7 rap CDs. Find each probability if she randomly selects 4 CDs. (Lesson 12-3)

- a. \( P(2 \text{ rock}) \)
- b. \( P(1 \text{ rap}) \)
- c. \( P(1 \text{ rock and 2 country}) \)
Study Guide

Key Concepts

Samples and Populations (Lessons 12-1 and 12-2)
- A sample is biased if its design favors certain outcomes.
- A sample is unbiased if it is random or unpredictable.

<table>
<thead>
<tr>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample</td>
</tr>
<tr>
<td>[ \sqrt{\frac{\sum(x_k - \bar{x})^2}{n-1}} ]</td>
</tr>
</tbody>
</table>

Conditional Probability (Lesson 12-3)
- The probability of an event given that another event has already occurred is the conditional probability.
- A contingency table records data in which different possible situations result in different possible outcomes.

Probability Distributions (Lessons 12-4, 12-5, and 12-7)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>uniform</td>
<td>All probabilities are equal.</td>
</tr>
<tr>
<td>discrete</td>
<td>finite number of possible outcomes</td>
</tr>
<tr>
<td>continuous</td>
<td>infinite number of possible outcomes</td>
</tr>
<tr>
<td>normal</td>
<td>symmetric curves</td>
</tr>
<tr>
<td>skewed</td>
<td>non-symmetric curves</td>
</tr>
<tr>
<td>binomial</td>
<td>Outcomes are one of two simple events.</td>
</tr>
</tbody>
</table>

Hypothesis Testing (Lesson 12-6)
- Inferential statistics draw conclusions about a population by using a sample.
- A hypothesis is an assumption that can be verified by testing

Key Vocabulary

alternative hypothesis (p. 781)  
biased (p. 745)  
binomial distribution (p. 786)  
binomial experiment (p. 786)  
causation (p. 747)  
conditional probability (p. 759)  
confidence interval (p. 780)  
continuous probability distribution (p. 770)  
control group (p. 746)  
correlation (p. 746)  
discrete probability distribution (p. 767)  
expected value (p. 767)  
experiment (p. 746)  
inferential statistics (p. 780)  
margin of sampling error (p. 753)  
measure of central tendency (p. 752)  
measure of variation (p. 754)  
normal distribution (p. 773)  
null hypothesis (p. 781)  
observational study (p. 746)  
parameter (p. 752)  
population (p. 745)  
probability (p. 764)  
probability distribution (p. 766)  
random variable (p. 766)  
relative frequency (p. 760)  
sample (p. 745)  
skewed distribution (p. 773)  
standard deviation (p. 754)  
statistic (p. 752)  
survey (p. 745)  
theoretical probability (p. 767)  
treatment group (p. 746)  
two-way table (p. 760)  
unbiased (p. 745)  
variable (p. 752)

Vocabulary Check

Choose a word or term from the list above that best completes each statement.

1. A(n) ________ for a particular random variable is a function that maps the sample space to the probabilities of the outcomes of the sample space.

2. When two events are related, there is a(n) ____________

3. A survey is ____________ if its design favors certain outcomes.

4. The group given the placebo is the ____________.

5. The ____________ provides the interval that shows how much responses from the sample would differ from the population.

6. A probability distribution with only a finite number of possible outcomes is a(n) ____________.
Lesson-by-Lesson Review

12-1 Experiments, Surveys, and Observational Studies (pp. 745–750)

State whether each survey would produce a random sample. Write yes or no. Explain.

7. Every tenth shopper coming out of a hardware store is surveyed to determine his or her satisfaction with the store.
8. Every tenth person coming out of a high school is asked what their favorite class is.
9. A fast food restaurant asks their customers to complete a survey asking what their favorite fast food restaurant is.

Determine whether each situation calls for a survey, an observational study, or an experiment. Explain the process.

10. Find 100 students, half of which have part-time jobs, and compare their grade-point averages.
11. Find 100 people and randomly split them into two equal groups. One group eats a specific diet while the other group does not. Compare the results.

Example 1
A car dealership selects 100 random customers who recently took their vehicles in for work and asks them how the service was. Would this produce a random sample? Explain.
Yes. Everyone in the population of customers has an equal chance to be part of the sample.

Example 2
A teacher has his first class take their test while listening to headphones. His second class does not. He compares their test results. Is this a survey, an observational study, or an experiment? Explain the process.
Experiment. The treatment group is the first class and the control is the second class. This is a biased experiment because the treated group knows who they are.

Example 3
A national poll estimates that the average number of hours spent per week sitting in traffic is four. Is this a population or a sample?
This represents a sample because only a fraction of the residents of the United States are polled.

Example 4
In a random survey of 2645 people, 12% said that hockey is their favorite sport. What is the margin of sampling error?

Margin of sampling error = \[ \pm \frac{1}{\sqrt{n}} \]
= \[ \pm \frac{1}{\sqrt{2645}} \]
\[ \approx \pm 0.0194 \]
The margin of sampling error \( \approx \pm 1.9\% \).

12-2 Statistical Analysis (pp. 752–758)

Determine whether each of the following represents a population or a sample.

12. Jarred conducts an online survey on cancer.
13. The French club wants to compare their AP test scores with the national average.
14. The field hockey team wants to compare their scoring average with everyone else in the league.
15. SEASONS In a random survey of 3446 people, 34% said that spring is their favorite season. What is the margin of sampling error?
16. SWIMMING While practicing, Kelly kept track of her times in the 400-meter individual medley. Find the standard deviation of her practice times.

<table>
<thead>
<tr>
<th>Times in Seconds</th>
<th>301</th>
<th>311</th>
<th>320</th>
<th>308</th>
<th>312</th>
<th>307</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>313</td>
<td>315</td>
<td>309</td>
<td>308</td>
<td>304</td>
<td>302</td>
</tr>
<tr>
<td></td>
<td>311</td>
<td>313</td>
<td>313</td>
<td>316</td>
<td>314</td>
<td>306</td>
</tr>
<tr>
<td></td>
<td>329</td>
<td>326</td>
<td>319</td>
<td>310</td>
<td>306</td>
<td>309</td>
</tr>
<tr>
<td></td>
<td>320</td>
<td>318</td>
<td>315</td>
<td>318</td>
<td>314</td>
<td>309</td>
</tr>
</tbody>
</table>

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= \[ \pm \frac{1}{\sqrt{2645}} \]
\[ \approx \pm 0.0194 \]
The margin of sampling error \( \approx \pm 1.9\% \).
12-3 Conditional Probability (pp. 759–763)

17. **CARDS** Jillian chooses a card at random from a standard deck. What is the probability that the card is a jack, given that it is not an ace, two, or three?

18. **BASEBALL** The results of who made the varsity baseball team are listed in the table below. Find the probability of each.

<table>
<thead>
<tr>
<th></th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left-Handed</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>Right-Handed</td>
<td>15</td>
<td>22</td>
</tr>
</tbody>
</table>

   a. Peter made the team given that he is left-handed.
   b. Paul is right-handed given he did not make the team.

Example 5

**MEDICINE** Find the probability that Lori has a Health class, given that she is a freshman.

<table>
<thead>
<tr>
<th></th>
<th>Health Class</th>
<th>No Health Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freshmen</td>
<td>126</td>
<td>84</td>
</tr>
<tr>
<td>Sophomores</td>
<td>98</td>
<td>72</td>
</tr>
</tbody>
</table>

\[ P(H \mid F) = \frac{P(H \text{ and } F)}{P(F)} \]

\[ = \frac{126}{380} \div \frac{210}{380} \]

\[ = \frac{126}{210} \text{ or } \frac{3}{5} \]

Simplify.

Example 6

**SPORTS CARDS** Bob is moving and all of his sports cards are mixed up in a box. Twelve cards are baseball, eight are football, and five are basketball. If he reaches in the box and selects them at random, find each probability.

19. \( P(3 \text{ football}) \)
20. \( P(3 \text{ baseball}) \)
21. \( P(1 \text{ basketball, 2 football}) \)
22. \( P(2 \text{ basketball, 1 baseball}) \)

23. **MARBLES** Sammy has a sack of 25 marbles. Eight are black, 10 are red, 4 are green, and the rest are white. He pulls two marbles out of the bag.

   a. Create a frequency table and a relative-frequency graph of the data.
   b. Which outcome is the most likely to occur?
   c. Find \( P(\text{black and green}) \).

24. **CARDS** Three nines, 4 tens, 5 sixes, 4 fives, 2 twos, and a three are pulled from a deck of cards. If one card is drawn from the cards that were pulled out, what is its expected value?
# 12-5 The Normal Distribution (pp. 773–778)

A normal distribution of data has each mean and standard deviation. Find each probability.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.</td>
<td>$\mu = 121, \sigma = 9, P(x &gt; 103)$</td>
</tr>
<tr>
<td>26.</td>
<td>$\mu = 84, \sigma = 8, P(x &gt; 108)$</td>
</tr>
<tr>
<td>27.</td>
<td>$\mu = 181, \sigma = 12, P(x &gt; 169)$</td>
</tr>
</tbody>
</table>

28. **RUNNING TIMES** The times in the 40-meter dash for a select group of professional football players is normally distributed with a mean of 4.7 and a standard deviation of 0.15.

   a. About what percent of the players have times below 4.4?
   b. About how many of the 800 players have times between 4.55 and 4.85?

---

# 12-6 Hypothesis Testing (pp. 780–784)

Find a 95% confidence interval for each of the following.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>29.</td>
<td>$\bar{x} = 23.3, s = 2.4, \text{ and } n = 80$</td>
</tr>
<tr>
<td>30.</td>
<td>$\bar{x} = 72.2, s = 5.8, \text{ and } n = 120$</td>
</tr>
<tr>
<td>31.</td>
<td>$\bar{x} = 81.4, s = 6.1, \text{ and } n = 200$</td>
</tr>
</tbody>
</table>

32. **INTERNET** A sample of 300 students was asked for the average amount of time they spend online during a week night. The mean time was 64.3 minutes with a standard deviation of 7.3 minutes. Determine a 95% confidence interval.

Test each null hypothesis. Write accept or reject.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>33.</td>
<td>$H_0: 60, H_1 &lt; 60, n = 100, \bar{x} = 59.4, \text{ and } s = 3.1$</td>
</tr>
<tr>
<td>34.</td>
<td>$H_0: 5.5, H_1 &gt; 5.5, n = 80, \bar{x} = 5.8, \text{ and } s = 0.7$</td>
</tr>
<tr>
<td>35.</td>
<td>$H_0: 32, H_1 &lt; 32, n = 60, \bar{x} = 31.5, \text{ and } s = 1.8$</td>
</tr>
</tbody>
</table>

36. **INTERSECTIONS** A light at an intersection is timed to let 10 cars turn left each rotation. Danny believes it is less than 10 and tests the light. After collecting data for 50 rotations, he arrives at a mean of 9.1 cars and a standard deviation of 0.8 cars. Test the hypothesis that the average number of cars is less than 10.

---

Example 7

A normal distribution of data has a mean of 78 and standard deviation of 5. Find the probability that random value $x$ is greater than 83.

$\mu = 78 \text{ and } \sigma = 5$

The probability that a randomly selected value in the distribution is greater than $\mu + \sigma$, that is, 78 + 5 or 83, is $13.5% + 2% + 0.5% = 16%$

In the normal curve, this includes the area that is greater than $\mu + \sigma$.

---

Example 8

Find a 95% confidence interval for $\bar{x} = 65, s = 1.6, \text{ and } n = 100$.

$CI = \bar{x} \pm \frac{s}{\sqrt{n}}$

$\approx 65 \pm 0.31$

The 95% confidence interval is $64.69 \leq \mu \leq 65.31$.

Example 9

Test the null hypothesis. Write accept or reject. $H_0 = 8$, $H_1 < 8, n = 90, \bar{x} = 8.1, \text{ and } s = 0.7$

$CI = \bar{x} \pm \frac{s}{\sqrt{n}}$

$\approx 8.1 \pm 0.15$

The 95% confidence interval is $7.95 \leq \mu \leq 8.25$. The confidence interval includes $H_0$, so we accept the null hypothesis.
Binomial Distributions (pp. 786–793)

A binomial distribution has a 40% rate of success. There are 10 trials. Calculate the probability of each.

37. exactly 3 successes
38. less than 8 successes
39. no more than 3 successes
40. at least 4 successes

In a certain dice game, a player tries to roll a total of 3 or 10 with two dice. Kevin designed and conducted a binomial experiment for 7 rolls of the dice.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>Total Successes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>f</td>
<td>s</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>s</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>f</td>
<td>f</td>
<td>s</td>
<td>f</td>
<td>f</td>
<td>s</td>
<td>f</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>f</td>
<td>s</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>f</td>
<td>s</td>
<td>2</td>
</tr>
</tbody>
</table>

From Kevin’s experiment, what is the experimental probability of rolling a 3 or 10 twice in seven rolls?

42. **SENIOR TRIP** A poll of students at Ryan’s school determined that 76% of the students wanted to go to a theme park for their senior trip. Ryan surveyed 180 random students from the school. What is the probability that at least 60 of the polled students did not want to go to a theme park?

43. **WORK** According to an online poll, 28% of adults feel that the standard 40-hour work-week should be increased. Sheila thinks the number should be lower, so she conducts a survey of 250 random adults. What is the probability that more than 55 of the surveyed adults feel that the standard 40-hour work-week should be increased?

44. **WATCHES** According to an online poll, 74% of adults wear watches. Timmy surveyed 200 random adults. What is the probability that at least 160 of the adults surveyed wear a watch?

Example 10

A binomial distribution has a 55% rate of success. There are 8 trials. What is the probability that there will be at least 2 successes?

Calculate the probability of 0 and 1 successes.

\[ P(1) = \binom{8}{1}(0.55)^1(0.45)^7 = 0.01644 \]

\[ P(0) = \binom{8}{0}(0.55)^0(0.45)^8 = 0.00168 \]

\[ P(\geq 2 \text{ successes}) = 1 - P(< 2 \text{ successes}) = 1 - P(1) - P(0) = 1 - 0.01644 - 0.00168 = 0.98188 \]

There is about a 98% probability that there will be at least 2 successes.

Example 11

**VACATIONS** According to an online poll, 70% of high school students take a vacation during the summer. Louie thinks the number should be lower so he conducts a survey of 650 random students. What is the probability that no more than 420 of the surveyed students go on a vacation in the summer?

The number of people surveyed has a binomial distribution with \( n = 650, s = 0.70, f = 0.30 \).

Use a normal distribution to approximate the probability.

Mean of a normal approximation:

\[ \mu = ns = 650(0.70) = 455 \]

Standard deviation of a normal approximation:

\[ \sigma = \sqrt{nsf} = \sqrt{650(0.7)(0.3)} \approx 11.68 \]

420 is about 3 standard deviations less than the mean, so the probability that no more than 420 responses agree is 0.5%.
Determine whether the following statements show correlation or causation. Explain.

1. When a baseball player hits the ball over the outfielder’s head and into the bleachers, he has hit a home run.
2. When Jimmy is running in the hallways, he is late for class.

State whether each survey would produce a random sample. Write yes or no. Explain.

3. An online store surveys its customers asking how much money they spend online per month.
4. A teacher selects the names of 5 students from a hat to determine who gives their speeches in class that day.

Which measure of central tendency best represents the data, and why?

5. AP Test Scores

| 4 | 4 | 3 | 3 | 3 |
| 4 | 5 | 5 | 4 | 4 |
| 3 | 3 | 3 | 3 | 4 |
| 3 | 3 | 3 | 4 | 4 |
| 4 | 5 | 3 | 4 | 3 |

6. Height in Inches

| 61 | 64 | 62 | 61 | 64 |
| 63 | 65 | 61 | 66 | 73 |
| 74 | 63 | 62 | 65 | 61 |
| 61 | 62 | 66 | 63 | 61 |

Determine whether each of the following represents a population or a sample.

7. Olivia records the addresses of every student at her high school.
8. Bridgette compares her class’s test results to the national average.
9. Joey separates the candy in his bag by color.
10. Paul asks 100 random people what their favorite movie is.

11. MULTIPLE CHOICE A survey of 6225 random people found that 48% eat fast food at least once per week. What is the likely interval that contains the percentage of the population that eats fast food at least once per week?

A 0.78%  B 1.27%  C ±1.27%  D ±0.78%

A normal distribution of data has each mean and standard deviation. Find each probability.

12. \(\mu = 54, \sigma = 5, P(x > 44)\)
13. \(\mu = 35, \sigma = 2.4, P(x < 37.4)\)

14. TESTS Mr. Holt’s class was given the opportunity to retake a test. He also held an optional review session at school the Sunday before. Some students improved and some did not.

<table>
<thead>
<tr>
<th></th>
<th>Improved</th>
<th>Did not Improve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attended</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>Did not Attend</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

a. Find the probability that Michael improved, given he attended the session.
b. Find the probability that Melissa did not attend the session, given she did not improve.

A bag contains 10 blue marbles, 4 yellow marbles, 3 pink marbles, 8 red marbles, and 12 green marbles. A marble is drawn at random. Find each probability.

15. The marble is red, given that it is not blue.
16. The marble is blue, given that it is not green or pink.

17. MULTIPLE CHOICE In a box of paper clips, 45 are red, 25 are yellow, and 30 are green. If 12 paperclips are drawn, what is the expected number of green paperclips?

F 2.5  G 4.8  H 3.0  J 3.6

18. DRAWINGS Ten male and 12 female students have been selected for a drawing for 5 free mp3 players. If the five names will be drawn at random, what is the probability that 3 winners will be male and 2 will be female?

Test each null hypothesis. Write accept or reject.

19. \(H_0 = 77, H_1 > 77, n = 150, \bar{x} = 78.1, \text{ and } s = 1.3\)
20. \(H_0 = 65, H_1 < 65, n = 120, \bar{x} = 64.8, \text{ and } s = 2.1\)

21. A binomial distribution has a 65% rate of success. There are 15 trials. What is the probability that there will be at least 10 successes?

22. WEATHER The weatherman says that there is a 40% chance of snow for each of the next seven days. Find the probability that it snows at least 2 of those days.
Solve Multi-Step Problems

Some problems that you will encounter on standardized tests require you to solve multiple parts in order to come up with the final solution. Use this lesson to practice these types of problems.

Strategies for Solving Multi-Step Problems

Step 1
Read the problem statement carefully.
Ask yourself:
• What am I being asked to solve? What information is given?
• Are there any intermediate steps that need to be completed before I can solve the problem?

Step 2
Organize your approach.
• List the steps you will need to complete in order to solve the problem.
• Remember that there may be more than one possible way to solve the problem.

Step 3
Solve and check.
• Work as efficiently as possible to complete each step and solve.
• If time permits, check your answer.

Test Practice Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

There are 15 boys and 12 girls in Mrs. Lawrence’s homeroom. Suppose a committee is to be made up of 6 randomly selected students. What is the probability that the committee will contain 3 boys and 3 girls? Round your answer to the nearest tenth of a percent.

A 27.2%  C 31.5%
B 29.6%  D 33.8%
Read the problem statement carefully. You are asked to find the probability that a committee will be made up of 3 boys and 3 girls. Finding this probability involves successfully completing several steps.

**Step 1** Find the number of possible successes.

There are $C(15, 3)$ ways to choose 3 boys from 15, and there are $C(12, 3)$ to choose 3 girls from 12. Use the Fundamental Counting Principle to find $s$, the number of possible successes.

$$s = C(15, 3) \times C(12, 3) = \frac{15!}{12!3!} \times \frac{12!}{9!3!} = 100,100$$

**Step 2** Find the total number of possible outcomes.

Compute the number of ways 6 people can be chosen from a group of 27 students.

$$C(27, 6) = 296,010$$

**Step 3** Compute the probability.

Find the probability by comparing the number of successes to the number of possible outcomes.

$$P(3 \text{ boys, 3 girls}) = \frac{100,100}{296,010} \approx 0.33816$$

So, there is about a 33.8% chance of selecting 3 boys and 3 girls for the committee. The answer is D.

---

**Exercises**

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

1. There are 52 cards in a standard deck. Of these, 4 of the cards are Aces. What is the probability of a randomly dealt 5-card hand containing a pair of Aces? Round your answer to the nearest whole percent.

   - **A** 4%
   - **B** 5%
   - **C** 6%
   - **D** 7%

2. According to the table, what is the probability that a randomly selected camper went on the horse ride, given that the camper is an 8th grader?

<table>
<thead>
<tr>
<th>Grade</th>
<th>Canoe Trip</th>
<th>Horse Ride</th>
<th>Nature Hike</th>
</tr>
</thead>
<tbody>
<tr>
<td>6th</td>
<td>8</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>7th</td>
<td>5</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>8th</td>
<td>11</td>
<td>9</td>
<td>6</td>
</tr>
</tbody>
</table>

   - **F** 0.731
   - **G** 0.441
   - **H** 0.346
   - **J** 0.153
Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Suppose the test scores on a final exam are normally distributed with a mean of 74 and a standard deviation of 3. What is the probability that a randomly selected test has a score higher than 77?
   A 2.5%  
   B 13.5%  
   C 16%  
   D 34%

2. A diameter of a circle has endpoints A(4, 6) and B(−3, −1). Find the approximate length of the radius.
   F 2.5 units  
   G 4.9 units  
   H 5.1 units  
   J 9.9 units

3. An equation can be used to find the total cost of a pizza with a certain diameter. Using the table below, find the equation that best represents y, the total cost, as a function of x, the diameter in inches.

<table>
<thead>
<tr>
<th>Diameter, x (in.)</th>
<th>Total Cost, y</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>$10.80</td>
</tr>
<tr>
<td>12</td>
<td>$14.40</td>
</tr>
<tr>
<td>20</td>
<td>$24.00</td>
</tr>
</tbody>
</table>

   A \( y = 1.2x \)  
   B \( x = 1.2y \)  
   C \( y = 0.83x \)  
   D \( y = x + 1.80 \)

4. Which shows the functions correctly listed in order from widest to narrowest graph?
   F \( y = 8x^2, y = 2x^2, y = \frac{1}{2}x^2, y = -\frac{4}{5}x^2 \)  
   G \( y = -\frac{4}{5}x^2, y = \frac{1}{2}x^2, y = 2x^2, y = 8x^2 \)  
   H \( y = \frac{1}{2}x^2, y = -\frac{4}{5}x^2, y = 2x^2, y = 8x^2 \)  
   J \( y = 8x^2, y = 2x^2, y = -\frac{4}{5}x^2, y = \frac{1}{2}x^2 \)

5. The table at the right shows the grades earned by students on a science test. Calculate the standard deviation of the test scores.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>76</td>
<td>8</td>
</tr>
<tr>
<td>82</td>
<td>6</td>
</tr>
<tr>
<td>92</td>
<td>4</td>
</tr>
<tr>
<td>98</td>
<td>2</td>
</tr>
</tbody>
</table>

   A 7.82  
   B 8.03  
   C 8.23  
   D 8.75

6. Which expression is equivalent to \((6a - 2b) - \frac{1}{4}(4a + 12b)\)?
   F \( 5a + 10b \)  
   G \( 10a + 10b \)  
   H \( 5a + b \)  
   J \( 5a - 5b \)

7. Simplify \( \sqrt[4]{27x^3} \).
   A \( 3x^2 \)  
   B \( 3x \)  
   C \( \sqrt{3x} \)  
   D \( \sqrt[4]{3x} \)

8. In the figure below, lines a and b are parallel. What are the measures of the angles in the triangle?

   \( 132^\circ \)
   \( a \)
   \( b \)

   F 42, 48, 90  
   G 42, 90, 132  
   H 48, 52, 90  
   J 48, 90, 132

9. Which of the following functions represents exponential decay?
   A \( y = 0.2(7)^x \)  
   B \( y = (0.5)^x \)  
   C \( y = 4(9)^x \)  
   D \( y = 5(\frac{4}{3})^x \)

10. Using the table below, which expression can be used to determine the \( n \)th term of the sequence?

<table>
<thead>
<tr>
<th>( n )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
</tr>
</tbody>
</table>

   F \( y = 6n \)  
   G \( y = n + 5 \)  
   H \( y = 2n + 1 \)  
   J \( y = 2(2n + 1) \)

Test-Taking Tip

Question 5 You can use a scientific calculator to find the standard deviation. Enter the data values as a list and calculate the 1-Var statistics.
Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. Determine whether each of the following situations calls for a survey, an observational study, or an experiment. Explain the process.

a. Caroline wants to find out if a particular plant food helps plants to grow faster than just water.

b. Allison wants to find opinions on favorite candidates in the upcoming student council elections.

c. Manuel wants to find out if people who get regular exercise sleep better at night.

12. GRIDDED RESPONSE Carla received a map of some walking paths through her college campus. Paths A, B, and C are parallel. What is the length \( x \) to the nearest tenth of a foot?

13. Alex wants to find the area of a triangle. He draws the triangle on a coordinate plane and finds that it has vertices at \((2, 1), (3, 4),\) and \((1, 4)\). Find the area of the triangle using determinants.

14. GRIDDED RESPONSE Perry drove to the gym at an average rate of 30 miles per hour. It took him 45 minutes. Going home, he took the same route, but drove at a rate of 45 miles per hour. How many miles is it to his house from the gym?

Extended Response

Record your answers on a sheet of paper. Show your work.

15. Martin is taking a multiple choice test that has 8 questions. Each question has four possible answers: A, B, C, or D. Martin forgot to study for the test, so he must guess at each answer.

a. What is the probability of guessing a correct answer on the test?

b. What is the expected number of correct answers if Martin guesses at each question?

c. What is the probability that Martin will get at least half of the questions correct? Round your answer to the nearest tenth of a percent.

16. Christine had one dress and three sweaters cleaned at the dry cleaner and the charge was $19.50. The next week, she had two dresses and two sweaters cleaned for a total charge of $23.00.

a. Let \( d \) represent the price of cleaning a dress and \( s \) represent the price of cleaning a sweater. Write a system of linear equations to represent the prices of cleaning each item.

b. Solve the system of equations using substitution or elimination. Explain your choice of method.

c. What will the charge be if Christine takes two dresses and four sweaters to be cleaned?