In Chapter 1, you solved equations and inequalities.

In Chapter 2, you will:
- Use equations of relations and functions.
- Determine the slope of a line.
- Use scatter plots and prediction equations.
- Graph linear inequalities.

RECREATION Linear functions can be used to model many aspects of recreational activities such as distance ridden on a bicycle, the amount of money a group of people would spend at a state fair, the height of a water slide at various points, or the amount of money you could earn from a hobby.

Algebra2 SE 2012 [TN]
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APR 13 2010
CHAPTER 2
PDF Pass
Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option  Take the Quick Check below. Refer to the Quick Review for help.

Quick Check

Write the ordered pair for each point. Then name the quadrant in which it is located. (Lesson 0-1)


6. BABYSITTING  Aliza earns $6 per hour babysitting. Make a table in which the x-coordinate represents the number of hours Aliza babysits, and the y-coordinate represents the amount of money she earns.

Evaluate each expression if \( a = -3 \), \( b = 4 \), and \( c = -2 \). (Lesson 1-1)

7. \( 4a - 3 \)  8. \( 2b - 5c \)  9. \( b^2 - 3b + 6 \)  10. \( \frac{2a + 4b}{c} \)

11. PHONE SERVICE  A cell phone company uses the expression \( 20 + 0.25m \) to determine the monthly charge for \( m \) minutes of air time. Find the monthly charge for 80 minutes of air time.

Solve each equation for the given variable. (Lesson 1-3)

12. \( 4x + 2y = 12 \) for \( y \)
13. \( a = 3b + 9 \) for \( b \)
14. \( 15w - 10 = 5v \) for \( v \)
15. \( 3x - 4y = 8 \) for \( x \)
16. \( \frac{d}{6} + \frac{f}{3} = 4 \) for \( d \)

Quick Review

Example 1

Write the ordered pair for point \( M \). Then name the quadrant in which it is located.

Step 1  Follow a vertical line through the point to find the x-coordinate on the x-axis.

Step 2  Follow a horizontal line through the point to find the y-coordinate on the y-axis.

Step 3  The ordered pair for point \( M \) is \((-4, 2)\). It can also be written as \( M(-4, 2) \).

The x-coordinate of \( M \) is negative, while the y-coordinate is positive. So \( M \) lies in Quadrant II.

Example 2

Evaluate \( 3a^2 - 2ab + b^2 \) if \( a = 4 \) and \( b = -3 \).

\[
3a^2 - 2ab + b^2 = 3(4^2) - 2(4)(-3) + (-3)^2 \\
= 3(16) - 2(4)(-3) + 9 \\
= 48 + 24 + 9 \\
= 81
\]

Example 3

Solve \( 3x + 6y = 24 \) for \( y \).

\[
3x + 6y = 24 \quad \text{Original equation} \\
3x + 6y - 3x = 24 - 3x \\
6y = 24 - 3x \quad \text{Subtract } 3x \text{ from each side.} \\
\frac{6y}{6} = \frac{24}{6} - \frac{3x}{6} \quad \text{Simplify.} \\
y = 4 - \frac{1}{2}x \quad \text{Divide each side by } 6.
\]
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 2. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

New Vocabulary

**English**
- one-to-one function  p. 61
- onto function  p. 61
- discrete relation  p. 62
- continuous relation  p. 62
- vertical line test  p. 62
- independent variable  p. 64
- dependent variable  p. 64
- linear equation  p. 69
- linear function  p. 69
- rate of change  p. 76
- bivariate data  p. 92
- positive correlation  p. 92
- negative correlation  p. 92
- line of fit  p. 94
- regression line  p. 94
- piecewise-linear function  p. 102
- absolute value function  p. 103
- parent function  p. 109
- quadratic function  p. 109
- linear inequality  p. 117

**Español**
- función biunívoca  p. 61
- función  p. 61
- relación discreta  p. 62
- relación continua  p. 62
- prueba de la recta vertical  p. 62
- variable independiente  p. 64
- variable dependiente  p. 64
- ecuación lineal  p. 69
- función lineal  p. 69
- tasa de cambio  p. 76
- datos bivariados  p. 92
- correlación positiva  p. 92
- correlación negativa  p. 92
- recta de ajuste  p. 94
- línea de regresión  p. 94
- función a intervalos lineal  p. 102
- función del valor absoluto  p. 103
- función madre  p. 109
- función cuadrática  p. 109
- desigualdad lineal  p. 117

Review Vocabulary

- **equation** p. 18  ecuación  a mathematical sentence stating that two mathematical expressions are equal
- **function** p. 7  función  a relation in which each x-coordinate is paired with exactly one y-coordinate
- **relation** p. 7  relación  a set of ordered pairs

---

**Relation**

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

**Function**

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>-3</td>
</tr>
<tr>
<td>6</td>
<td>-4</td>
</tr>
</tbody>
</table>
Recall that a function is a relation in which each element of the domain is paired with exactly one element in the range. All functions map elements of the domain to elements of the range, but they may differ in the way the elements of the domain and range are paired.

**Key Concept Functions**

- **one-to-one function**: Each element of the domain pairs to exactly one unique element of the range.

- **onto function**: Each element of the range corresponds to an element of the domain.

- **both one-to-one and onto**: Each element of the domain is paired to exactly one element of the range, and each element of the range corresponds to a unique element of the domain.

---

**Example 1 **

State the domain and range of each relation. Then determine whether each relation is a function. If it is a function, determine if it is one-to-one, onto, both, or neither.

a. \( \{(-6, -1), (-5, -9), (-3, -7), (-1, 7), (6, -9)\} \)

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>D</td>
</tr>
<tr>
<td>-5</td>
<td>C</td>
</tr>
<tr>
<td>-3</td>
<td>B</td>
</tr>
<tr>
<td>-1</td>
<td>A</td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

**function**: yes, because each element of the domain is paired with one element of the range

**one-to-one**: no, because each element of the domain is not paired with a unique element of the range

**onto**: yes, because each element of the range corresponds to an element of the domain
b. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>-1</th>
<th>-2</th>
<th>-1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Domain: $\{-2, -1, 2\}$  
Range: $\{-2, -1, 0, 1, 2\}$

The relation is not a function because 2 is mapped to both -2 and 2, and -1 is mapped to both -1 and 1.

**Guided Practice**

State the domain and range of each relation. Then determine whether each relation is a function. If it is a function, determine if it is one-to-one, onto, both, or neither.

1A. 

A relation in which the domain is a set of individual points, like the relation in Graph A, is said to be a **discrete relation**. Notice that its graph consists of points that are not connected. When the domain of a relation has an infinite number of elements and the relation can be graphed with a line or smooth curve, the relation is a **continuous relation**.

With both discrete and continuous graphs, you can use the **vertical line test** to determine whether the relation is a function.

**Key Concept  Vertical Line Test**

<table>
<thead>
<tr>
<th>Words</th>
<th>Models</th>
<th>Words</th>
<th>Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>If no vertical line intersects a graph in more than one point, the graph represents a function.</td>
<td></td>
<td>If a vertical line intersects a graph in two or more points, the graph does not represent a function.</td>
<td></td>
</tr>
</tbody>
</table>
Real-World Example 2

**BICYCLING** The graph shows the length of the Tour de France in kilometers each year from 2000 through 2009. Is the relation **discrete** or **continuous**? Does the graph represent a function?

Because the graph consists of distinct points, the function is discrete. Use the vertical line test. No vertical line can be drawn that contains more than one of the data points. Therefore, the relation is a function.

Guided Practice

2. The number of employees a company had in each year from 2004 to 2009 were 25, 28, 34, 31, 27, and 29. Graph this information and determine whether the relation is **discrete** or **continuous**. Does the graph represent a function?

Equations of Relations and Functions

Relations and functions can also be represented by equations. The solutions of an equation in $$x$$ and $$y$$ are the set of ordered pairs $$(x, y)$$ that make the equation true. To determine whether an equation represents a function, it is often simplest to look at the graph of the relation.

Example 3 Graph a Relation

Graph $$y = \frac{1}{2}x - 3$$, and determine the domain and range. Then determine whether the equation is a **function**, is **one-to-one**, **onto**, **both**, or **neither**. State whether it is **discrete** or **continuous**.

Make a table of values that satisfy the equation. Then graph the equation.

<table>
<thead>
<tr>
<th>$$x$$</th>
<th>$$y$$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$$-4$$</td>
<td>$$-5$$</td>
</tr>
<tr>
<td>$$-2$$</td>
<td>$$-4$$</td>
</tr>
<tr>
<td>$$0$$</td>
<td>$$-3$$</td>
</tr>
<tr>
<td>$$2$$</td>
<td>$$-2$$</td>
</tr>
<tr>
<td>$$4$$</td>
<td>$$-1$$</td>
</tr>
</tbody>
</table>

Every real number is the $$x$$-coordinate of some point on the line, and every real number is the $$y$$-coordinate of some point on the line. So the domain and range are both all real numbers.

The graph passes the vertical line test, so the equation is a function. Every $$x$$-value is paired with exactly one unique $$y$$-value, and every $$y$$-value corresponds to an $$x$$-value. Thus, the function is both one-to-one and onto.

Because the graph is a solid line without breaks, the function is continuous.

Guided Practice

3. Graph $$y = x^2 + 1$$, and determine the domain and range. Then determine whether the equation is a **function**, is **one-to-one**, **onto**, **both**, or **neither**. State whether it is **discrete** or **continuous**.
When an equation represents a function, the variable, often \( x \), with values making up the domain is called the independent variable. The other variable, often \( y \), is called the dependent variable because its values depend on \( x \).

Equations that represent functions are often written in function notation. The equation \( y = 5x - 1 \) can be written as \( f(x) = 5x - 1 \). Suppose you want to find the value in the range that corresponds to the element \(-6\) in the domain of the function. The value \( f(-6) \) is found by substituting \(-6\) for each \( x \) in the equation. Therefore, \( f(-6) = 5(-6) - 1 \) or \(-31\).

**Example 4 Evaluate a Function**

Given \( f(x) = 2x^2 - 8 \), find each value.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( f(6) )</td>
<td>b. ( f(2y) )</td>
</tr>
<tr>
<td>( f(x) = 2x^2 - 8 )</td>
<td>( f(x) = 2x^2 - 8 )</td>
</tr>
<tr>
<td>( f(6) = 2(6)^2 - 8 )</td>
<td>( f(2y) = 2(2y)^2 - 8 )</td>
</tr>
<tr>
<td>( = 2(36) - 8 )</td>
<td>( = 2(2y)^2 - 8 )</td>
</tr>
<tr>
<td>( = 72 - 8 = 64 )</td>
<td>( = 8y^2 - 8 )</td>
</tr>
</tbody>
</table>

Simplify.

**Guided Practice**

Given \( g(x) = 0.5x^2 - 5x + 3.5 \), find each value.

4A. \( g(2.8) \)  
4B. \( g(4a) \)

---

**Check Your Understanding**

**Example 1** State the domain and range of each relation. Then determine whether each relation is a function. If it is a function, determine if it is one-to-one, onto, both, or neither.

1. \[ \begin{array}{c|c}
   x & y \\
   \hline
   5 & 3 \\
   6 & 1 \\
   -2 & -8
\end{array} \]

2. \[ \begin{array}{c|c}
   x & y \\
   \hline
   1 & 2.5 \\
   -2 & 1.5 \\
   0 & 0 \\
   1 & -2.5
\end{array} \]

3. \[ \begin{array}{c|c}
   x & y \\
   \hline
   -2 & -4 \\
   1 & -4 \\
   4 & -2 \\
   8 & 6
\end{array} \]

**Example 2**  
**BASKETBALL** The table shows the average points per game for Dwayne Wade of the Miami Heat for four seasons.

<table>
<thead>
<tr>
<th>Season</th>
<th>Dwayne Wade’s Age</th>
<th>Average Points Per Game</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005–2006</td>
<td>24</td>
<td>27.2</td>
</tr>
<tr>
<td>2006–2007</td>
<td>25</td>
<td>27.4</td>
</tr>
<tr>
<td>2007–2008</td>
<td>26</td>
<td>24.6</td>
</tr>
<tr>
<td>2008–2009</td>
<td>27</td>
<td>30.6</td>
</tr>
</tbody>
</table>

Source: Basketball-Reference

**Example 3** Graph each equation, and determine the domain and range. Determine whether the equation is a function, is one-to-one, onto, both, or neither. Then state whether it is discrete or continuous.

5. \( y = 5x + 4 \)  
6. \( y = -4x - 2 \)  
7. \( y = 3x^2 \)  
8. \( x = 7 \)
Example 4  Evaluate each function.
9. \( f(-3) \) if \( f(x) = -4x - 8 \)  
10. \( g(5) \) if \( g(x) = -2x^2 - 4x + 1 \)

Practice and Problem Solving  Extra Practice begins on page 947.

Example 1  State the domain and range of each relation. Then determine whether each relation is a function. If it is a function, determine if it is one-to-one, onto, both, or neither.
11. \[ \begin{array}{c|c} x & y \\ \hline -0.3 & -6 \\ 0.4 & -3 \\ 1.2 & -1 \\ 1.2 & 4 \end{array} \]
12. \[ \begin{array}{c|c|c|c} x & y & z & w \\ \hline 2 & -6 & -4 & 12 \\ 4 & -4 & -4 & 14 \end{array} \]
13. \{(-3, -4), (-1, 0), (3, 0), (5, 3)\}

Example 2  14. POLITICS  The table below shows the population of several states and the number of U.S. representatives from those states.

<table>
<thead>
<tr>
<th>State</th>
<th>Population (millions)</th>
<th>Number of Representatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>California</td>
<td>33.93</td>
<td>53</td>
</tr>
<tr>
<td>Florida</td>
<td>16.03</td>
<td>25</td>
</tr>
<tr>
<td>Illinois</td>
<td>12.44</td>
<td>19</td>
</tr>
<tr>
<td>New York</td>
<td>19.00</td>
<td>29</td>
</tr>
<tr>
<td>North Carolina</td>
<td>8.07</td>
<td>13</td>
</tr>
<tr>
<td>Texas</td>
<td>20.90</td>
<td>32</td>
</tr>
</tbody>
</table>

Source: U.S. Bureau of the Census

Example 3  Graph each equation, and determine the domain and range. Determine whether the equation is a function, is one-to-one, onto, both, or neither. Then state whether it is discrete or continuous.
15. \( y = -3x + 2 \)
16. \( y = 0.5x - 3 \)
17. \( y = 2x^2 \)
18. \( y = -5x^2 \)
19. \( y = 4x^2 - 8 \)
20. \( y = -3x^3 - 1 \)

Example 4  Evaluate each function.
21. \( f(-8) \) if \( f(x) = 5x^3 + 1 \)
22. \( f(2.5) \) if \( f(x) = 16x^2 \)

23. DIVING  The table below shows the pressure on a diver at various depths.

<table>
<thead>
<tr>
<th>Depth (ft)</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure (atm)</td>
<td>1</td>
<td>1.6</td>
<td>2.2</td>
<td>2.8</td>
<td>3.4</td>
<td>4</td>
</tr>
</tbody>
</table>

a. Write a relation to represent the data.

b. Graph the relation.

c. Identify the domain and range. Is the relation discrete or continuous?

d. Is the relation a function? Explain your reasoning.

Find each value if \( f(x) = 3x + 2 \), \( g(x) = -2x^2 \), and \( h(x) = -4x^2 - 2x + 5 \).
24. \( f(-5) \)
25. \( f(9) \)
26. \( g(-3) \)
27. \( g(-6) \)
28. \( h(3) \)
29. \( h(8) \)
30. \( f\left(\frac{2}{3}\right) \)
31. \( g\left(\frac{3}{2}\right) \)
32. \( h\left(\frac{1}{5}\right) \)
**PODCASTS** Chaz has a collection of 15 podcasts downloaded on his digital audio player. He decides to download 3 more podcasts each month. The function \( P(t) = 15 + 3t \) counts the number of podcasts \( P(t) \) he has after \( t \) months. How many podcasts will he have after 8 months?

**34. MULTIPLE REPRESENTATIONS** In this problem you will investigate one-to-one and onto functions.

a. **Graphical** Graph each function on a separate graphing calculator screen.
   
   \[
   f(x) = x^2 \quad g(x) = 2^x \quad h(x) = x^3 - 3x^2 - 5x + 6 \quad j(x) = x^3
   \]

b. **Tabular** Use the graphs to create a table showing the number of times a horizontal line could intersect the graph of each function. List all possibilities.

c. **Analytical** For a function to be one-to-one, a horizontal line on the graph of the function can intersect the function at most once. Which functions meet this condition? Which do not? Explain your reasoning.

d. **Analytical** For a function to be onto, every possible horizontal line on the graph of the function must intersect the function at least once. Which functions meet this condition? Which do not? Explain your reasoning.

e. **Graphical** Create a table showing whether each function is one-to-one and/or onto.

**H.O.T. Problems** Use Higher-Order Thinking Skills

35. **ERROR ANALYSIS** Omar and Madison are finding \( f(3d) \) for the function \( f(x) = -4x^2 - 2x + 1 \). Is either of them correct? Explain your reasoning.

   **Omar**
   
   \[
   f(3d) = -4(3d)^2 - 2(3d) + 1
   \]
   
   \[
   = -36d^2 - 6d + 1
   \]

   **Madison**
   
   \[
   f(3d) = -4(3d)^2 - 2(3d) + 1
   \]
   
   \[
   = 12d^2 - 6d + 1
   \]

36. **CHALLENGE** Consider the functions \( f(x) \) and \( g(x) \). \( f(a) = 19 \) and \( g(a) = 33 \), while \( f(b) = 31 \) and \( g(b) = 51 \). If \( a = 5 \) and \( b = 8 \), find two possible functions to represent \( f(x) \) and \( g(x) \).

37. **REASONING** If the graph of a relation crosses the \( y \)-axis at more than one point, is the relation sometimes, always, or never a function? Explain your reasoning.

38. **OPEN ENDED** Graph a relation that can be used to represent each of the following.
   
   a. the height of a baseball that is hit into the outfield
   
   b. the speed of a car that travels to the store, stopping at two lights along the way
   
   c. the height of a person from age 5 to age 80
   
   d. the temperature on a typical day from 6 A.M. to 11 P.M.

39. **REASONING** Determine whether the following statement is true or false. Explain your reasoning.

   *If a function is onto, then it must be one-to-one as well.*

40. **WRITING IN MATH** Explain why the vertical line test can determine if a relation is a function.
**Standardized Test Practice**

41. Patricia’s swimming pool contains 19,500 gallons of water. She drains the pool at a rate of 6 gallons per minute. Which of these equations represents the number of gallons of water \( g \) remaining in the pool after \( m \) minutes?

A \( g = 19,500 - 6m \)  
B \( g = 19,500 + 6m \)  
C \( g = \frac{19,500}{6m} \)  
D \( g = \frac{19,500}{19,500} \)

42. **SHORT RESPONSE** Look at the pattern below.

\[ \frac{5}{2}, -2, -\frac{3}{2}, -1, \ldots \]

If the pattern continues, what will the next term be?

**43. GEOMETRY** Which set of dimensions represents a triangle similar to the triangle shown below?

\[
\begin{array}{c}
5 \\
12 \\
13
\end{array}
\]

F 1 unit, 2 units, 3 units  
G 7 units, 11 units, 12 units  
H 10 units, 23 units, 24 units  
J 20 units, 48 units, 52 units

44. **SAT/ACT** If \( g(x) = x^2 \), which expression is equal to \( g(x + 1) \)?

A \( 1 \)  
B \( x^2 + 1 \)  
C \( x^2 + 2x + 1 \)  
D \( x^2 - x \)  
E \( x^2 + x + 1 \)

**Spiral Review**

**Solve each inequality.** *(Lesson 1-6)*

45. \( 48 > 7y + 6 > 20 \)  
46. \( z + 12 > 18 \) or \( -2z + 16 > 12 \)  
47. \( 2|4x + 2| + 3 > 21 \)

48. **CLUBS** Mr. Willis is starting a chess club at his high school. He sent an advertisement at the right to all of the homerooms. Write an absolute value inequality representing the situation. *(Lesson 1-6)*

49. **SALES** Ling can spend no more than $120 at the summer sale of a department store. She wants to buy shirts on sale for $15 each. Write and solve an inequality to determine the number of shirts she can buy. *(Lesson 1-5)*

**Solve each equation. Check your solutions.** *(Lesson 1-4)*

50. \( 18 = 2|2a + 6| - 2 \)  
51. \( 2 = -3|4c - 5| + 8 \)  
52. \( -5 = 2|3b + 4| - 9 \)

**Simplify each expression.** *(Lesson 1-2)*

53. \( 6(3a - 2b) + 3(5a + 4b) \)  
54. \( -4(5x - 3y) + 2(y + 3x) \)  
55. \( -7(2c - 4d) + 8(3c + d) \)

**Skills Review**

**Solve each equation. Check your solutions.** *(Lesson 1-3)*

56. \( 5x + 2 = 32 \)  
57. \( 6a - 3 = 21 \)  
58. \( -2x + 5 = 5x + 19 \)

59. \( 6b + 4 = -2b - 28 \)  
60. \( 2(x + 5) - 3(x - 4) = 19 \)  
61. \( 4(2y - 3) + 5(3y + 1) = -99 \)

62. \( 5c - 8 + 2c = 4c + 10 \)  
63. \( 8d - 4 + 3d = 2d - 100 - 7d \)  
64. \( 10y - 5 - 3y = 4(2y + 3) - 20 \)

**connectED.mcgraw-hill.com**
OBJECTIVE Use discrete and continuous functions to solve real-world problems.

A cup of frozen yogurt costs $2 at the Yogurt Shack. We might describe the cost of \( x \) cups of yogurt using the continuous function \( y = 2x \), where \( y \) is the total cost in dollars. The graph of that function is shown at the right.

From the graph, you can see that 2 cups of yogurt cost $4, 3 cups cost $6, and so on. The graph also shows that 1.5 cups of yogurt cost 2(1.5) or $3. However, the Yogurt Shack probably will not sell partial cups of yogurt. This function is more accurately modeled with a discrete function.

The graph of the discrete function at the right also models the cost of buying cups of frozen yogurt. The domain in this graph makes sense in this situation.

When choosing a discrete function or a continuous function to model a real-world situation, consider whether all real numbers make sense as part of the domain.

Exercises

Determine whether each function is correctly modeled using a discrete or continuous function. Explain your reasoning.

1. **Converting Units**

2. **E-Mails Received**

3. \( y \) represents the distance a car travels in \( x \) hours.

4. \( y \) represents the total number of riders who have ridden on a roller coaster after \( x \) rides.

5. **WRITING IN MATH** Give an example of a real-world function that is discrete and a real-world function that is continuous. Explain your reasoning.
Linear Relations and Functions

The points on the graph above lie along a straight line. Relations that have straight line graphs are called linear relations. Relations that are not linear are called nonlinear relations. An equation such as $x + y = 5$ is called a linear equation. A linear equation has no operations other than addition, subtraction, and multiplication of a variable by a constant. The variables may not be multiplied together or appear in a denominator. A linear equation does not contain variables with exponents other than 1. The graph of a linear equation is always a line.

<table>
<thead>
<tr>
<th>Linear equations</th>
<th>Nonlinear equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4x - 5y = 16$</td>
<td>$2x + 6y^2 = -25$</td>
</tr>
<tr>
<td>$x = 10$</td>
<td>$y = \sqrt{x} + 2$</td>
</tr>
<tr>
<td>$y = -\frac{2}{3}x - 1$</td>
<td>$x + xy = -\frac{5}{8}$</td>
</tr>
<tr>
<td>$y = \frac{1}{2}x$</td>
<td>$y = \frac{1}{x}$</td>
</tr>
</tbody>
</table>

A linear function is a function with ordered pairs that satisfy a linear equation. Any linear function can be written in the form $f(x) = mx + b$, where $m$ and $b$ are real numbers.

Example 1 Identify Linear Functions

State whether each function is a linear function. Write yes or no. Explain.

a. $f(x) = 8 - \frac{3}{4}x$
   Yes; it can be written as $f(x) = -\frac{3}{4}x + 8$.
   $m = -\frac{3}{4}, b = 8$

b. $f(x) = \frac{2}{x}$
   No; the expression includes division by the variable.

c. $g(x, y) = 3xy - 4$
   No; the two variables are multiplied together.

Guided Practice

1A. $f(x) = \frac{5}{x + 6}$
1B. $g(x) = -\frac{3}{2}x + \frac{1}{3}$

You can evaluate linear functions by substituting values for $x$ or $f(x)$. 
Real-World Example 2: Evaluate a Linear Function

PLANTS The growth rate of a sample of Bermuda grass is given by the function \( f(x) = 5.9x + 3.25 \), where \( f(x) \) is the total height in inches \( x \) days after an initial measurement.

a. How tall is the sample after 3 days?

\[
\begin{align*}
  f(x) &= 5.9x + 3.25 & \text{Original function} \\
  f(3) &= 5.9(3) + 3.25 & \text{Substitute 3 for } x \\
        &= 20.95 & \text{Simplify.}
\end{align*}
\]

The height of the sample after 3 days is 20.95 inches.

b. The term 3.25 in the function represents the height of the grass when it was initially measured. The sample is how many times as tall after 3 days?

Divide the height after 3 days by the initial height.

\[
\frac{20.95}{3.25} \approx 6.4
\]

The height after 3 days is about 6.4 times as great as the initial height.

Guided Practice

2A. If the Bermuda grass is 50.45 inches tall, how many days has it been since it was last cut?

2B. Is it reasonable to think that this rate of growth can be maintained for long periods of time? Explain.

Standard Form Any linear equation can be written in standard form, \( Ax + By = C \), where \( A \), \( B \), and \( C \) are integers with a greatest common factor of 1.

Key Concept Standard Form of a Linear Equation

Words The standard form of a linear equation is \( Ax + By = C \), where \( A \), \( B \), and \( C \) are integers with a greatest common factor of 1, \( A \geq 0 \), and \( A \) and \( B \) are not both zero.

Examples \( 3x + 5y = 12; A = 3, B = 5, \) and \( C = 12 \)

Example 3 Standard Form

Write \( \frac{-3}{10}x = 8y - 15 \) in standard form. Identify \( A, B, \) and \( C \).

\[
\begin{align*}
  \frac{-3}{10}x &= 8y - 15 & \text{Original equation} \\
  \frac{-3}{10}x - 8y &= -15 & \text{Subtract 8y from each side.} \\
  3x + 80y &= 150 & \text{Multiply each side by } -10.
\end{align*}
\]

\( A = 3, B = 80, \) and \( C = 150 \)

Guided Practice

Write each equation in standard form. Identify \( A, B, \) and \( C. \)

3A. \( 2y = 4x + 5 \)

3B. \( 3x - 6y - 9 = 0 \)
Example 4 Use Intercepts to Graph a Line

Find the x-intercept and the y-intercept of the graph of \(2x - 3y + 8 = 0\). Then graph the equation.

The x-intercept is the value of \(x\) when \(y = 0\).

\[
2x - 3y + 8 = 0 \quad \text{Original equation}
\]
\[
2x - 3(0) + 8 = 0 \quad \text{Substitute 0 for } y.
\]
\[
2x = -8 \quad \text{Subtract 8 from each side.}
\]
\[
x = -4 \quad \text{Divide each side by 2.}
\]

The x-intercept is \(-4\).

Likewise, the y-intercept is the value of \(y\) when \(x = 0\).

\[
2x - 3y + 8 = 0 \quad \text{Original equation}
\]
\[
2(0) - 3y + 8 = 0 \quad \text{Substitute 0 for } x.
\]
\[
-3y = -8 \quad \text{Subtract 8 from each side.}
\]
\[
y = \frac{8}{3} \quad \text{Divide each side by 3.}
\]

The y-intercept is \(\frac{8}{3}\).

Use these ordered pairs to graph the equation.

Guided Practice

4. Find the x-intercept and the y-intercept of the graph of \(2x + 5y - 10 = 0\). Then graph the equation.

Check Your Understanding

Example 1 State whether each function is a linear function. Write yes or no. Explain.

1. \(f(x) = \frac{x + 12}{5}\) 2. \(g(x) = \frac{7 - x}{x}\) 3. \(p(x) = 3x^2 - 4\) 4. \(q(x) = -8x - 21\)

Example 2 RECREATION You want to make sure that you have enough music for a car trip. If each CD is an average of 45 minutes long, the linear function \(m(x) = 0.75x\) could be used to find out how many CDs you need to bring.

a. If you have 4 CDs, how many hours of music is that?

b. If the trip you are taking is 6 hours, how many CDs should you bring?

Example 3 Write each equation in standard form. Identify \(A\), \(B\), and \(C\).

6. \(y = -4x - 7\) 7. \(y = 6x + 5\) 8. \(3x = -2y - 1\)

9. \(-8x = 9y - 6\) 10. \(12y = 4x + 8\) 11. \(4x - 6y = 24\)

Example 4 Find the x-intercept and the y-intercept of the graph of each equation. Then graph the equation using the intercepts.

12. \(y = 5x + 12\) 13. \(y = 4x - 10\) 14. \(2x + 3y = 12\) 15. \(3x - 4y - 6 = 15\)
Example 1  State whether each equation or function is a linear function. Write yes or no. Explain.
   16. \(3y - 4x = 20\)  17. \(y = x^2 - 6\)  18. \(h(x) = 6\)
   19. \(j(x) = 2x^2 + 4x + 1\)  20. \(g(x) = 5 + \frac{6}{x}\)  21. \(f(x) = \sqrt{7 - x}\)
   22. \(4x + \sqrt{y} = 12\)  23. \(\frac{1}{x} + \frac{1}{y} = 1\)  24. \(f(x) = \frac{4x}{5} + \frac{8}{3}\)

Example 2  25. ROLLER COASTERS  The speed of the Steel Dragon 2000 roller coaster in Mie Prefecture, Japan, can be modeled by \(y = 10.4x\), where \(y\) is the distance traveled in meters in \(x\) seconds.
   a. How far does the coaster travel in 25 seconds?
   b. The speed of Kingda Ka in Jackson, New Jersey, can be described by \(y = 33.9x\). Which coaster travels faster? Explain your reasoning.

Example 3  Write each equation in standard form. Identify \(A, B,\) and \(C\).
   26. \(-7x - 5y = 35\)  27. \(8x + 3y + 6 = 0\)  28. \(10y - 3x + 6 = 11\)
   29. \(-6x - 3y - 12 = 21\)  30. \(3y = 9x - 12\)  31. \(2.4y = -14.4x\)
   32. \(\frac{2}{3}y - \frac{3}{4}x + \frac{1}{6} = 0\)  33. \(\frac{4}{5}y + \frac{1}{8}x = 4\)  34. \(-0.08x = 1.24y - 3.12\)

Example 4  Find the \(x\)-intercept and the \(y\)-intercept of the graph of each equation. Then graph the equation using the intercepts.
   35. \(y = -8x - 4\)  36. \(5y = 15x - 90\)  37. \(-4y + 6x = -42\)
   38. \(-9x - 7y = -30\)  39. \(\frac{1}{3}y - \frac{2}{9}x = 4\)  40. \(\frac{3}{4}y - \frac{2}{3}x = 12\)

41. FINANCIAL LITERACY  Latonya earns a commission of \(1.75\) for each magazine subscription she sells and \(1.50\) for each newspaper subscription she sells. Her goal is to earn a total of \(525\) in commissions in the next two weeks.
   a. Write an equation that is a model for the different numbers of magazine and newspaper subscriptions that can be sold to meet the goal.
   b. Graph the equation. Does this equation represent a function? Explain.
   c. If Latonya sells 100 magazine subscriptions and 200 newspaper subscriptions, will she meet her goal? Explain.

42. SNAKES  Suppose the body length \(L\) in inches of a baby snake is given by \(L(m) = 1.5 + 2m\), where \(m\) is the age of the snake in months until it becomes 12 months old.
   a. Find the length of an 8-month-old snake.
   b. Find the snake’s age if the length of the snake is 25.5 inches.

43. STATE FAIR  The Ohio State Fair charges \$8 for admission and \$5 for parking. After Joey pays for admission and parking, he plans to spend all of his remaining money at the ring game, which costs \$3 per game.
   a. Write an equation representing the situation.
   b. How much did Joey spend at the fair if he paid \$6 for food and drinks and played the ring game 4 times?
Write each equation in standard form. Identify \( A \), \( B \), and \( C \).

44. \( \frac{x + 5}{3} = -2y + 4 \)  
45. \( \frac{4x - 1}{5} = 8y - 12 \)  
46. \( \frac{-2x - 8}{3} = -12y + 18 \)

Find the \( x \)-intercept and the \( y \)-intercept of the graph of each equation.

47. \( \frac{6x + 15}{4} = 3y - 12 \)  
48. \( \frac{-8x + 12}{3} = 16y + 24 \)  
49. \( \frac{15x + 20}{4} = \frac{3y + 6}{5} \)

50. **FUNDRAISING** The Freshman Class Student Council wanted to raise money by giving car washes. The students spent $10 on supplies and charged $2 per car wash.
   a. Write an equation to model the situation.
   b. Graph the equation.
   c. How much money did they earn after 20 car washes?
   d. How many car washes are needed for them to earn $100?

51. **MULTIPLE REPRESENTATIONS** Consider the following linear functions.
   
   \( f(x) = -2x + 4 \)  
   \( g(x) = 6 \)  
   \( h(x) = \frac{1}{3}x + 5 \)

   a. **Graphical** Graph the linear functions on separate graphs.
   b. **Tabular** Use the graphs to complete the table.

<table>
<thead>
<tr>
<th>Function</th>
<th>One-to-One</th>
<th>Onto</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = -2x + 4 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( g(x) = 6 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( h(x) = \frac{1}{3}x + 5 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. **Verbal** Are all linear functions one-to-one and/or onto? Explain your reasoning.

**H.O.T. Problems** Use Higher-Order Thinking Skills

52. **CHALLENGE** Write a function with an \( x \)-intercept of \((a, 0)\) and a \( y \)-intercept of \((0, b)\).

53. **OPEN ENDED** Write an equation of a line with an \( x \)-intercept of 3.

54. **REASONING** Determine whether an equation of the form \( x = a \), where \( a \) is a constant, is sometimes, always, or never a function. Explain your reasoning.

55. **WHICH ONE DOESN’T BELONG?** Of the four equations shown, identify the one that does not belong. Explain your reasoning.

   \[ y = 2x + 3 \]  
   \[ 2x + y = 5 \]  
   \[ y = 5 \]  
   \[ y = 2xy \]

56. **WRITING IN MATH** Consider the graph of the relationship between hours worked and earnings.
   a. When would this graph represent a linear relationship? Explain your reasoning.
   b. Provide another example of a linear relationship in a real-world situation.
57. Tom bought \( n \) DVDs for a total cost of \( 15n - 2 \) dollars. Which expression represents the cost of each DVD?

A \( n(15n - 2) \)  
B \( n + (15n - 2) \)  
C \( (15n - 2) \div n; n \neq 0 \)  
D \( (15n - 2) - n \)

58. SHORT RESPONSE What is the complete solution of the equation?

\[ |9 - 3x| = 18 \]

59. NUMBER THEORY If \( a, b, c, \) and \( d \) are consecutive odd integers and \( a < b < c < d \), how much greater is \( c + d \) than \( a + b \)?

F 2  
H 6  
G 4  
J 8

60. SAT/ACT Which function is linear?

A \( f(x) = x^2 \)  
B \( g(x) = \sqrt{x - 1} \)  
C \( f(x) = \sqrt{9 - x^2} \)  
D \( g(x) = \frac{2.7}{x} \)  
E \( f(x) = 2x \)

Spiral Review

State the domain and range of each relation. Then determine whether each relation is a function. If it is a function, determine if it is one-to-one, onto, both, or neither. (Lesson 2-1)

61.

62.

63.

64. SHOPPING Claudio is shopping for a new television. The average price of the televisions he likes is $800, and the actual prices differ from the average by up to $350. Write and solve an absolute value inequality to determine the price range of the televisions. (Lesson 1-6)

Evaluate each expression if \( a = -6, b = 5, \) and \( c = 3.6 \). (Lesson 1-1)

65. \( \frac{6a - 3c}{2ab} \)  
66. \( \frac{a + 7b}{4bc} \)  
67. \( \frac{b - c}{a + c} \)

68. FOOD Brandi can order a small, medium, or large pizza with pepperoni, mushrooms, or sausage. How many different one-topping pizzas can she order? (Lesson 0-4)

Skills Review

Evaluate each expression. (Lesson 1-1)

69. \( \frac{12 - 8}{4 - (-2)} \)  
70. \( \frac{5 - 9}{3 - (-6)} \)  
71. \( \frac{-2 - 8}{3 - (-5)} \)  
72. \( \frac{-2 - (-6)}{-1 - (-8)} \)

73. \( \frac{-7 - (-11)}{-3 - 9} \)  
74. \( \frac{-1 - 8}{7 - (-3)} \)  
75. \( \frac{-12 - (-3)}{-6 - (-5)} \)  
76. \( \frac{4 - 3}{2 - 5} \)
The solution of an equation is called the root of the equation.

Example Determine Roots

Find the root of \( 0 = 5x - 10 \).

\[
\begin{align*}
0 &= 5x - 10 \quad & \text{Original equation} \\
10 &= 5x \quad & \text{Add 10 to each side.} \\
2 &= x \quad & \text{Divide each side by 5.}
\end{align*}
\]

The root of the equation is 2.

You can also find the root of an equation by finding the zero of its related function. Values of \( x \) for which \( f(x) = 0 \) are called zeros of the function \( f \).

Linear Equation  Related Linear Function
\[
\begin{align*}
0 &= 5x - 10 \\
f(x) &= 5x - 10 \text{ or } y = 5x - 10
\end{align*}
\]

The zero of a function is the \( x \)-intercept of its graph. Since the graph of \( y = 5x - 10 \) intersects the \( x \)-axis at 2, the zero of the function is 2.

Exercises

1. Use \( 0 = 4x + 10 \) and \( f(x) = 4x + 10 \) to distinguish among roots, solutions, and zeros.

2. Relate solutions of equations and \( x \)-intercepts of graphs.

Determine whether each statement is true or false. Explain your reasoning.

3. The function graphed at the right has two zeros, \(-2\) and \(-1\).

4. The root of \( 6x + 9 = 0 \) is \(-1.5\).

5. \( f(0) \) is a zero of the function \( f(x) = -\frac{2}{3}x + 12 \).

6. **FUNDRAISERS** The function \( y = 2x - 150 \) represents the money raised \( y \) when the Boosters sell \( x \) soft drinks at a basketball game. Find the zero and describe what it means in the context of this situation. Make a connection between the zero of the function and the root of \( 0 = 2x - 150 \).
Lesson 2-3
Rate of Change and Slope

Then

• You graphed linear relations. (Lesson 2-2)

Now

1. Find rate of change.
2. Determine the slope of a line.

Why?

• The table shows the total distance a car traveled over various time intervals. The distance formula, \( rt = d \) or \( r = \frac{d}{t} \), relates time and distance.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Distance (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>68</td>
</tr>
<tr>
<td>2.5</td>
<td>170</td>
</tr>
<tr>
<td>3</td>
<td>204</td>
</tr>
<tr>
<td>4.5</td>
<td>306</td>
</tr>
<tr>
<td>5</td>
<td>340</td>
</tr>
</tbody>
</table>

1 Rate of Change

Rate of change is a ratio that compares how much one quantity changes, on average, relative to the change in another quantity. If \( x \) is the independent variable and \( y \) is the dependent variable, then rate of change \( \frac{\Delta y}{\Delta x} \).

This is sometimes referred to as \( \frac{\Delta y}{\Delta x} \).

Real-World Example 1

CHEMISTRY

The table shows the temperature of a solution after it has been removed from a heat source. Find the rate of change in temperature for the solution.

Use the ordered pairs (2, 139.4) and (5, 133.1).

rate of change = \( \frac{\text{change in } y}{\text{change in } x} \)

= \( \frac{\text{change in temperature}}{\text{change in time}} \)

= \( \frac{133.1 - 139.4}{5 - 2} \)

= \( \frac{-6.3}{3} \) or \( -2.1 \)

The rate of change is -2.1. This means that the temperature is decreasing by 2.1°C each minute.

Guided Practice

1. RECREATION

The graph at the right shows the number of gallons of water in a swimming pool as it is being filled. At what rate is the pool being filled?
Up to this point, you have used rates of change that are constant. Many real-world situations involve rates of change that are not constant. These situations are often described using an average rate of change over a specified interval.

### Real-World Example 2  Average Rate of Change

**MUSIC** Refer to the graph at the right. Find the average rate of change of the percent of total music sales for both CDs and downloads from 2001 to 2008. Compare the rates.

**CDs:**
rate of change \[= \frac{\text{change in } y}{\text{change in } x}\]
\[= \frac{\text{change in percent}}{\text{change in time}}\]
\[= \frac{77.8 - 89.2}{2008 - 2001} = \frac{-11.4}{7} = -1.63\]

**Downloads:**
rate of change \[= \frac{\text{change in } y}{\text{change in } x}\]
\[= \frac{\text{change in percent}}{\text{change in time}}\]
\[= \frac{12.8 - 0.2}{2008 - 2001} = \frac{12.6}{7} = 1.8\]

The percent of CD music sales declined at an average rate of 1.63% per year, while the percent of downloaded music sales increased at an average rate of 1.8% per year.

### Guided Practice

2. **EDUCATION** In 2000, 23,142 students applied to State College, and 34,689 students applied to Central University. In 2008, 29,563 students applied to State College, and 36,107 applied to Central University. Determine the average rate of change in applicants for both schools from 2000 to 2008.

2 **Slope** The **slope** of a line is the ratio of the change in the \( y \)-coordinates to the corresponding change in the \( x \)-coordinates. The slope of a line is the same as its rate of change.

Suppose a line passes through points at \((x_1, y_1)\) and \((x_2, y_2)\).

\[
\text{Slope} = \frac{\text{change in } y \text{-coordinates}}{\text{change in } x \text{-coordinates}} = \frac{y_2 - y_1}{x_2 - x_1}
\]
Key Concept  Slope of a Line

**Words**  The slope of a line is the ratio of the change in y-coordinates to the change in x-coordinates.

**Symbols**  The slope \( m \) of a line passing through \((x_1, y_1)\) and \((x_2, y_2)\) is given by \( m = \frac{y_2 - y_1}{x_2 - x_1} \), where \( x_1 \neq x_2 \).

Example 3  Find Slope Using Coordinates

Find the slope of the line that passes through \((-4, 3)\) and \((2, 5)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope Formula}
\]

\[
= \frac{5 - 3}{2 - (-4)} \quad (x_1, y_1) = (-4, 3), (x_2, y_2) = (2, 5)
\]

\[
= \frac{2}{6} \text{ or } \frac{1}{3} \quad \text{Simplify.}
\]

Guided Practice

Find the slope of the line that passes through each pair of points.

3A. \((1, -3)\) and \((3, 5)\)  
3B. \((-8, 11)\) and \((24, -9)\)

Study Tip  Slope is Constant  The slope of a line is the same, no matter what two points on the line are used.

Example 4  Find Slope Using a Graph

Find the slope of the line shown at the right.

The line passes through \((-2, 0)\) and \((0, -3)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope Formula}
\]

\[
= \frac{-3 - 0}{0 - (-2)} \quad (x_1, y_1) = (-2, 0), (x_2, y_2) = (0, -3)
\]

\[
= \frac{-3}{2} \text{ or } -\frac{3}{2} \quad \text{Simplify.}
\]

Guided Practice

Find the slope of each line.

4A.  
4B.
Check Your Understanding

Example 1  Find the rate of change for each set of data.

1.  | Time (min) | 2 | 4 | 6 | 8 | 10 |
    | Distance (ft) | 12 | 24 | 36 | 48 | 60 |

2.  | Time (sec) | 5 | 10 | 15 | 20 | 25 |
    | Volume (cm$^3$) | 16 | 32 | 48 | 64 | 80 |

Example 2  Find the rate of change for each set of data.

3. **CAMERAS** The graph shows the number of digital still cameras and film cameras sold by Yellow Camera Stores in recent years.
   a. Find the average rate of change of the number of digital cameras sold from 2004 to 2009.
   b. Find the average rate of change of the number of film cameras sold from 2004 to 2009.
   c. What do the signs of each rate of change represent?

Example 3  Find the slope of the line that passes through each pair of points.

4. (3, 2), (8, 12)  5. (−1, 4), (3, −8)  6. (−2, −5), (−7, 10)

Example 4  Determine the rate of change of each graph.

7. ![Graph 1](image1.png)

8. ![Graph 2](image2.png)

Practice and Problem Solving

Example 1  Find the rate of change for each set of data.

9.  | Time (day) | 3 | 6 | 9 | 12 | 15 |
    | Height (mm) | 20 | 40 | 60 | 80 | 100 |

10.  | Weight (lb) | 11 | 22 | 33 | 44 | 55 |
    | Cost ($) | 8 | 16 | 24 | 32 | 40 |
Example 2  

11. **HEALTH** The table below shows Lisa’s temperature during an illness over a 3-day period.

<table>
<thead>
<tr>
<th>Day</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>8:00 A.M.</td>
<td>8:00 P.M.</td>
<td>8:00 A.M.</td>
</tr>
<tr>
<td>Temp (°F)</td>
<td>100.5</td>
<td>102.3</td>
<td>103.1</td>
</tr>
</tbody>
</table>

**a.** What was the average rate of change in Lisa’s temperature from 8:00 A.M. on Monday to 8:00 P.M. on Monday?

**b.** What was the average rate of change in Lisa’s temperature from 8:00 A.M. on Tuesday to 8:00 P.M. on Wednesday? Is your answer reasonable? What does the sign of the rate mean?

**c.** During which 12-hour period was the average rate of change in Lisa’s temperature the greatest?

Example 3  

Find the slope of the line that passes through each pair of points. Express as a fraction in simplest form.

12. (−2, 11), (5, 6)  
13. (−9, −11), (6, 3)  
14. (−1.5, 3.5), (4.5, 6)  
15. (−4.5, 9.5), (−1, 2.5)  
16. (−8, −0.5), (−4, 5)  
17. (−6, −2), (−1.5, 5.5)

Example 4  

Determine the rate of change of each graph.

22. **RECREATION** The table shows your height on a water slide at various time intervals.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Height (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>120</td>
</tr>
<tr>
<td>1</td>
<td>90</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

**a.** Graph the height versus the time on the water slide.

**b.** Find the average rate of change in a rider’s height between 1 and 3 seconds.

**c.** Find the average rate of change in a rider’s height between 0 and 5 seconds.

**d.** What is another word for rate of change in this situation?

Determine the rate of change for each equation.

23. \(6y = 8x - 40\)  
24. \(-2y - 16x = 41\)  
25. \(12x - 4y + 5 = 18\)  
26. \(20x + 85y = 120\)  
27. \(\frac{3}{2}y - \frac{5}{4}y = 15\)  
28. \(\frac{1}{6}y + \frac{3}{8}y = 24\)
29 WASHINGTON MONUMENT  The Washington Monument is 555 feet $\frac{51}{8}$ inches tall and weighs 90,854 tons. The monument is topped by an aluminum square pyramid. The sides of the pyramid’s base measure 5.6 inches, and the pyramid is 8.9 inches tall. Estimate the slope that a face of the pyramid makes with its base.

30. MARINE LIFE  The illustrations show the growth of a starfish over time.
   a. Find the average rate of change in the measure over time.
   b. Predict the size of the starfish in 2009.

Find the value of $r$ so that the line that passes through each pair of points has the given slope.

31. $(6, r), (3, 3), m = 2$
32. $(8, 1), (5, r), m = \frac{1}{3}$
33. $(10, r), (4, -3), m = \frac{4}{3}$
34. $(8, -2), (r, -6), m = -4$

35. MULTIPLE REPRESENTATIONS  In this problem, you will explore the rate of change for the function $f(x) = x^2$.
   a. Graphical  Graph $f(x) = x^2$.
   b. Tabular  Copy and complete the table. To complete the slope row, find the slope of the line containing two consecutive points such as $(4, 16)$ and $(3, 9)$. The first one is completed for you.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>16</td>
<td>9</td>
<td></td>
<td></td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>slope</td>
<td></td>
<td></td>
<td></td>
<td>-7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
   c. Verbal  Describe what happens to the rate of change for $f(x) = x^2$ as $x$ increases.

H.O.T. Problems  Use Higher-Order Thinking Skills

36. ERROR ANALYSIS  Patty and Tim are asked to find the slope of the line passing through the points $(4, 3)$ and $(7, 9)$. Is either of them correct? Explain.

<table>
<thead>
<tr>
<th>Patty</th>
<th>Tim</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m = \frac{9 - 3}{7 - 4} = \frac{6}{3} = 2$</td>
<td>$m = \frac{7 - 4}{9 - 3} = \frac{3}{2}$</td>
</tr>
</tbody>
</table>

37. CHALLENGE  The graph of a line passes through the points $(2, 3)$ and $(5, 8)$. Explain how you would find the $y$-coordinate of the point $(11, y)$ on the same line. Then find $y$.

38. OPEN ENDED  Write an example of a function with a rate of change four times as large as its $x$-intercept.

39. REASONING  Determine whether the statement A line has a slope that is a real number is sometimes, always, or never true. Explain your reasoning.

40. WRITING IN MATH  Describe the process of finding the rate of change for each.
   a. a table of values   b. a graph   c. an equation
41. **GRIDDED RESPONSE** What is the slope of the line shown in the graph?

![Graph with a line and grid]

42. **SAT/ACT** In the figure below, the large square contains two smaller squares. If the areas of the two smaller squares are 4 and 25, what is the sum of the perimeters of the two shaded rectangles?

![Figure with a large square and two smaller squares]

A 14  
B 20  
C 24  
D 28  
E 49

43. **GEOMETRY** In \( \triangle ABC \) shown, \( AC = 16 \) and \( m \angle DAB = 60^\circ \). What is the measure of \( \overline{BD} \)?

F \( 9\sqrt{2} \)  
G \( 4\sqrt{3} \)  
H 9  
J 4

44. The table shows the cost of bananas depending on the amount purchased. Which conclusion can be made based on information in the table?

<table>
<thead>
<tr>
<th>Number of Pounds</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.45</td>
</tr>
<tr>
<td>20</td>
<td>4.60</td>
</tr>
<tr>
<td>50</td>
<td>10.50</td>
</tr>
<tr>
<td>100</td>
<td>19.00</td>
</tr>
</tbody>
</table>

A The cost of 10 pounds of bananas would be more than $4.  
B The cost of 200 pounds of bananas would be at most $38.  
C The cost of bananas is always more than $0.20 per pound.  
D The cost of bananas is always less than $0.28 per pound.

45. \( 6y - 8x = 19 \)  
46. \( 4x^2 = 2y - 9 \)  
47. \( 18 = 2xy + 6 \)

Evaluate each function.

48. \( f(-9) \) if \( f(x) = -7x + 8 \)  
49. \( g(-4) \) if \( g(x) = -3x^2 + 2 \)  
50. \( h(12) \) if \( h(x) = 4x^2 - 10x \)

51. **RACING** There are 8 contestants in a 400-meter race. In how many different ways can the top three runners finish?  

52. \((-4, -8)\)  
53. \((-2, 6)\)  
54. \((3, -1)\)

55. \( 8 = 4m - 6 \)  
56. \( -6 = 3(8) + b \)  
57. \( -2 = -3x + 5 \)
1. **Forms of Equations** Consider the line through \( A(0, b) \) and \( C(x, y) \). Notice that \( b \) is the \( y \)-intercept. You can use these two points to find the slope of \( AC \).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope Formula}
\]

\[
= \frac{y - b}{x - 0} \quad (x_1, y_1) = (0, b), (x_2, y_2) = (x, y)
\]

\[
= \frac{y - b}{x} \quad \text{Simplify.}
\]

Now solve the equation for \( y \).

\[
mx = y - b \quad \text{Multiply each side by} \, x.
\]

\[
mx + b = y \quad \text{Add} \, b \, \text{to each side.}
\]

\[
y = mx + b \quad \text{Symmetric Property of Equality}
\]

Equations written in this format are in **slope-intercept form**.

### Key Concept: Slope-Intercept Form

**Words**

The slope-intercept form of the equation of a line is \( y = mx + b \), where \( m \) is the slope and \( b \) is the \( y \)-intercept.

**Symbols**

\[
y = mx + b
\]

If you are given the slope and \( y \)-intercept of a line, you can find an equation of the line by substituting the values of \( m \) and \( b \) into the slope-intercept form.
Sometimes it is necessary to calculate the slope before you can write an equation.

**Example 1 Write an Equation in Slope-Intercept Form**

Write an equation in slope-intercept form for the line.

The graph intersects the \( y \)-axis at \(-2\). So \( b = -2\).

**Step 1** Find the slope.

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

\[
= \frac{-2 - (-1)}{0 - (-4)}
= \frac{-1}{4}
\]

\( \text{Slope Formula} \)

\( (x_1, y_1) = (-4, -1), (x_2, y_2) = (0, -2) \)

\( m = \frac{-1}{4} \) or \( b = -2 \)

**Step 2** Substitute the values into the slope-intercept equation.

\[
y = mx + b \quad \text{Slope-intercept form}
\]

\[
y = \frac{-1}{4}x - 2
\]

\( m = \frac{-1}{4}, b = -2 \)

**Guided Practice**

Write an equation in slope-intercept form for the line described.

1A. slope \( \frac{4}{3} \), passes through \((0, 4)\)
1B. passes through \((0, -6)\) and \((-4, 10)\)

If you know the slope of a line and the coordinates of a point on the line, you can use the **point-slope form** to find an equation of the line.

**Key Concept Point-Slope Form**

<table>
<thead>
<tr>
<th>Words</th>
<th>The point-slope form of the equation of a line is (y - y_1 = m(x - x_1)), where ((x_1, y_1)) are the coordinates of a point on the line and (m) is the slope of the line.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td>(y - y_1 = m(x - x_1))</td>
</tr>
<tr>
<td>slope</td>
<td>(y - y_1 = m(x - x_1))</td>
</tr>
<tr>
<td>coordinates of a point on the line</td>
<td>(y - y_1 = m(x - x_1))</td>
</tr>
</tbody>
</table>

**Example 2 Write an Equation Given Slope and One Point**

Write an equation of the line through \((6, -2)\) with a slope of \(-4\).

\[
y - y_1 = m(x - x_1)
\]

\[
y - (-2) = -4(x - 6)
\]

\[
y + 2 = -4x + 24
\]

\[
y = -4x + 22
\]

**Guided Practice**

Write an equation in slope-intercept form for the line described.

2A. passes through \((2, 3); m = \frac{1}{2}\)
2B. passes through \((-2, -1); m = -3\)
You can use any two points on a line to write an equation.

**Test Example 3**

Which is an equation of the line that passes through \((-2, 7)\) and \((3, -3)\)?

A) \(y = -\frac{1}{2}x - \frac{3}{2}\)  
B) \(y = -2x + 3\)  
C) \(y = \frac{1}{2}x + 8\)  
D) \(y = 2x + 11\)

**Definitions**  
Be certain to review key vocabulary, such as \(y\)-intercept, so that you understand what is being asked in a question.

**Read the Test Item**

You are given the coordinates of two points on the line.

**Solve the Test Item**

**Step 1**  
Find the slope of the line.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]  
\[ = \frac{-3 - 7}{3 - (-2)} \]  
\[ = \frac{-10}{5} \text{ or } -2 \]  
Simplify.

**Step 2**  
Write an equation. Use either ordered pair for \((x_1, y_1)\).

\[ y - y_1 = m(x - x_1) \]  
\[ y - (-3) = -2(x - 3) \]  
\[ y + 3 = -2x + 6 \]  
\[ y = -2x + 3 \]  
Simplify. Subtract 3 from each side.

The answer is B.

**Guided Practice**

3. Which is an equation of the line that passes through \((4, -9)\) and \((2, -4)\)?

F) \(y = -\frac{5}{2}x + 1\)  
G) \(y = -\frac{5}{2}x - 1\)  
H) \(y = \frac{2}{5}x + \frac{37}{5}\)  
J) \(y = \frac{2}{5}x - \frac{37}{5}\)

**Parallel and Perpendicular Lines**  
Slopes can help you determine whether two lines are parallel or perpendicular.

**Key Concept**  
**Parallel and Perpendicular Lines**

<table>
<thead>
<tr>
<th>Parallel Lines</th>
<th>Perpendicular Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two nonvertical lines are <strong>parallel</strong> if and only if they have the same slope. All vertical lines are parallel.</td>
<td>Two nonvertical lines are <strong>perpendicular</strong> if and only if the product of the slopes is (-1). Vertical lines and horizontal lines are perpendicular.</td>
</tr>
</tbody>
</table>
Example 4  Write an Equation of a Perpendicular Line

Write an equation in slope-intercept form for the line that passes through (5, –6) and is perpendicular to the line with equation \( y = -\frac{3}{2}x + 7 \).

The slope of the given line is \(-\frac{3}{2}\). Because the slopes of perpendicular lines are opposite reciprocals, the slope of the line perpendicular to the given line is \( \frac{2}{3} \).

Use the point-slope form and the ordered pair (5, –6).

\[
    y - y_1 = m(x - x_1) \quad \text{Point-slope form}
\]

\[
    y - (-6) = \frac{2}{3}(x - 5) \quad (x_1, y_1) = (5, -6) \quad \text{and} \quad m = \frac{2}{3}
\]

\[
    y + 6 = \frac{2}{3}x - \frac{10}{3} \quad \text{Distributive Property}
\]

\[
    y = \frac{2}{3}x - \frac{28}{3} \quad \text{Subtract 6 from each side and simplify.}
\]

CHECK  Graph both equations to verify the solution.

Guided Practice

4. Write an equation in slope-intercept form for the line that passes through (3, 7) and is parallel to the line with equation \( y = \frac{3}{4}x - 5 \).
Example 1 Write an equation in slope-intercept form for the line described.

8. slope 3, passes through (0, −2)
9. slope $\frac{1}{2}$, passes through (0, 5)
10. slope $\frac{6}{5}$, passes through (0, 8)
11. slope $\frac{9}{2}$, passes through $\left(0, \frac{13}{2}\right)$

Example 2 Write an equation of the line passing through each pair of points.

12. slope −2, passes through (−3, 14)
13. slope 4, passes through (6, 9)
14. slope $\frac{3}{5}$, passes through (−6, −8)
15. slope $\frac{1}{4}$, passes through (12, −4)

16. **PART-TIME JOB** Each week, Carmen earns a base pay of $15 plus $0.17 for every pamphlet that she delivers. Write an equation that can be used to find how much Carmen earns each week. How much will she earn the week that she delivers 300 pamphlets?

Example 3 Write an equation of the line passing through each pair of points.

17. (−2, −6), (4, 6)
18. (−8, −5), (−3, 10)
19. (−4, 12), (−2, −4)
20. (4.6, 3.4), (2.2, 2.8)
21. (5.5, 0.6), (1.1, 2.8)
22. (−25, −16), (−29, 12)

Example 4 Write an equation in slope-intercept form for the line that satisfies each set of conditions.

23. passes through (4, 2), perpendicular to $y = −2x + 3$
24. passes through (−6, −6), parallel to $y = \frac{4}{3}x + 8$
25. passes through (12, 0), parallel to $y = −\frac{1}{2}x − 3$
26. passes through (10, 2), perpendicular to $y = 4x + 6$

27. **FINANCIAL LITERACY** Julio buys a used car for $5900. Monthly expenses for the car—which include insurance, maintenance, and gas—average $180 per month. Write an equation that represents the total cost of buying and owning the car for $x$ months.

28. **DELI** The sales of a sandwich store increased approximately linearly from $52,000 to $116,000 during the first five years of business. Write an equation that models the sales $y$ after $x$ years. Determine what the sales will be at the end of 12 years if the pattern continues.

29. **WHALES** In 2009, it was estimated that there were 300 northern right whales in existence. The population of northern right whales is expected to decline by at least 25 whales each generation. Write an equation that represents the number of northern right whales that will be in existence in $x$ generations.

Write an equation in slope-intercept form for each graph.
33. **ROSES** Brad wants to send his girlfriend Kelli a dozen roses. He visits two stores. For what distance do the two stores charge the same amount to deliver a dozen roses?

<table>
<thead>
<tr>
<th></th>
<th>Full Bloom</th>
<th>Flowers R US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dozen roses</td>
<td>$30</td>
<td>$40</td>
</tr>
<tr>
<td>Delivery:</td>
<td>$3 per mile</td>
<td>$2 per mile</td>
</tr>
</tbody>
</table>

34. **TYPING** The equation \( y = 55(23 - x) \) can be used to model the number of words \( y \) you have left to type after \( x \) minutes.
   a. Write this equation in slope-intercept form.
   b. Identify the slope and \( y \)-intercept.
   c. Find the number of words you have left to type after 20 minutes.

35. **RECRUITING** As an army recruiter, Ms. Cooper is paid a daily salary plus commission. When she recruits 10 people, she earns $100. When she recruits 14 people, she earns $120.
   a. Write a linear equation to model this situation.
   b. What is Ms. Cooper’s daily salary?
   c. How much would Ms. Cooper earn in a day if she recruits 20 people?

36. **TRAVEL** Refer to the table at the right.
   a. Write and graph the linear equation that gives the distance \( y \) in kilometers in terms of the number \( x \) in miles.
   b. What distance in kilometers corresponds to 20 miles?
   c. What number is the same in kilometers and miles? Explain your reasoning.

<table>
<thead>
<tr>
<th>Miles</th>
<th>Kilometers</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>161</td>
</tr>
<tr>
<td>50</td>
<td>80.5</td>
</tr>
</tbody>
</table>

**H.O.T. Problems** Use Higher-Order Thinking Skills

37. **REASONING** Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

   *The quadrilateral formed by any two parallel lines and two lines perpendicular to those lines is a square.*

38. **CHALLENGE** Given \( \square ABCD \) with vertices \( A(a, b), B(c - a, d), C(c + a, d), \) and \( D(c, b) \), write an equation of a line perpendicular to diagonal \( \overline{BD} \) that contains \( A \).

39. **REASONING** Write \( y = ax + b \) in point-slope form.

40. **OPEN ENDED** Write the equations of two parallel lines with negative slopes.

41. **REASONING** Write an equation in point-slope form of a line with an \( x \)-intercept of \( c \) and \( y \)-intercept of \( d \).

42. **WRITING IN MATH** What is an advantage of representing a linear function in both standard form and slope-intercept form?
43. The total cost \( c \) in dollars to go to a water park and ride \( n \) water rides is given by the equation \( c = 15 + 3n \).

If the total cost was $33, how many water rides were ridden?

A 6  B 7  C 8  D 9

44. SHORT RESPONSE To raise money, the service club bought 1000 candy bars for $0.60 each. If the club sells all of the candy bars for $1 each, what will be their total profit?

45. PROBABILITY A fair six-sided die is tossed. What is the probability that a number less than 3 will show on the face of the die?

F \( \frac{1}{2} \)  G \( \frac{1}{3} \)  H \( \frac{2}{3} \)  J \( \frac{1}{6} \)

46. SAT/ACT What is an equation of the line through \( (\frac{1}{2}, -\frac{3}{2}) \) and \( (\frac{-1}{2}, \frac{1}{2}) \)?

A \( y = -2x - \frac{1}{2} \)  B \( y = -3x \)  C \( y = 2x - 5 \)  D \( y = \frac{1}{2}x + 1 \)  E \( y = -2x - \frac{5}{2} \)

50. RECREATION Scott is currently on page 210 of an epic novel that is 980 pages long. He plans to read 30 pages per day until he finishes the novel. Write and solve a linear equation to determine how many days it will take Scott to complete the novel. (Lesson 2-2)
An equation of a direct variation is a special case of a linear equation. A direct variation can be expressed in the form \( y = kx \). This means that \( y \) is a multiple of \( x \). The \( k \) in this equation is a constant and is called the constant of variation.

Notice that the graph of \( y = 4x \) is a straight line through the origin. An equation of a direct variation is a special case of an equation written in slope-intercept form, \( y = mx + b \). When \( m = k \) and \( b = 0 \), \( y = mx + b \) becomes \( y = kx \). So the slope of a direct variation equation is its constant of variation.

To express a direct variation, we say that \( y \) varies directly as \( x \). In other words, as \( x \) increases, \( y \) increases or decreases at a constant rate.

**Key Concept** Direct Variation

\( y \) varies directly as \( x \) if there is some nonzero constant \( k \) such that \( y = kx \). \( k \) is called the constant of variation.

**Activity**

**GOLD** The karat rating \( r \) of a gold object varies directly as the percentage \( p \) of gold in the object. A 14-karat ring is 58.25% gold.

**a.** Write and graph a direct variation equation relating \( r \) and \( p \).

Use the point \((0.5825, 14)\) to find the constant of variation.

\[
\begin{align*}
y &= kx \\
14 &= k(0.5825) \\
\end{align*}
\]

Divide each side by 0.5825.

\[
\begin{align*}
24.03 &\approx k \\
\end{align*}
\]

The direct variation equation is \( r = 24.03p \).

**b.** Find the karat rating of a ring that is 75% gold.

Use the calculator to find the karat rating.

**KEYSTROKES:**

\[2^\text{nd} \ [\text{CALC}] \ 0.75 \ \text{ENTER} \ 18.0225\]

The karat rating of a ring that is 75% gold is 18 karats.

**Mental Check**

75% of 24 = \( \frac{3}{4} \) of 24

Think \( \frac{3}{4} \) of 24 is 18.

**Exercises**

1. **SWIMMING** When you swim underwater, the pressure on your ears varies directly with the depth at which you are swimming. If you are swimming in 8 feet of water, the pressure on your ears is 3.44 pounds per square inch. Write and graph a direct variation equation relating pressure and depth. Then find the pressure at a depth of 65 feet.

2. Graph the direct variation equations \( y = -4x, y = -2x, y = 4x, \) and \( y = 2x \). Compare and contrast the graphs of the equations.
1. State the domain and range of the relation \{(-3, 2), (4, 1), (0, 3), (5, -2), (2, 7)\}. Then determine whether the relation is a function.

2. Graph \( y = 2x - 3 \) and determine whether the equation is a function, is one-to-one, onto, both, or neither. State whether it is discrete or continuous.

Given \( f(x) = 3x^3 - 2x + 7 \), find each value.

3. \( f(-2) \)  
4. \( f(2y) \)  
5. \( f(1.4) \)

6. State whether \( f(x) = 2x^2 - 9 \) is a linear function. Explain.

7. MULTIPLE CHOICE The daily pricing for renting a mid-sized car is given by the function \( f(x) = 0.35x + 49 \), where \( f(x) \) is the total rental price for a car driven \( x \) miles. Find the rental cost for a car driven 250 miles.

   A $84  
   B $112.50  
   C $136.50  
   D $215

Write each equation in standard form. Identify \( A, B, \) and \( C. \)

8. \( y = -6x + 5 \)  
9. \( y = 10x \)
10. \( -\frac{5}{8}x = 2y + 11 \)  
11. \( 0.5x = 3 \)

Find the \( x \)-intercept and the \( y \)-intercept of the graph of each equation. Then graph the equation using the intercepts.

12. \( 4x - 3y + 12 = 0 \)  
13. \( 10 - x = 2y \)

14. SPEED The table shows the distance traveled by a car after each time given in minutes. Find the rate of change in distance for the car.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Distance (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>45</td>
<td>60</td>
</tr>
<tr>
<td>60</td>
<td>80</td>
</tr>
<tr>
<td>75</td>
<td>120</td>
</tr>
</tbody>
</table>

Find the slope of the line that passes through each pair of points. Express as a fraction in simplest form.

15. \((-2, 6), (1, 15)\)  
16. \((3, 5), (7, 15)\)
17. \((4, 8), (4, -3)\)  
18. \((-2.5, 4), (1.5, -2)\)

19. Find the slope of the line shown.

Write an equation for the line that satisfies each set of conditions.

20. slope \( \frac{2}{3} \), passes through \((3, -4)\)  
21. slope \(-2.5\), passes through \((1, 2)\)  

Write an equation of the line through each set of points.

22. \((-2, 3), (4, 1)\)  
23. \((4.2, 3.6), (1.8, -1.2)\)

24. MULTIPLE CHOICE Each week, Jaya earns $32 plus $0.25 for each newspaper she delivers. Write an equation that can be used to determine how much Jaya earns each week. How much will she earn during a week in which she delivers 240 papers?

   F $75  
   G $92  
   H $148  
   J $212

25. PART-TIME JOB Jesse is a pizza delivery driver. Each day his employer gives him $20 plus $0.50 for every pizza that he delivers.

   a. Write an equation that can be used to determine how much Jesse earns each day if he delivers \( x \) pizzas.

   b. How much will he earn the day he delivers 20 pizzas?
**Scatter Plots and Lines of Regression**

**Then**
- You wrote linear equations. (Lesson 2-4)

**Now**
- Use scatter plots and prediction equations.
- Model data using lines of regression.

**Why?**
- The scatter plot shows the number of visitors to Isle Royale National Park in Michigan per year.

**Visitors to Isle Royale National Park**

<table>
<thead>
<tr>
<th>Year</th>
<th>Visitors (thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>3</td>
</tr>
<tr>
<td>1996</td>
<td>6</td>
</tr>
<tr>
<td>2000</td>
<td>9</td>
</tr>
<tr>
<td>2004</td>
<td>12</td>
</tr>
<tr>
<td>2008</td>
<td>15</td>
</tr>
</tbody>
</table>

Source: National Park Service

**New Vocabulary**
- bivariate data
- scatter plot
- dot plot
- positive correlation
- negative correlation
- line of fit
- prediction equation
- regression line
- correlation coefficient

**Tennessee Curriculum Standards**
- SPI 3103.5.3 Analyze patterns in a scatter-plot and describe relationships in both linear and non-linear data.
- SPI 3103.5.7 Determine/recognize when the correlation coefficient measures goodness of fit.
- Also addresses ✓ 3103.1.5, ✓ 3103.1.10, ✓ 3103.5.1, ✓ 3103.5.3, ✓ 3103.5.6, ✓ 3103.5.7, and ✓ 3103.5.8.

**Key Concept Scatter Plots**

<table>
<thead>
<tr>
<th>Positive Correlation</th>
<th>Negative Correlation</th>
<th>No Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Positive Correlation Diagram" /></td>
<td><img src="image2" alt="Negative Correlation Diagram" /></td>
<td><img src="image3" alt="No Correlation Diagram" /></td>
</tr>
</tbody>
</table>

- **Strong Positive Correlation**
  - The slope of the line is positive and the points are close to the line.

- **Weak Negative Correlation**
  - The slope of the line is negative and the points are not close to the line.

- **No Relative Correlation**
  - There is no obvious pattern of increase or decrease for the given data.

1. **Scatter Plots and Prediction Equations** Data with two variables, such as year and number of visitors, are called **bivariate data**. A set of bivariate data graphed as ordered pairs in a coordinate plane is called a **scatter plot** or **dot plot**.

A scatter plot can show whether there is a positive, negative, or no correlation between two variables. Correlations are usually described as **strong** or **weak**. In a strong correlation, the points of the scatterplot are closer to the graph of a line than the points representing a weak correlation.

When you find a line that closely approximates a set of data, you are finding a **line of fit** for the data. An equation of such a line is often called a **prediction equation** because it can be used to predict one of the variables given the other variable.

To find a line of fit and a prediction equation for a set of data, select two points that appear to represent the data well. This is a matter of personal judgment, so your line and prediction equation may be different from someone else’s.
Review Vocabulary

**outlier** a data point that does not appear to belong to the rest of the set.

---

**Real-World Example 1 Use a Scatter Plot and Prediction Equation**

**TECHNOLOGY** The table shows the percent of U.S. households with Internet access.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent</td>
<td>18.0</td>
<td>41.5</td>
<td>50.4</td>
<td>54.7</td>
<td>61.7</td>
</tr>
</tbody>
</table>

Source: U.S. Census Bureau

a. Make a scatter plot and a line of fit, and describe the correlation. Let \( x \) be the number of years since 1995.

Graph the data as ordered pairs with the number of years since 1995 on the horizontal axis and the percent of households on the vertical axis.

The points (5, 41.5) and (12, 61.7) appear to represent the data well. Draw a line through these two points. The data show a strong positive correlation.

b. Use two ordered pairs to write a prediction equation.

Find an equation of the line through (5, 41.5) and (12, 61.7).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{61.7 - 41.5}{12 - 5} = \frac{20.2}{7} \approx 2.89
\]

Point-Slope form

\[
y - y_1 = m(x - x_1) \rightarrow y - 41.5 = 2.89(x - 5)
\]

Substitute.

\[
y - 41.5 \approx 2.89x - 14.45 \rightarrow y \approx 2.89x + 27.05
\]

Distributive Property

One prediction equation is \( y = 2.89x + 27.05 \).

c. Predict the percent of households with Internet access in 2020.

The year 2020 is 25 years after 1995, so find \( y \) when \( x = 25 \).

\[
y \approx 2.89x + 27.05 \rightarrow y \approx 2.89(25) + 27.05 \rightarrow y \approx 99.3
\]

Simplify.

The model predicts that 99.3% of U.S. households will have Internet access in 2020.

d. How accurate does your prediction appear to be?

Except for the outlier at (2, 18.0), the line fits the data well, so the prediction value should be fairly accurate.

---

Guided Practice

1. **HOUSING** The table shows the mean selling price of new, privately-owned, single-family homes for six consecutive years.

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price($1000)</td>
<td>154.5</td>
<td>166.4</td>
<td>181.9</td>
<td>207.0</td>
<td>228.7</td>
<td>273.5</td>
</tr>
</tbody>
</table>

A. Make a scatter plot and a line of fit, and describe the correlation.

B. Write a prediction equation.

C. Predict the selling price of a new home for year 8.

D. How accurate does your prediction appear to be?
2 Lines of Regression  Another method for writing a line of fit is to use a line of regression. A regression line is determined through complex calculations to ensure that the distance of all data points to the line of fit are at a minimum. Most graphing calculators and spreadsheets can perform these calculations easily.

The correlation coefficient \( r \), \(-1 \leq r \leq 1\), is a measure that shows how well data are modeled by a linear equation.

- When \( r \) is close to \(-1\), the data have a negative correlation.
- When \( r = 0 \), the data have no correlation.
- When \( r \) is close to 1, the data have a positive correlation.

Real-World Example 2  Regression Line

The table shows the life expectancy for people born in the United States.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Life Expectancy (yr)</td>
<td>73.7</td>
<td>74.6</td>
<td>75.4</td>
<td>75.8</td>
<td>76.8</td>
<td>77.7</td>
</tr>
</tbody>
</table>

Source: U.S. CDC

Use a graphing calculator to make a scatter plot of the data. Find an equation for and graph a line of regression. Then use the equation to predict the life expectancy of a person born in 2025.

Step 1  Make a scatter plot.

- Enter the years of birth in L1 and the ages in L2.

  KEYS: [STAT] ENTER 1980 ENTER 1983 ENTER 1990 ENTER ...

- Set the viewing window to fit the data.

  KEYS: [WINDOW] 1975 ENTER 2010 ENTER 5 ENTER 70 ENTER 90 ENTER 2

- Use [STAT PLOT] to graph the scatter plot.

  KEYS: [2nd] [STAT PLOT] ENTER ENTER Graph

Step 2  Find the equation of the line of regression.

- Find the regression equation by selecting \( \text{LinReg}(ax + b) \) on the [STAT] [CALC] menu.

  KEYS: [STAT] [4] ENTER

  The regression equation is about \( y = 0.14x - 211.43 \). The slope indicates that the life expectancy increases at a rate of about 0.14 per year. The correlation coefficient \( r \) is about 0.99, which is very close to 1. So, the data fit the regression line very well.

Step 3  Graph the regression equation.

- Copy the equation to the \( Y= \) list and graph.

  KEYS: \( Y= \) VARS 5 1 Graph

  Notice that the regression line comes close to all of the data points. As the correlation coefficient indicated, the line fits the data very well.
Predictions

When you are predicting an $x$-value greater than or less than any in the data set, the process is known as extrapolation.

When you are predicting an $x$-value between the least and greatest in the data set, the process is known as interpolation.

Example 1

The table shows the percent of sales that were made in music stores in the United States for the period 1999–2008. Use a graphing calculator to make a scatter plot of the data. Find and graph a line of regression. Then use the function to predict the percent of sales made in a music store in 2018.

Example 2

The table shows the median income of families in North Carolina by family size in a recent year. Use a graphing calculator to make a scatter plot of the data. Find an equation for and graph a line of regression. Then use the equation to predict the median income of a North Carolina family of 9.
Example 1
For Exercises 3–6, complete parts a–c.

a. Make a scatter plot and a line of fit, and describe the correlation.
b. Use two ordered pairs to write a prediction equation.
c. Use your prediction equation to predict the missing value.

3. COMPACT DISC SALES The table shows the number of CDs sold in recent years at Jerome’s House of Music. Let $x$ be the number of years since 2000.

<table>
<thead>
<tr>
<th>Year</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2017</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of CDs sold</td>
<td>49,300</td>
<td>47,280</td>
<td>43,450</td>
<td>40,125</td>
<td>35,792</td>
<td>?</td>
</tr>
</tbody>
</table>

4. BASKETBALL The table shows the number of field goals and assists for some of the members of the Miami Heat in a recent NBA season.

| Field Goals | 472 | 353 | 278 | 283 | 238 | 265 | 186 | 162 | 144 |
| Assists     | 384 | 97  | 81  | 79  | 18  | 130 | 94  | 95  | ?  |

Source: NBA

5. ICE CREAM The table shows the amount of ice cream Sunee’s Homemade Ice Creams sold for eight months. Let $x = 1$ for January.

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>June</th>
<th>July</th>
<th>Aug</th>
<th>Sept</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gallons sold</td>
<td>37</td>
<td>44</td>
<td>72</td>
<td>80</td>
<td>105</td>
<td>110</td>
<td>119</td>
<td>131</td>
<td>?</td>
</tr>
</tbody>
</table>

6. DRAMA CLUB The table shows the total revenue of all of Central High School’s plays in recent school years. Let $x$ be the number of years since 2003.

<table>
<thead>
<tr>
<th>School Year</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2016</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue ($)</td>
<td>603</td>
<td>666</td>
<td>643</td>
<td>721</td>
<td>771</td>
<td>?</td>
</tr>
</tbody>
</table>

Example 2

7. SALES The table shows the sales of Chayton’s Computers. Let $x$ be the number of years since 2002 and use a graphing calculator to make a scatter plot of the data. Find an equation for and graph a line of regression. Then use the function to predict the sales in 2018.

<table>
<thead>
<tr>
<th>Year</th>
<th>Sales ($ thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>640</td>
</tr>
<tr>
<td>2005</td>
<td>715</td>
</tr>
<tr>
<td>2006</td>
<td>791</td>
</tr>
<tr>
<td>2007</td>
<td>852</td>
</tr>
<tr>
<td>2008</td>
<td>910</td>
</tr>
<tr>
<td>2009</td>
<td>944</td>
</tr>
</tbody>
</table>

8. BUSINESS The table shows the number of employees of a small company. Let $x$ be the number of years since 2000 and use a graphing calculator to make a scatter plot of the data. Find an equation for and graph a line of regression. Then use the function to predict the number of employees in 2025.

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Employees</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td>4</td>
</tr>
<tr>
<td>2003</td>
<td>7</td>
</tr>
<tr>
<td>2004</td>
<td>11</td>
</tr>
<tr>
<td>2005</td>
<td>14</td>
</tr>
<tr>
<td>2006</td>
<td>20</td>
</tr>
<tr>
<td>2007</td>
<td>23</td>
</tr>
</tbody>
</table>

**BASEBALL** The table at the right shows the total attendance and wins for the Cleveland Indians in some recent years.

<table>
<thead>
<tr>
<th>Year</th>
<th>Wins</th>
<th>Attendance</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>65</td>
<td>1,776,904</td>
</tr>
<tr>
<td>2008</td>
<td>81</td>
<td>2,182,087</td>
</tr>
<tr>
<td>2007</td>
<td>96</td>
<td>2,275,916</td>
</tr>
<tr>
<td>2006</td>
<td>78</td>
<td>1,997,936</td>
</tr>
<tr>
<td>2005</td>
<td>93</td>
<td>2,014,220</td>
</tr>
<tr>
<td>2004</td>
<td>80</td>
<td>1,814,401</td>
</tr>
<tr>
<td>2003</td>
<td>68</td>
<td>1,730,001</td>
</tr>
</tbody>
</table>

a. Make a scatter plot of the years and attendance.

b. If a linear shape is apparent, find a regression equation to summarize the trend.

c. Interpret the slope of the regression line in the context of the data.

d. Is the relationship between years and attendance stronger than the one between wins and attendance? Explain.

**CLASS SIZE** The table at the right shows the relationship between the number of students in a mathematics class and the average grade for each class.

<table>
<thead>
<tr>
<th>Class Size</th>
<th>Class Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>81.2</td>
</tr>
<tr>
<td>19</td>
<td>80.5</td>
</tr>
<tr>
<td>24</td>
<td>82.5</td>
</tr>
<tr>
<td>25</td>
<td>79.9</td>
</tr>
<tr>
<td>27</td>
<td>78.6</td>
</tr>
<tr>
<td>29</td>
<td>79.3</td>
</tr>
<tr>
<td>32</td>
<td>77.7</td>
</tr>
</tbody>
</table>

a. Make a scatter plot and find a regression equation for the data. Then graph the regression line.

b. What is the correlation coefficient of the data?

c. Describe the correlation. How accurate is the regression equation?

**FINANCIAL LITERACY** Jocelyn is analyzing the sales of her company. The table at the right shows the total sales for each of six years.

<table>
<thead>
<tr>
<th>Year</th>
<th>Sales ($ millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>31.2</td>
</tr>
<tr>
<td>2004</td>
<td>34.6</td>
</tr>
<tr>
<td>2005</td>
<td>18.6</td>
</tr>
<tr>
<td>2006</td>
<td>37.7</td>
</tr>
<tr>
<td>2007</td>
<td>41.3</td>
</tr>
<tr>
<td>2008</td>
<td>45.1</td>
</tr>
</tbody>
</table>

a. Find a regression equation and correlation coefficient for the data. Let \( x \) be the year.

b. Use the regression equation to predict the 2020 sales.

c. Remove the outlier from the data set and find a new regression equation and correlation coefficient.

d. Use the new regression equation to predict the sales in 2020.

e. Compare the correlation coefficients for the two equations. Which function fits the data better? Which prediction should Jocelyn expect to be more accurate?

**H.O.T. Problems** Use Higher-Order Thinking Skills

12. **REASONING** What is the relevance of the correlation coefficient of a linear regression line? Explain your reasoning.

13. **CHALLENGE** If statements \( a \) and \( b \) have a positive correlation, \( b \) and \( c \) have a negative correlation, and \( c \) and \( d \) have a positive correlation, what can you determine about the correlation between statements \( a \) and \( d \)? Explain your reasoning.

14. **OPEN ENDED** Provide real-world quantities that represent each of the following.

   a. positive correlation
   b. negative correlation
   c. no correlation

15. **CHALLENGE** Draw a scatter plot for the following data set.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.8</th>
<th>3.2</th>
<th>4.0</th>
<th>4.8</th>
<th>5.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>3.5</td>
<td>4.7</td>
<td>5.1</td>
<td>6.8</td>
<td>7.1</td>
<td>7.5</td>
<td>8.8</td>
<td>10.3</td>
</tr>
</tbody>
</table>

Which of the following best represents the correlation coefficient \( r \) for the data? Justify your answer.

a. 0.99   b. \(-0.98\)   c. 0.62   d. 0.08

16. **WRITING IN MATH** Explain why a linear equation is useful when working with data.
17. SHORT RESPONSE What is the value of the expression below?

\[17 - 3[-1 + 2(7 - 4)]\]

18. Anna took brownies to a club meeting. She gave half of her brownies to Selena. Selena gave a third of her brownies to Randall. Randall gave a fourth of his brownies to Trina. If Trina has 3 brownies, how many brownies did Anna have in the beginning?

A 12  
B 36  
C 72  
D 144  

19. GEOMETRY Which is always true?

F A parallelogram is a square.  
G A parallelogram is a rectangle.  
H A quadrilateral is a trapezoid.  
J A square is a rectangle.  

20. SAT/ACT Which line best fits the data in the graph?

A \(y = x\)  
B \(y = -0.5x + 4\)  
C \(y = -0.5x - 4\)  
D \(y = 0.5x + 0.5\)  
E \(y = 1.5x - 1.5\)  

Spiral Review

Write an equation in slope-intercept form for each graph. (Lesson 2-4)

21.  
22.  
23.  

Find the rate of change for each set of data. (Lesson 2-3)

24.  
25.  
26.  
27.  

28. RECREATION Ramona estimates that she will need 50 tennis balls for every player that signs up for the tennis club and at least 150 more just in case. Write an inequality to express the situation. (Lesson 1-5)

29. DODGEBALL Six teams played in a dodgeball tournament. In how many ways can the top three teams finish? (Lesson 0-5)

Skills Review

Solve each equation. (Lesson 1-4)

30. \(-4|x - 2| = -12\)  
31. \(|3x + 4| = 21\)  
32. \(2|4x - 1| + 3 = 9\)
OBJECTIVE Use median fit lines to make predictions.

Many teens would love to own a car. Of course, you know there is more to owning a car than just the cost of the car. Over the years, the cost of gas has been rising. What might the cost per gallon be when you get a car? Graphing data can help you predict the future cost. The table below shows the cost of one gallon of unleaded gas from 1990 through 2005.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$1.35</td>
<td>$1.32</td>
<td>$1.32</td>
<td>$1.30</td>
<td>$1.31</td>
<td>$1.34</td>
<td>$1.41</td>
<td>$1.42</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$1.25</td>
<td>$1.36</td>
<td>$1.69</td>
<td>$1.66</td>
<td>$1.56</td>
<td>$1.78</td>
<td>$2.07</td>
<td>$2.49</td>
</tr>
</tbody>
</table>

Source: The World Almanac and Book of Facts

Activity

Follow the steps to find a median-fit line for a data set and make a prediction.

Step 1 As accurately as possible, graph the ordered pairs (year, cost). For example, one ordered pair is (1990, 1.35).

Step 2 Divide the data points into 3 groups as equal in size as possible. If there is one extra point, place it in the middle group. If there are two extra points, place one in each of the two outer groups. Separate the groups with dashed lines.

Step 3 For each group, find the median of the x- and y-values.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 1.35)</td>
<td>(5, 1.34)</td>
<td>(11, 1.66)</td>
</tr>
<tr>
<td>(1, 1.32)</td>
<td>(6, 1.41)</td>
<td>(12, 1.56)</td>
</tr>
<tr>
<td>(2, 1.32)</td>
<td>(7, 1.42)</td>
<td>(13, 1.78)</td>
</tr>
<tr>
<td>(3, 1.30)</td>
<td>(8, 1.25)</td>
<td>(14, 2.07)</td>
</tr>
<tr>
<td>(4, 1.31)</td>
<td>(9, 1.36)</td>
<td>(15, 2.49)</td>
</tr>
<tr>
<td>median: (2, 1.32)</td>
<td>median: (7.5, 1.335)</td>
<td>median: (13, 1.78)</td>
</tr>
</tbody>
</table>

(continued on the next page)
**Step 4** Plot the median points on the graph. Draw a dashed line through the median points for Groups 1 and 3. Then about one third of the way down toward the median for Group 2, draw a solid line parallel to the dashed line. This is the median-fit line.

**Step 5** When two different people draw a median-fit line, the results may vary slightly due to inaccuracy in measurements and estimation of coordinates. Pick two points on the solid line, for example, (4, 1.32) and (14, 1.76). Then find the slope of the median-fit line and write an equation.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.76 - 1.32}{14 - 4} = \frac{0.44}{10} = 0.044
\]

\[
(y - y_1) = m(x - x_1) = 0.044(x - 4)
\]

\[
y - 1.32 = 0.044x - 0.176
\]

\[
y = 0.044x + 1.144
\]

**Exercises**

1. Suppose you buy your own car in 2020. Use the equation in Step 5 to predict the price per gallon of unleaded gasoline for that year. Explain whether you think the prediction is reasonable.

2. Write an equation for another type of best-fit line, and use the equation to predict for the year 2020.

3. The table shows the number of licensed drivers in the U.S., in millions, for various years. Make a scatter plot of the data, draw a median-fit line, and predict the number of licensed drivers in 2018. Then explain whether you think the prediction is reasonable.

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>130 145 157 167 177 185 187 191</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year</td>
<td>2001</td>
<td>2002</td>
<td>2003</td>
<td>2004</td>
<td>2005</td>
<td>2006</td>
<td>2007</td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>191 195 196 199 201 203 205</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: U.S. Department of Transportation

4. The table shows the median sale price for a house in the U.S. for various years. Make a scatter plot of the data, draw a median-fit line, and predict the median house sales price in 2020. Then explain whether you think the prediction is reasonable.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>$92,000</td>
<td>$110,500</td>
<td>$128,400</td>
<td>$133,300</td>
<td>$139,000</td>
<td>$158,100</td>
</tr>
<tr>
<td>Year</td>
<td>2003</td>
<td>2004</td>
<td>2005</td>
<td>2006</td>
<td>2007</td>
<td>2008</td>
</tr>
<tr>
<td>Price</td>
<td>$180,200</td>
<td>$195,200</td>
<td>$219,000</td>
<td>$221,900</td>
<td>$219,000</td>
<td>$198,100</td>
</tr>
</tbody>
</table>

Source: National Association of Realtors
Piecewise-Defined Functions

The function relating income and tax is not a linear function because each interval, or piece, of the function is defined by a different expression. A function that is written using two or more expressions is called a piecewise-defined function.

On the graph of a piecewise-defined function, a dot indicates that the point is included in the graph. A circle indicates that the point is not included in the graph.

Example 1

**Piecewise-Defined Function**

Graph \( f(x) = \begin{cases} x - 2 & \text{if } x < -1 \\ x + 3 & \text{if } x \geq -1 \end{cases} \). Identify the domain and range.

**Step 1**

Graph \( f(x) = x - 2 \) for \( x < -1 \).

\[
f(x) = x - 2 \\
= (-1) - 2 \\
= -3
\]

Because \(-1\) does not satisfy the inequality, begin with a circle at \((-1, -3)\).

**Step 2**

Graph \( f(x) = x + 3 \) for \( x \geq -1 \).

\[
f(x) = x + 3 \\
= (-1) + 3 \\
= 2
\]

Because \(-1\) satisfies the inequality, begin with a dot at \((-1, 2)\).

The function is defined for all values of \( x \), so the domain is all real numbers.

The \( f(x) \)-coordinates of points on the graph are all real numbers less than \(-3\) and all real numbers greater than or equal to \( 2 \), so the range is \( \{ f(x) | f(x) < -3 \text{ or } f(x) \geq 2 \} \).

Guided Practice

1. Graph \( f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases} \). Identify the domain and range.
Piecewise-defined functions are often defined by several linear functions.

**Example 2 Write a Piecewise-Defined Function**

Write the piecewise-defined function shown in the graph.

Examine and write a function for each portion of the graph.

The left portion of the graph is the graph of \( f(x) = 2x + 3 \). There is a circle at (1, 5), so the linear function is defined for \( \{x | x < 1\} \).

The center portion of the graph is the graph of \( f(x) = -x + 2 \). There are dots at (1, 1) and (2, 0), so the linear function is defined for \( \{x | 1 \leq x \leq 2\} \).

The right portion of the graph is the constant function \( f(x) = 3 \). There is a circle at (2, 3), so the constant function is defined for \( \{x | x > 2\} \).

Write the piecewise-defined function.

\[
\begin{align*}
f(x) &= \begin{cases} 
2x + 3 & \text{if } x < 1 \\
-x + 2 & \text{if } 1 \leq x \leq 2 \\
3 & \text{if } x > 2 
\end{cases}
\end{align*}
\]

**CHECK** The graph shows a portion of a line with positive slope for \( x < 1 \).

The graph has negative slope for \( 1 \leq x \leq 2 \) and constant slope for \( x > 2 \). The function is reasonable for the graph.

**Guided Practice**

Write the piecewise-defined function shown in each graph.

2A.

2B.

**Step Functions and Absolute Value Functions** Unlike a piecewise-defined function, a piecewise-linear function contains a single expression. A common piecewise-linear function is the step function. The graph of a step function consists of line segments.

The greatest integer function, written \( f(x) = \lfloor x \rfloor \), is one kind of step function. The symbol \( \lfloor x \rfloor \) means *the greatest integer less than or equal to* \( x \). For example, \( \lfloor 3.25 \rfloor = 3 \) and \( \lfloor -4.6 \rfloor = -5 \).
Real-World Example 3 Use a Step Function

**BUSINESS** An automotive repair center charges $50 for any part of the first hour of labor, and $35 for any part of each additional hour. Draw a graph that represents this situation.

**Understand** The total labor charge is $50 for the first hour plus $35 for each additional fraction of an hour, so the graph will be a step function.

**Plan** If the time spent on labor is greater than 0 hours, but less than or equal to 1 hour, then the labor charge is $50. If the time is greater than 1 hour but less than 2 hours, then the labor charge is $85, and so on.

**Solve** Use the pattern of times and costs to make a table, where \( x \) is the number of hours of labor and \( T(x) \) is the total labor charge. Then graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( T(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>0 &lt; ( x \leq 1 )</td>
<td>85</td>
</tr>
<tr>
<td>1 &lt; ( x \leq 2 )</td>
<td>120</td>
</tr>
<tr>
<td>2 &lt; ( x \leq 3 )</td>
<td>155</td>
</tr>
<tr>
<td>3 &lt; ( x \leq 4 )</td>
<td>190</td>
</tr>
<tr>
<td>4 &lt; ( x \leq 5 )</td>
<td></td>
</tr>
</tbody>
</table>

**Check** Since the repair center rounds any fraction of an hour up to the next whole number, each segment of the graph has a circle at the left endpoint and a dot at the right endpoint.

**Guided Practice**

3. **RECYCLING** A recycling company pays $5 for every full box of newspaper. They do not give any money for partial boxes. Draw a graph that shows the amount of money \( P(x) \) for the number of boxes \( x \) brought to the recycling center.

Another piecewise-linear function is the absolute value function. An **absolute value function** is a function that contains an algebraic expression within absolute value symbols.

**Math History Link**

Karl Weierstrass (1815–1897) At the wishes of his father, Weierstrass studied law, economics, and finance at the University of Bonn, but then dropped out to study his true interest, mathematics, at the University of Münster. In an 1841 essay, Weierstrass first used \( | | \) to denote absolute value.

Photo: The Granger Collection
Example 4 Absolute Value Functions

Graph \( f(x) = |2x| - 4 \). Identify the domain and range.

Create a table of values.

| \( x \) | \(|2x| - 4\) |
|---|---|
| -3 | 2 |
| -2 | 0 |
| -1 | -2 |
| 0  | -4 |
| 1  | -2 |
| 2  | 0 |
| 3  | 2 |

Graph the points and connect them.

The domain is the set of all real numbers. The range is \( \{f(x) | f(x) \geq -4\} \).

Guided Practice

Graph each function. Identify the domain and range.

4A. \( f(x) = |x - 2| \)

4B. \( f(x) = -|x| + 1 \)

Check Your Understanding

Example 1

Graph each function. Identify the domain and range.

1. \( g(x) = \begin{cases} 
-3 & \text{if } x \leq -4 \\
0 & \text{if } -4 < x < 2 \\
-\frac{x}{6} & \text{if } x \geq 2 
\end{cases} \)

2. \( f(x) = \begin{cases} 
8 & \text{if } x \leq -1 \\
2x & \text{if } -1 < x < 4 \\
-4 - x & \text{if } x \geq 4 
\end{cases} \)

Example 2

Write the piecewise-defined function shown in each graph.

3.

4.

Example 3

THEATER Springfield High School’s theater can hold 250 students. The drama club is performing a play in the theater. Draw a graph of a step function that shows the relationship between the number of tickets sold \( x \) and the minimum number of performances \( y \) that the drama club must do.

Graph each function. Identify the domain and range.

6. \( g(x) = -2[x] \)

7. \( h(x) = \lfloor x - 5 \rfloor \)

Example 4

Graph each function. Identify the domain and range.

8. \( g(x) = | -3x | \)

9. \( f(x) = 2|x| \)

10. \( h(x) = |x + 4| \)

11. \( s(x) = | -2x | + 6 \)
Example 1
Graph each function. Identify the domain and range.

12. \( f(x) = \begin{cases} 
-3x & \text{if } x \leq -4 \\
x & \text{if } 0 < x \leq 3 \\
8 & \text{if } x > 3 
\end{cases} \)

13. \( f(x) = \begin{cases} 
2 & \text{if } x \leq -6 \\
5 & \text{if } -6 < x \leq 2 \\
-2x + 1 & \text{if } x > 4 
\end{cases} \)

14. \( g(x) = \begin{cases} 
2x + 2 & \text{if } x < -6 \\
x & \text{if } -6 \leq x \leq 2 \\
-3 & \text{if } x > 2 
\end{cases} \)

15. \( g(x) = \begin{cases} 
-2 & \text{if } x < -4 \\
x - 3 & \text{if } -1 \leq x \leq 5 \\
2x - 15 & \text{if } x > 7 
\end{cases} \)

Example 2
Write the piecewise-defined function shown in each graph.

Example 3
Graph each function. Identify the domain and range.

20. \( f(x) = \lfloor x \rfloor - 6 \)

21. \( h(x) = \lceil x \rceil - 8 \)

22. \( f(x) = \lfloor 3x + 2 \rfloor \)

23. \( g(x) = 2\lfloor 0.5x + 4 \rfloor \)

Example 4
Graph each function. Identify the domain and range.

24. \( f(x) = |x - 5| \)

25. \( g(x) = |x + 2| \)

26. \( h(x) = |2x| - 8 \)

27. \( k(x) = |-3x| + 3 \)

28. \( f(x) = 2|x - 4| + 6 \)

29. \( h(x) = -3|0.5x + 1| - 2 \)

30. **GIVING** Patrick is donating money and volunteering his time to an organization that restores homes for the needy. His employer will match his monetary donations up to $100.

   a. Identify the type of function that models the total amount of money received by the charity when Patrick donates \( x \) dollars.

   b. Write and graph a function for the situation.

31. **CARS** A car’s speedometer reads 60 miles an hour.

   a. Write an absolute value function for the difference between the car’s actual speed \( a \) and the reading on the speedometer.

   b. What is an appropriate domain for the function? Explain your reasoning.

   c. Use the domain to graph the function.
32. **RECREATION**  The charge for renting a bicycle from a rental shop for different amounts of time is shown at the right.

a. Identify the type of function that models this situation.

b. Write and graph a function for the situation.

<table>
<thead>
<tr>
<th>Time</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2 hour</td>
<td>$6</td>
</tr>
<tr>
<td>1 hour</td>
<td>$10</td>
</tr>
<tr>
<td>2 hours</td>
<td>$16</td>
</tr>
<tr>
<td>Daily</td>
<td>$24</td>
</tr>
</tbody>
</table>

Use each graph to write the absolute value function.

Graph each function. Identify the domain and range.

35. \( f(x) = \left\lfloor 0.5x \right\rfloor \)

36. \( g(x) = \left\lceil \frac{x}{2} \right\rceil \)

37. \( g(x) = \begin{cases} \left\lfloor x \right\rfloor & \text{if } x < -4 \\ x + 1 & \text{if } -4 \leq x \leq 5 \\ -\left\lfloor x \right\rfloor & \text{if } x > 3 \end{cases} \)

38. \( h(x) = \begin{cases} -\left\lfloor x \right\rfloor & \text{if } x < -6 \\ \left\lfloor x \right\rfloor & \text{if } -6 \leq x \leq 2 \\ \left\lfloor x \right\rfloor & \text{if } x > 2 \end{cases} \)

39. **MULTIPLE REPRESENTATIONS**  Consider the following absolute value functions.

\[ f(x) = |x| - 4 \]
\[ g(x) = |3x| \]

a. **Tabular**  Use a graphing calculator to create a table of \( f(x) \) and \( g(x) \) values for \( x = -4 \) to \( x = 4 \).

b. **Graphical**  Graph the functions on separate graphs.

c. **Numerical**  Determine the slope between each two consecutive points in the table.

d. **Verbal**  Describe how the slopes of the two sections of an absolute value graph are related.

**H.O.T. Problems**  Use Higher-Order Thinking Skills

40. **OPEN ENDED**  Write an absolute value relation in which the domain is all nonnegative numbers and the range is all real numbers.

41. **CHALLENGE**  Graph \( |y| = 2|x + 3| - 5 \).

42. **REASONING**  Find a counterexample to the statement and explain your reasoning.

\[ \text{In order to find the greatest integer function of } x \text{ when } x \text{ is not an integer, round } x \text{ to the nearest integer.} \]

43. **OPEN ENDED**  Write an absolute value function in which \( f(5) = -3 \).

44. **WRITING IN MATH**  Explain how piecewise functions can be used to accurately represent real-world problems.
45. SHORT RESPONSE What expression gives the \( n \)th term of the linear pattern defined by the table?

\[
\begin{array}{c|c|c|c|c}
2 & 4 & 6 & 8 & n \\
\hline
7 & 13 & 19 & 25 & ? \\
\end{array}
\]

46. Solve: \( 5(x + 4) = x + 4 \)

Step 1: \( 5x + 20 = x + 4 \)

Step 2: \( 4x + 20 = 4 \)

Step 3: \( 4x = 24 \)

Step 4: \( x = 6 \)

Which is the first incorrect step in the solution shown above?

A Step 4  
B Step 3  
C Step 2  
D Step 1

47. NUMBER THEORY Twelve consecutive integers are arranged in order from least to greatest. If the sum of the first six integers is 381, what is the sum of the last six integers?

F 345  
G 381  
H 387  
J 417

48. SAT/ACT For which function does \( f \left( -\frac{1}{2} \right) \neq -1? \)

A \( f(x) = 2x \)  
B \( f(x) = | -2x | \)  
C \( f(x) = [x] \)  
D \( f(x) = \left[ 2x \right] \)  
E \( f(x) = -[2x] \)

49. FOOTBALL The table shows the relationship between the total number of male students per school and the number of students who tried out for the football team. (Lesson 2-5)

a. Find a regression equation for the data.

b. Determine the correlation coefficient.

c. Predict how many students will try out for football at a school with 800 male students.

Write an equation in slope-intercept form for the line described. (Lesson 2-4)

50. passes through \((-3, -6)\), perpendicular to \( y = -2x + 1 \)

51. passes through \((4, 0)\), parallel to \(3x + 2y = 6 \)

52. passes through the origin, perpendicular to \(4x - 3y = 12 \)

Find each value if \( f(x) = -4x + 6 \), \( g(x) = -x^2 \), and \( h(x) = -2x^2 - 6x + 9 \). (Lesson 2-1)

53. \( f(2c) \)  
54. \( g(a + 1) \)  
55. \( h(6) \)

56. Determine whether the figures below are similar. (Lesson 0-6)

\[
\begin{array}{c|c|c}
33 & 26.4 & 9.6 \\
12 & & \\
\end{array}
\]

Skills Review

Graph each equation. (Lesson 2-1)

57. \( y = -0.25x + 8 \)  
58. \( y = \frac{4}{3}x + 2 \)  
59. \( 8x + 4y = 32 \)
The parent function of the family of linear functions is \( f(x) = x \). You can use a graphing calculator to investigate how changing the parameters \( m \) and \( b \) in \( f(x) = mx + b \) affects the graphs as compared to the parent function.

**Activity 1  \( b \) in \( f(x) = mx + b \)**

Graph \( f(x) = x, f(x) = x + 3, \) and \( f(x) = x - 5 \) in the standard viewing window.

Enter the equations in the \( Y= \) list as \( Y_1, Y_2, \) and \( Y_3 \). Then graph the equations.

**KEYSTROKES:**

\[
Y= X,T,\theta,n \quad \text{ENTER} \quad X,T,\theta,n \quad + \quad 3 \\
\text{ENTER} \quad X,T,\theta,n \quad - \quad 5 \quad \text{ENTER} \quad \text{ZOOM} \quad 6
\]

1A. Compare and contrast the graphs.

1B. How would you obtain the graphs of \( f(x) = x + 3 \) and \( f(x) = x - 5 \) from the graph of \( f(x) = x \)?

The parameter \( m \) in \( f(x) = mx + b \) affects the graphs in a different way than \( b \).

**Activity 2  \( m \) in \( f(x) = mx + b \)**

Graph \( f(x) = x, f(x) = 3x, \) and \( f(x) = \frac{1}{2}x \) in the standard viewing window.

Enter the equations in the \( Y= \) list and graph.

2A. How do the graphs compare?

2B. Which graph is steepest? Which graph is the least steep?

2C. Graph \( f(x) = -x, f(x) = -3x, \) and \( f(x) = -\frac{1}{2}x \) in the standard viewing window.

How do these graphs compare?

**Analyze the Results**

Graph each set of equations on the same screen. Describe the similarities or differences among the graphs.

1. \( f(x) = 3x \)  
   \( f(x) = 3x + 1 \)  
   \( f(x) = 3x - 2 \)

2. \( f(x) = x + 2 \)  
   \( f(x) = 5x + 2 \)  
   \( f(x) = \frac{1}{2}x + 2 \)

3. \( f(x) = x - 3 \)  
   \( f(x) = 2x - 3 \)  
   \( f(x) = 0.75x - 3 \)

4. What do the graphs of equations of the form \( f(x) = mx + b \) have in common?

5. How do the values of \( b \) and \( m \) affect the graph of \( f(x) = mx + b \) as compared to the parent function \( f(x) = x \)?

6. Summarize your results. How can knowing about the effects of \( m \) and \( b \) help you sketch the graph of a function?
**Parent Functions and Transformations**

**New Vocabulary**
- family of graphs
- parent graph
- parent function
- constant function
- identity function
- quadratic function
- translation
- reflection
- line of reflection
- dilation

**KeyConcept Parent Functions**

**Constant Function**

The general equation of a constant function is $f(x) = a$, where $a$ is any number. The domain is all real numbers, and the range consists of a single real number $a$.

**Identity Function**

The identity function $f(x) = x$ passes through all points with coordinates $(a, a)$. It is the parent function of most linear functions. Its domain and range are all real numbers.

**Absolute Value Function**

Recall that the parent function of absolute value functions is $f(x) = |x|$. The domain of $f(x) = |x|$ is the set of real numbers, and the range is the set of real numbers greater than or equal to 0.

**Quadratic Function**

The parent function of quadratic functions is $f(x) = x^2$. The domain of $f(x) = x^2$ is the set of real numbers, and the range is the set of real numbers greater than or equal to 0.

---

**Why?**

1. **Parent Graphs** A family of graphs is a group of graphs that display one or more similar characteristics. The parent graph, which is the graph of the parent function, is the simplest of the graphs in a family. This is the graph that is transformed to create other members in a family of graphs.

   Nick makes $8 an hour working at a pizza shop. The red line represents his wages. If he also delivers the pizzas, he is paid $2 more per hour. The blue line represents Nick’s wages when he delivers. These graphs are examples of transformations.
Example 1  Identify a Function Given the Graph

Identify the type of function represented by each graph.

a. The graph is in the shape of a V. The graph represents an absolute value function.

b. The graph is a horizontal line that crosses the y-axis at 4. The graph represents a constant function.

Guided Practice

1A.

1B.

2 Transformations  Transformations of a parent graph may appear in a different location, flip over an axis, or appear to have been stretched or compressed. The transformed graph may resemble the parent graph, or it may not.

A **translation** moves a figure up, down, left, or right.

- When a constant \( k \) is added to or subtracted from a parent function, the result \( f(x) \pm k \) is a translation of the graph up or down.
- When a constant \( h \) is added to or subtracted from \( x \) before evaluating a parent function, the result, \( f(x \pm h) \), is a translation left or right.

Example 2  Describe and Graph Translations

Describe the translation in \( y = |x| + 2 \). Then graph the function.

The graph of \( y = |x| + 2 \) is a translation of the graph of \( y = |x| \) up 2 units.

Guided Practice

Describe the translation in each function. Then graph the function.

2A. \( y = |x + 3| \)  
2B. \( y = x^2 - 4 \)
A reflection flips a figure over a line called the line of reflection.

- When a parent function is multiplied by $-1$, the result $-f(x)$ is a reflection of the graph in the $x$-axis.
- When only the variable is multiplied by $-1$, the result $f(-x)$ is a reflection of the graph in a line of reflection through the vertex.

**Example 3** Describe and Graph Reflections

Describe the reflection in $y = -x^2$. Then graph the function.

The graph of $y = -x^2$ is a reflection of the graph of $y = x^2$ in the $x$-axis.

![Graph of Reflection](image)

**Guided Practice**

Describe the reflection in each function. Then graph the function.

3A. $y = -|x|$

3B. $y = -x$

A dilation shrinks or enlarges a figure proportionally. When the variable in a linear parent function is multiplied by a nonzero number, the slope of the graph changes.

- When a nonlinear parent function is multiplied by a nonzero number, the function is stretched or compressed vertically.
- Coefficients greater than 1 cause the graph to be stretched vertically, and coefficients between 0 and 1 cause the graph to be compressed vertically.

**Example 4** Describe and Graph Dilations

Describe the dilation in $y = 4x$. Then graph the function.

The graph of $y = 4x$ is a dilation of the graph of $y = x$. The slope of the graph of $y = 4x$ is steeper than that of the graph of $y = x$.

![Graph of Dilation](image)

**Guided Practice**

Describe the dilation in each function. Then graph the function.

4A. $y = 2x^2$

4B. $y = \frac{1}{3}x$
LANDSCAPING  Ethan is going to add a brick walkway around the perimeter of his vegetable garden. The area of the walkway can be represented by the function $f(x) = 4(x + 2.5)^2 - 25$. Describe the transformations in the function. Then graph the function.

The graph of $f(x) = 4(x + 2.5)^2 - 25$ is a combination of transformations of the parent graph $f(x) = x^2$. Determine how each transformation affects the parent graph.

$$f(x) = 4(x + 2.5)^2 - 25$$

+ 2.5 translates $f(x) = x^2$ left 2.5 units.

− 25 translates $f(x) = x^2$ down 25 units.

4 stretches $f(x) = x^2$ vertically.

Guided Practice

5. SCIENCE  The function $C(x) = \frac{5}{9}(x - 32)$ can be used to determine the temperature in degrees Celsius when given the temperature in degrees Fahrenheit. Describe the transformations in the function. Then graph the function.

The table summarizes the changes to the parent graph under different transformations.

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Change to Parent Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Translation</strong></td>
<td></td>
</tr>
<tr>
<td>$f(x + h), h &gt; 0$</td>
<td>Translates graph $h$ units left.</td>
</tr>
<tr>
<td>$f(x - h), h &gt; 0$</td>
<td>Translates graph $h$ units right.</td>
</tr>
<tr>
<td>$f(x) + k, k &gt; 0$</td>
<td>Translates graph $k$ units up.</td>
</tr>
<tr>
<td>$f(x) - k, k &gt; 0$</td>
<td>Translates graph $k$ units down.</td>
</tr>
<tr>
<td><strong>Reflection</strong></td>
<td></td>
</tr>
<tr>
<td>$-f(x)$</td>
<td>Reflects graph in the $x$-axis.</td>
</tr>
<tr>
<td>$f(-x)$</td>
<td>Reflects graph in the $y$-axis.</td>
</tr>
<tr>
<td><strong>Dilation</strong></td>
<td></td>
</tr>
<tr>
<td>$a \cdot f(x),</td>
<td>a</td>
</tr>
<tr>
<td>$a \cdot f(x), 0 &lt;</td>
<td>a</td>
</tr>
<tr>
<td>$f(bx),</td>
<td>b</td>
</tr>
<tr>
<td>$f(bx), 0 &lt;</td>
<td>b</td>
</tr>
</tbody>
</table>
Check Your Understanding

Example 1  Identify the type of function represented by each graph.

1. 

2. 

Example 2  Describe the translation in each function. Then graph the function.

3. $y = x^2 - 4$

4. $y = |x + 1|$

Example 3  Describe the reflection in each function. Then graph the function.

5. $y = -|x|$

6. $y = (-x)^2$

Example 4  Describe the dilation in each function. Then graph the function.

7. $y = \frac{3}{5}x$

8. $y = 3x^2$

Example 5  9. **FOOD** The manager of a coffee shop is randomly checking cups of coffee drinks prepared by employees to ensure that the correct amount of coffee is in each cup. Each 12-ounce drink should contain half coffee and half steamed milk. The amount of coffee by which each drink varies can be represented by $f(x) = \frac{1}{2}|x - 12|$. Describe the transformations in the function. Then graph the function.

Practice and Problem Solving

Example 1  Identify the type of function represented by each graph.

10. 

11. 

12. 

13. 

Extra Practice begins on page 947.
Example 2  Describe the translation in each function. Then graph the function.
14. \( y = x^2 + 4 \)  
15. \( y = |x| - 3 \)  
16. \( y = x - 1 \)  
17. \( y = x + 2 \)  
18. \( y = (x - 5)^2 \)  
19. \( y = |x + 6| \)

Example 3  Describe the reflection in each function. Then graph the function.
20. \( y = -x \)  
21. \( y = -x^2 \)  
22. \( y = (-x)^2 \)  
23. \( y = |-x| \)  
24. \( y = -|x| \)  
25. \( y = (-x) \)

Example 4  Describe the dilation in each function. Then graph the function.
26. \( y = (3x)^2 \)  
27. \( y = 6x \)  
28. \( y = 4|x| \)  
29. \( y = |2x| \)  
30. \( y = \frac{2}{3}x \)  
31. \( y = \frac{1}{2}x^2 \)

Example 5  32. HEALTH A non-impact workout can burn up to 7.5 Calories per minute. The equation to represent how many Calories a person burns after \( m \) minutes of the workout is \( C(m) = 7.5m \). Identify the transformation in the function. Then graph the function.

Write an equation for each function.

33. \[
\begin{align*}
    y &= \frac{3}{4}x \\
    \text{Graph} \\
    O & \quad x
\end{align*}
\]

34. \[
\begin{align*}
    y &= \frac{3}{4}x \\
    \text{Graph} \\
    O & \quad x
\end{align*}
\]

35. \[
\begin{align*}
    y &= \frac{3}{4}x \\
    \text{Graph} \\
    O & \quad x
\end{align*}
\]

36. \[
\begin{align*}
    y &= \frac{3}{4}x \\
    \text{Graph} \\
    O & \quad x
\end{align*}
\]

37. \[
\begin{align*}
    y &= \frac{3}{4}x \\
    \text{Graph} \\
    O & \quad x
\end{align*}
\]

38. \[
\begin{align*}
    y &= \frac{3}{4}x \\
    \text{Graph} \\
    O & \quad x
\end{align*}
\]

39. BUSINESS The graph of the cost of producing \( x \) widgets is represented by the blue line in the graph. After hiring a consultant, the cost of producing \( x \) widgets is represented by the red line in the graph. Write the equations of both lines and describe the transformation from the blue line to the red line.
40. **ROCKETRY** Kenji launched a toy rocket from ground level. The height \( h(t) \) of Kenji’s rocket after \( t \) seconds is shown in blue. Emily believed that her rocket could fly higher and longer than Kenji’s. The flight of Emily’s rocket is shown in red.

   a. Identify the type of function shown.
   b. How much longer than Kenji’s rocket did Emily’s rocket stay in the air?
   c. How much higher than Kenji’s rocket did Emily’s rocket go?
   d. Describe the type of transformation between the two graphs.

Write an equation for each function.

![Graphs](image)

**H.O.T. Problems** Use Higher-Order Thinking Skills

43. **CHALLENGE** Explain why performing a horizontal translation followed by a vertical translation ends up being the same transformation as performing a vertical translation followed by a horizontal translation.

44. **ERROR ANALYSIS** Carla and Kimi are determining if \( f(x) = 2x \) is the identity function. Is either of them correct? Explain your reasoning.

   **Carla**
   \[ f(x) = 2x \text{ is the identity function because it is linear and goes through the origin.} \]

   **Kimi**
   \[ f(x) = 2x \text{ is not the identity function because the values in the domain do not correspond to their duplicates in the range.} \]

45. **OPEN ENDED** Draw a figure in Quadrant II. Use any of the transformations you learned in this lesson to move your figure to Quadrant IV. Describe your transformation.

46. **REASONING** Study the parent graphs at the beginning of this lesson. Select a parent graph with positive \( y \)-values at its leftmost points and positive \( y \)-values at its rightmost points.

47. **WRITING IN MATH** Explain why the reflection of the graph of \( f(x) = x^2 \) in the \( y \)-axis is the same as the graph of \( f(x) = x^2 \). Is this true for all reflections of quadratic equations? If not, describe a case when it is false.
48. What is the solution set of the inequality?
\[ 6 - |x + 7| \leq -2 \]
A \( \{x|-15 \leq x \leq 1\} \)
B \( \{x|x \leq -1 \text{ or } x \geq 3\} \)
C \( \{x|-1 \leq x \leq 3\} \)
D \( \{x|x \leq -15 \text{ or } x \geq 1\} \)

49. GEOMETRY The measures of two angles of a triangle are \( x \) and \( 4x \). Which of these expressions represents the measure of the third angle?
F \( 180 + x + 4x \)
G \( 180 - x - 4x \)
H \( 180 - x + 4x \)
J \( 180 + x - 4x \)

50. GRIDDED RESPONSE Find the value of \( x \) that makes \( \frac{1}{2} = \frac{x - 2}{x + 2} \) true.

51. SAT/ACT Which could be the equation for the graph?
A \( y = 3x + 2 \)
B \( y = 3x - 2 \)
C \( y = -3x + 2 \)

Spiral Review

Graph each function. Identify the domain and range. (Lesson 2-6)

52. \( f(x) = |x - 3| \)
53. \( h(x) = [x] - 5 \)
54. \( f(x) = \begin{cases} -2x & \text{if } x \leq -2 \\ x & \text{if } -2 < x \leq 1 \\ 4 & \text{if } x > 1 \end{cases} \)

55. ATTENDANCE The table shows the annual attendance to West High School’s Summer Celebration. (Lesson 2-5)

<table>
<thead>
<tr>
<th>Year</th>
<th>Attendance</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>61</td>
</tr>
<tr>
<td>2005</td>
<td>83</td>
</tr>
<tr>
<td>2006</td>
<td>85</td>
</tr>
<tr>
<td>2007</td>
<td>92</td>
</tr>
<tr>
<td>2008</td>
<td>97</td>
</tr>
<tr>
<td>2009</td>
<td>106</td>
</tr>
</tbody>
</table>

a. Find a regression equation for the data.
b. Determine the correlation coefficient.
c. Predict how many people will attend the Summer Celebration in 2010.

Solve each inequality. (Lesson 1-6)

56. \(-12 \leq 2x + 4 \leq 8\)
57. \(-4 < -3y + 2 < 11\)
58. \(|x - 3| > 7\)

59. CARS Loren is buying her first car. She is considering 4 different models and 5 different colors. How many different cars could she buy? (Lesson 0-4)

Determine if each relation is a function. (Lesson 0-1)

60.
61.
62.

Skills Review

Evaluate each expression if \( x = -4 \) and \( y = 6 \). (Lesson 1-1)

63. \( 4x - 8y + 12 \)
64. \( 5y + 3x - 8 \)
65. \( -12x + 10y - 24 \)
Graph Linear Inequalities

A linear inequality resembles a linear equation, but with an inequality symbol instead of an equals symbol. For example, \( y > -3x - 2 \) is a linear inequality and \( y = -3x - 2 \) is the related linear equation.

The graph of the inequality \( y > -3x - 2 \) is shown as a shaded region. Every point in the shaded region satisfies the inequality. The graph of \( y = -3x - 2 \) is the boundary of the region. It is drawn as a dashed line to show that points on the line do not satisfy the inequality. If the symbol were \( \leq \) or \( \geq \), then points on the boundary would satisfy the inequality, so the boundary would be drawn as a solid line.

**Example 1** Dashed Boundary

Graph \( x + 4y > 2 \).

**Step 1** The boundary of the graph is the graph of \( x + 4y = 2 \). Since the inequality symbol is \( > \), the boundary will be dashed.

**Step 2** Test the point \((0, 0)\) because it is not on the boundary.

\[
\begin{align*}
\quad \quad \quad \quad \quad x + 4y &> 2 & \text{Original inequality} \\
0 + 4(0) &> 2 & (x, y) = (0, 0) \\
0 &> 2 & \text{False}
\end{align*}
\]

The region that does not contain \((0, 0)\) is shaded.

**CHECK** The graph indicates that \((0, 3)\) is a solution.

\[
\begin{align*}
\quad \quad \quad \quad \quad x + 4y &> 2 & \text{Original inequality} \\
0 + 4(3) &> 2 & (x, y) = (0, 3) \\
12 &> 2 & \text{True}
\end{align*}
\]

The solution checks.

**Guided Practice**

1A. Graph \( 3x + \frac{1}{2}y < 2 \).

1B. Graph \( -x + 2y > 4 \).
RECREATION A recreation center offers various 30-minute and 60-minute art classes. The recreation director has allotted up to 20 hours per week for art classes.

a. Write an inequality to represent the number of classes that can be offered per week. Graph the inequality.

Let \( x \) represent the number of 30-minute or \( \frac{1}{2} \)-hour art classes, and let \( y \) represent the number of 60-minute or 1-hour art classes. Because the sum can equal the maximum, the inequality symbol is \( \leq \), and the boundary is solid. The inequality is \( \frac{1}{2}x + y \leq 20 \).

**Step 1** Graph the boundary \( \frac{1}{2}x + y = 20 \).

**Step 2** Test the point \((0, 0)\).

\[
\frac{1}{2}(0) + (0) \leq 20 \quad \text{(x, y) = (0, 0)}
\]

\[0 \leq 20 \quad \checkmark \quad \text{True}\]

The region that contains \((0, 0)\) is shaded.

b. Can the recreation director schedule 25 of the 30-minute classes and 15 of the 60-minute classes during a given week? Explain your reasoning.

The point \((25, 15)\) lies outside the shaded region, so it does not satisfy the inequality. Thus, the recreation director cannot schedule 25 30-minute and 15 60-minute classes.

Guided Practice

2. Manuel has $15 to spend at the county fair. The fair costs $5 for admission, $0.75 for each ride ticket, and $0.25 for each game ticket. Write an inequality, and draw a graph that represents the number of \( r \) ride and \( g \) game tickets that Manuel can buy.

2 Graph Absolute Value Inequalities Graphing absolute value inequalities is similar to graphing linear inequalities. First you graph the absolute value equation. Then you determine whether the boundary is dashed or solid and which region should be shaded.

Example 3 Absolute Value Inequality

Graph \( y \geq |x| - 4 \).

Since the inequality symbol is \( \geq \), the boundary is solid.

Graph the equation. Then test \((0, 0)\).

\[
y \geq |x| - 4 \quad \text{Original inequality}
\]

\[
0 \geq |0| - 4 \quad \text{(x, y) = (0, 0)}
\]

\[0 \geq -4 \quad \checkmark \quad \text{True}\]

The region that includes \((0, 0)\) is shaded.

Guided Practice

3A. Graph \( y \leq 2|x| + 3 \).

3B. Graph \( y \geq 3|x + 1| \).
Check Your Understanding

Example 1  
Graph each inequality.
1. \( y \leq 4 \)  
2. \( x \geq -6 \)  
3. \( x + 4y \leq 2 \)  
4. \( 3x + y > -8 \)

Example 2  
5. CAR MAINTENANCE  
Gregg needs to buy gas and oil for his car. Gas costs $3.45 a gallon, and oil costs $2.41 a quart. He has $50 to spend.
   a. Write an inequality to represent the situation, where \( g \) is the number of gallons of gas he buys and \( q \) is the number of quarts of oil.
   b. Graph the inequality.
   c. Can Gregg buy 10 gallons of gasoline and 8 quarts of oil? Explain.

Example 3  
Graph each inequality.
6. \( y \geq |x + 3| \)  
7. \( y - 6 < |x| \)

Practice and Problem Solving

Example 1  
Graph each inequality.
8. \( x + 2y > 6 \)  
9. \( y \geq -3x - 2 \)  
10. \( 2y + 3 \leq 11 \)  
11. \( 4x - 3y > 12 \)  
12. \( 6x + 4y \leq -24 \)  
13. \( y \geq \frac{3}{4}x + 6 \)

Example 2  
14. COLLEGE  
April’s guidance counselor says that she needs a combined score of at least 1700 on her college entrance exams to be eligible for the college of her choice. The highest possible score is 2400—1200 on the math portion and 1200 on the verbal portion.
   a. The inequality \( x + y \geq 1700 \) represents this situation, where \( x \) is the verbal score and \( y \) is the math score. Graph this inequality.
   b. Refer to your graph. If she scores a 680 on the math portion of the test and 910 on the verbal portion of the test, will April be eligible for the college of her choice?

Example 3  
Graph each inequality.
15. \( y > |3x| \)  
16. \( y + 4 \leq |x - 2| \)  
17. \( y - 6 < |-2x| \)  
18. \( y + 8 < 2|\frac{2}{3}x + 6| \)  
19. \( 2y > |4x - 5| \)  
20. \( -y \leq |3x - 4| \)

21. SCHOOL DANCE  
Carlos estimates that he will need to earn at least $700 to take his girlfriend to the prom. Carlos works two jobs as shown in the table.
   a. Write an inequality to represent this situation.
   b. Graph the inequality.
   c. Will he make enough working 50 hours at each job?

Graph each inequality.
22. \( y \geq |-2x - 6| \)  
23. \( y \leq |x - 3| + 4 \)  
24. \( y - 3 > -2|x + 4| \)  
25. \( |y| > |x| \)  
26. \( |x - y| > 5 \)  
27. \( |x + 3y| \geq 2 \)
28. **JEWELRY** Mei is making necklaces and bracelets to sell at a craft show. She has enough beads to make 50 pieces. Let \( x \) represent the number of bracelets and \( y \) represent the number of necklaces.
   a. Write an inequality that shows the possible number of necklaces and bracelets Mei can make.
   b. Graph the inequality.
   c. Give three possible solutions for the number of necklaces and bracelets that can be made.

29. **GIFT CARDS** Susan received a gift card from an electronics store for \$400. She wants to spend the money on DVDs, which cost \$20 each, and CDs, which cost \$15 each.
   a. Let \( d \) equal the number of DVDs, and let \( c \) equal the number of CDs. Write an inequality that shows the possible combinations of DVDs and CDs that Susan can purchase.
   b. Graph the inequality.
   c. Give three possible solutions for the number of DVDs and CDs she can buy.

Graph each inequality.
30. \( y \geq |x| \)  
31. \( y < |x + 2| \)  
32. \( y \geq |x| \)

**H.O.T. Problems** Use Higher-Order Thinking Skills

33. **OPEN ENDED** Create an absolute value inequality in which none of the possible solutions fall in the second or third quadrant.

34. **CHALLENGE** Graph the following inequality.
   \[
g(x) = \begin{cases} 
  |x + 1| & \text{if } x \leq -4 \\
  -|x| & \text{if } -4 < x < 2 \\
  |x - 4| & \text{if } x \geq 2
\end{cases}
\]

35. **ERROR ANALYSIS** Paulo and Janette are graphing \( x - y \geq 2 \). Is either of them correct? Explain your reasoning.

36. **REASONING** When will it be possible to shade two different areas when graphing a linear absolute value inequality? Explain your reasoning.

37. **WRITING IN MATH** Describe a situation in which there are no solutions to an absolute value inequality. Explain your reasoning.
38. **EXTENDED RESPONSE** Craig scored 85%, 96%, 79%, and 81% on his first four math tests. He hopes to score high enough on the final test to earn a 90% average. If the final test is worth twice as much as one of the other tests, determine if Craig can earn a 90% average. If so, what score does Craig need to get on the final test to accomplish this? Explain how you found your answer.

39. Which of the following sets of numbers represents an infinite set?
   - A [2, 4, 6]
   - B {whole numbers between -50 and 50}
   - C {integers}
   - D $\{\frac{1}{2}, 3, 4, 5\} \quad \{\frac{2}{4}, 5, 6\}$

40. **SHORT RESPONSE** Which theorem of congruence should be used to prove $\triangle ABC \cong \triangle XYZ$?

41. **SAT/ACT** For which function is the range $|f(x)| |f(x)\leq 0|$?
   - F $f(x) = -x$
   - J $f(x) = |x|
   - G $f(x) = [x]$
   - K $f(x) = -|x|
   - H $f(x) = [-x]$

### Spiral Review

Write an equation for each graph. (Lesson 2-7)

42. [Graph of a parabola]
43. [Graph of a line]
44. [Graph of a line]

Graph each function. (Lesson 2-6)

45. $f(x) = \begin{cases} x & \text{if } x < 1 \\ 3 & \text{if } 1 \leq x \leq 3 \\ -2x & \text{if } x > 3 \end{cases}$

46. $f(x) = \begin{cases} x + 3 & \text{if } x < -2 \\ 2x & \text{if } -2 \leq x \leq 2 \\ -3x & \text{if } x > 2 \end{cases}$

47. $f(x) = \begin{cases} -2x & \text{if } x \leq -2 \\ x + 1 & \text{if } 0 < x \leq 6 \\ x - 5 & \text{if } x > 6 \end{cases}$

Write each equation in standard form. Identify $A$, $B$, and $C$. (Lesson 2-2)

48. $-6y = 8x - 3$

49. $12y + x = -3y + 5x - 6$

50. $\frac{x + 3}{4} + \frac{y - 1}{2} = 3$

51. **TENNIS** Sixteen players signed up for tennis lessons. The instructor plans to use 50 tennis balls for every player and have 200 extra. How many tennis balls are needed for the lessons? (Lesson 1-3)

Multiply. (Lesson 0-2)

52. $(3x - 4)(2x + 1)$

53. $(6x + 5)(-x - 3)$

54. $(5x + 2)(-2x + 3)$

### Skills Review

Graph each linear equation. (Lesson 2-2)

55. $y = 2x - 8$

56. $y = -\frac{3}{4}x + 2$

57. $3y - 4x = 24$
Study Guide

KeyConcepts

Relations and Functions (Lesson 2-1)
- A function is a relation where each member of the domain is paired with exactly one member of the range.

Linear Equations and Slope (Lessons 2-2 to 2-4)
- Standard Form: $Ax + By = C$, where $A$, $B$, and $C$ are integers whose greatest common factor is 1, $A \geq 0$, and $A$ and $B$ are not both zero.
- Slope-Intercept Form: $y = mx + b$
- Point-Slope Form: $y - y_1 = m(x - x_1)$

Scatter Plots and Lines of Regression (Lesson 2-5)
- A prediction equation can be used to predict the value of one of the variables given the value of the other variable.
- A line of regression can be used to model data.

Special Functions and Parent Functions (Lessons 2-6 and 2-7)
- A piecewise-defined function is made up of two or more expressions.
- Translations, reflections, and dilations to a parent graph form a family of graphs.

Graphing Linear and Absolute Value Inequalities (Lesson 2-8)
- You can graph an inequality by following these steps.
  - **Step 1** Determine whether the boundary is solid or dashed. Graph the boundary.
  - **Step 2** Choose a point not on the boundary and test it in the inequality.
  - **Step 3** If a true inequality results, shade the region containing your test point. If a false inequality results, shade the other region.

VocabularyCheck

Choose the correct term to complete each sentence.

1. A function is (discrete, one-to-one) if each element of the domain is paired to exactly one unique element of the range.

2. The (domain, range) of a relation is the set of all first coordinates from the ordered pairs which determine the relation.

3. The (constant, identity) function is a linear function described by $f(x) = x$.

4. If you are given the coordinates of two points on a line, you can use the (slope-intercept, point-slope) form to find the equation of the line that passes through them.

5. A set of bivariate data graphed as ordered pairs in a coordinate plane is called a (scatter plot, line of fit).

6. A function that is written using two or more expressions is called a (linear, piecewise) function.
### Lesson-by-Lesson Review

#### 2-1 Relations and Functions (pp. 61–67)

**Example 1**

State the domain and range of the relation \{(-4, 3), (-1, 0), (-2, 4), (3, -1), (2, 6)\}. Then determine whether the relation is a function. If it is a function, determine if it is one-to-one, onto, both, or neither.

Domain: \{-4, -2, -1, 2, 3\}
Range: \{-1, 0, 3, 4, 6\}

Each element of the domain is paired with one element of the range, so the relation is a function. The function is both one-to-one, onto, both, or neither.

**Example 2**

Find \(f(-2)\) if \(f(x) = 4x - 3\).

\[
\begin{align*}
  f(-2) &= 4(-2) - 3 \\
         &= -8 - 3 \\
         &= -11
\end{align*}
\]

**Example 3**

State whether \(f(x) = 3x^2\) is a linear function. Write yes or no. Explain.

No, because the expression includes a variable raised to the second power.

**Example 4**

Write the equation \(y = -5x + 8\) in standard form. Identify \(A\), \(B\), and \(C\).

\[
y = -5x + 8
\]

Original equation

\[
5x + y = 8
\]

Add 5x to each side.

\[
A = 5, B = 1, \text{ and } C = 8
\]

#### 2-2 Linear Relations and Functions (pp. 69–74)

**Example 3**

State whether \(f(x) = 3x^2\) is a linear function. Write yes or no. Explain.

No, because the expression includes a variable raised to the second power.

**Example 4**

Write the equation \(y = -5x + 8\) in standard form. Identify \(A\), \(B\), and \(C\).

\[
y = -5x + 8
\]

Original equation

\[
5x + y = 8
\]

Add 5x to each side.

\[
A = 5, B = 1, \text{ and } C = 8
\]
2-3 Rate of Change and Slope (pp. 76–82)

25. **RETAIL** The table shows the number of DVDs sold each week at the Super Movie Store. Find the average rate of change of the number of DVDs sold from week 2 to week 5.

<table>
<thead>
<tr>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>DVDs Sold</td>
<td>76</td>
<td>58</td>
<td>94</td>
<td>83</td>
<td>112</td>
</tr>
</tbody>
</table>

Find the slope of the line that passes through each pair of points.

30. (2, 5), (6, −3)
31. (8, 2), (2, 8)

32. **Example 5**

Find the slope of the line that passes through each pair of points.

a. (−2, 9), (1, 4)
   
   \[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
   
   \[ = \frac{4 - 9}{1 - (-2)} \]
   
   \[ = \frac{-5}{3} \] Simplify.

b. (−3, 6), (4, 6)
   
   \[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
   
   \[ = \frac{6 - 6}{4 - (-3)} \]
   
   \[ = 0 \] or 0 Simplify.

2-4 Writing Linear Equations (pp. 83–89)

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

33. slope −2, passes through (−3, −5)
34. slope \(\frac{2}{3}\), passes through (4, −1)
35. passes through (−2, 4) and (0, 8)
36. passes through (3, 5) and (−1, 5)

37. (6, 1), (4, 9)
38. (−4, 2), (6, 8)

Write an equation of the line passing through each pair of points.

39. through (1, 2), parallel to \(y = 4x - 3\)
40. through (−3, 5), perpendicular to \(y = \frac{2}{3}x - 8\)

41. **PETS** Drew paid a $250 fee when he adopted a puppy. The average monthly cost of feeding and caring for the puppy is $32. Write an equation that represents the total cost of adopting and caring for the puppy for \(x\) months.

**Example 6**

Write an equation of the line through (−2, 5) and (0, −9).

Find the slope of the line.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ = \frac{-9 - 5}{0 - (-2)} \]

\[ = -14 \] or −7 Simplify.

Write an equation.

\[ y - y_1 = m(x - x_1) \] Point-slope form

\[ y - 5 = -7(x - (-2)) \] Substitute.

\[ y - 5 = -7(x + 2) \] Simplify.

\[ y - 5 = -7x - 14 \] Distributive Property

\[ y = -7x - 9 \] Add 5 to each side.

The equation is \(y = -7x - 9\).
2-5 Scatter Plots and Lines of Regression (pp. 92–98)

Make a scatter plot and a line of fit and describe the correlation for each set of data. Then, use two ordered pairs to write a prediction equation.

42. HEATING The table shows the monthly heating cost for a large home.

<table>
<thead>
<tr>
<th>Month</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Jan</th>
<th>Feb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bill ($)</td>
<td>72</td>
<td>114</td>
<td>164</td>
<td>198</td>
<td>224</td>
<td>185</td>
</tr>
</tbody>
</table>

43. AMUSEMENT PARK The table shows the annual attendance in thousands at an amusement park during the last 5 years.

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>People (thousands)</td>
<td>44</td>
<td>42</td>
<td>39</td>
<td>31</td>
<td>24</td>
</tr>
</tbody>
</table>

Example 7

SCHOOL ENROLLMENT The table shows the number of students each year at a school.

<table>
<thead>
<tr>
<th>Year's</th>
<th>'04</th>
<th>'05</th>
<th>'06</th>
<th>'07</th>
<th>'08</th>
<th>'09</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students</td>
<td>125</td>
<td>116</td>
<td>142</td>
<td>154</td>
<td>146</td>
<td>175</td>
</tr>
</tbody>
</table>

Use (2004, 125) and (2009, 175) to find a prediction equation.

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

Slope Formula

\[
= \frac{175 - 125}{2009 - 2004}
\]

Substitution

\[
= \frac{50}{5} = 10
\]

Simplify.

\[
y - y_1 = m(x - x_1)
\]

Point-slope form

\[
y - 125 = 10(x - 2004)
\]

Substitution

\[
y = 10x - 20,040
\]

Distributive Property

\[
y = 10x - 19,915
\]

Add 125 to each side.

2-6 Special Functions (pp. 101–107)

Graph each function. Identify the domain and range.

44. \( f(x) = \begin{cases} 
-2x & \text{if } x \leq -1 \\
1 + x & \text{if } -1 < x < 3 \\
x & \text{if } x \geq 3
\end{cases} \)

45. \( f(x) = \begin{cases} 
-3 & \text{if } x < -1 \\
4x - 3 & \text{if } -1 \leq x \leq 3 \\
x & \text{if } x > 3
\end{cases} \)

46. Write the piecewise function shown in the graph.

Graph each function. Identify the domain and range.

47. \( f(x) = \lfloor x \rfloor + 2 \)

48. \( f(x) = \lfloor x + 3 \rfloor \)

Example 8

Write the piecewise function shown in the graph.

The left portion of the graph is the graph of \( f(x) = 3 \). There is a circle at \((-2, 3)\), so the linear function is defined for \( x < -2 \).

The center portion of the graph is the graph of \( f(x) = x - 1 \). There is a dot at \((-2, -3)\) and a circle at \((2, 1)\), so the linear function is defined for \(-2 \leq x < 2\).

The right portion of the graph is the graph of \( f(x) = 2x \). There is a dot at \((2, 4)\), so the linear function is defined for \( x \geq 2 \).

\[
f(x) = \begin{cases} 
3 & \text{if } x < -2 \\
1 + x & \text{if } -2 \leq x < 2 \\
2x & \text{if } x \geq 2
\end{cases}
\]
2–7 Parent Functions and Transformations (pp. 109–116)

Identify the type of function represented by each graph.

49. ![Graph](image1)

50. ![Graph](image2)

51. Describe the translation in \( y = x^2 - 3 \).

52. Describe the reflection in \( y = -x^2 \).

53. **Construction** A large arch is being constructed at the entrance of a new city hall building. The shape of the arch resembles the graph of the function \( f(x) = -0.025x^2 + 3.64x - 0.038 \). Describe the shape of the arch.

Example 9

Identify the type of function represented by the graph.

The graph is in the shape of a V. The graph represents an absolute value function.

Example 10

Describe the translation in \( y = |x + 6| \).

The graph of \( y = |x + 6| \) is a translation of the graph of \( y = |x| \) 6 units left.

2–8 Graphing Linear and Absolute Value Inequalities (pp. 117–121)

Graph each inequality.

54. \( x - 3y < 6 \)  
55. \( y \geq 2x + 1 \)

56. \( 2x + 4y \leq 12 \)  
57. \( y < -3x - 5 \)

58. \( y > |2x| \)  
59. \( y \geq |2x - 2| \)

60. \( y + 3 < |x + 1| \)  
61. \( 2y \leq |x - 3| \)

62. **Books** Spencer has saved $96 for a trip to his favorite bookstore. Each paperback book costs $8 and each hardback book costs $12. Write and graph an inequality that shows the number of paperback books and hardback books Spencer can purchase.

Example 11

Graph \( x - 2y > 6 \).

Since the inequality symbol is >, the graph of the boundary line should be dashed. Graph \( x - 2y = 6 \).

Test

\[ x - 2y > 6 \at \( (0, 0) \)
\[
\begin{align*}
0 - 2(0) &> 6 \\
0 &> 6 \quad \times
\end{align*}
\]
1. State the domain and range of the relation shown in the table. Then determine if it is a function. If it is a function, determine if it is one-to-one, onto, both, or neither.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>−1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

Find each value if \( f(x) = −2x + 3 \).

2. \( f(−4) \)

3. \( f(3y) \)

4. Write \( 2y = −6x + 4 \) in standard form. Identify \( A, B \) and \( C \).

5. Find the \( x \)-intercept and the \( y \)-intercept for \( 3x − 4y = −24 \).

6. MULTIPLE CHOICE The cost of producing \( x \) pumpkin pies at a small bakery is given by \( C(x) = 49 + 1.75x \). Find the cost of producing 25 pies.
   
   A $74.00  
   B $81.50  
   C $92.75  
   D $108.25

Find the slope of the line that passes through each pair of points.

7. \( (1, 6), (3, 10) \)

8. \( (−2, 7), (3, −1) \)

9. MULTIPLE CHOICE Find the equation of the line that passes through \((0, −3)\) and \((4, 1)\).

   F \( y = −x + 3 \)  
   G \( y = −x − 3 \)  
   H \( y = x − 3 \)  
   J \( y = x + 3 \)

10. Write an equation in slope-intercept form for the line that has slope \( −2 \) and passes through the point \((3, −4)\).

11. Write an equation of the line that passes through \((2, −4)\) and \((1, 6)\).

12. Write an equation in slope-intercept form for the line that passes through \((-3, 5)\) and is parallel to \( y = −6x + 1 \).

13. EMERGENCY ROOM A hospital tracks the number of emergency room visits during the fall and winter months.

<table>
<thead>
<tr>
<th>Month</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Jan</th>
<th>Feb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Visits</td>
<td>124</td>
<td>163</td>
<td>155</td>
<td>171</td>
<td>192</td>
</tr>
</tbody>
</table>

   a. Make a scatter plot and describe the correlation.

   b. Use two ordered pairs to write a prediction equation.

   c. Use your prediction equation to predict the number of emergency room visits for March.

14. Graph \( f(x) = \begin{cases} −x & \text{if } x < −2 \\ x + 2 & \text{if } −2 ≤ x ≤ 2 \\ 5 & \text{if } x > 2 \end{cases} \).

15. Write the piecewise function shown.

16. Identify the domain and range of \( y = [x] + 2 \).

17. Describe the translation to \( y = x^2 + 5 \).

18. Describe the reflection in \( y = −|x| \).

Graph each inequality.

19. \( y ≥ 4x − 1 \)

20. \( 2x + 6y < −12 \)
Reading Math Problems

The first step to solving any math problem is to read the problem. When reading a math problem to get the information you need to solve, it is helpful to use special reading strategies.

Strategies for Reading Math Problems

**Step 1**

Read the problem quickly to gain a general understanding of it.

- **Ask yourself:** “What do I know?” “What do I need to find out?”
- **Think:** “Is there enough information to solve the problem? Is there extra information?”
- **Highlight:** If you are allowed to write in your test booklet, underline or highlight important information. Cross out any information you do not need.

**Step 2**

Reread the problem to identify relevant facts.

- **Analyze:** Determine how the facts are related.
- **Key Words:** Look for key words to solve the problem.
- **Vocabulary:** Identify mathematical terms. Think about the concepts and how they are related.
- **Plan:** Make a plan to solve the problem.
- **Estimate:** Quickly estimate the answer.

**Step 3**

Identify any obvious wrong answers.

- **Eliminate:** Eliminate any choices that are very different from your estimate.
- **Units of Measure:** Identify choices that are possible answers based on the units of measure in the question. For example, if the question asks for area, only answers in square units will work.

**Step 4**

Look back after solving the problem.

**Check:** Make sure you have answered the question.
Test Practice Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

Sandy heated a solution over a burner and then removed it from the heat source. The temperature of the solution decreased linearly as it cooled. The temperatures after 0, 2, 5, and 9 minutes are shown in the table. What is the rate of change in the temperature of the solution as it cools?

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>133.2</td>
</tr>
<tr>
<td>2</td>
<td>130.4</td>
</tr>
<tr>
<td>5</td>
<td>126.2</td>
</tr>
<tr>
<td>9</td>
<td>120.6</td>
</tr>
</tbody>
</table>

A $-1.4$ degrees per minute  
B $-0.8$ degrees per minute  
C $0.8$ degrees per minute  
D $1.4$ degrees per minute

Read the problem carefully. There is extra information in the problem. To determine the slope, you only need information from two points on the linear function. Use two of the points to find the slope.

\[ m = \frac{130.4 - 133.2}{2 - 0} = -1.4 \]

The correct answer is A.

Exercises

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. The graph shows the cost of shipping packages. How much would it cost to ship a package that weighs 2 pounds 8 ounces?

   ![Shipping Costs Graph](image)

   A $3.50  
   B $4.50  
   C $5.00  
   D $5.50

2. What is the slope of the line shown in the graph?

   ![Graph](image)

   F $-2$  
   G $-\frac{1}{2}$  
   H $\frac{1}{2}$  
   J $2$
Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. What is the domain of the relation shown below?

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>-3</td>
</tr>
</tbody>
</table>

A \{0, 1, 2, 4, 6\}  
B \{-3, -1, 0, 4\}  
C \{-3, 1, 2, 6\}  
D \{-3, -1\}

2. What is the slope of the line?

A \(-2\)  
B \(-\frac{1}{2}\)  
C \(\frac{1}{2}\)  
D \(2\)

3. The Robinson family bought their home in 1998 for $152,400. When they sold it in 2010, the value was $174,900. What was the annual rate of change in the value of the home?

A \$1225  
B \$1875  
C \$22,500  
D \$27,275

4. Carmen works for an electronics retailer. She earns a weekly salary of $450 plus a commission of 4.5% on her weekly sales. Write a linear equation for Carmen’s weekly earnings \(E\) if she has \(d\) dollars in sales.

A \(E = (450 + 4.5)d\)  
B \(E = (450 + 0.045)d\)  
C \(E = 450 + 0.045d\)  
D \(E = 450 + 4.5d\)

5. Which of the graphs represents the solution set for \(|x - 3| - 4 = 0\)?

A  
B  
C  
D

6. Karissa has $10 per month to spend text messaging on her cell phone. The phone company charges $4.95 for the first 100 messages and $0.10 for each additional message. How many text messages can Karissa afford to send each month?

A 50  
B 100  
C 150  
D 151

7. Given \(y = 2.24x + 16.45\), which statement best describes the effect of decreasing the \(y\)-intercept by 20.25?

A The \(x\)-intercept increases.  
B The \(y\)-intercept increases.  
C The new line has a greater rate of change.  
D The new line is perpendicular to the original.
**Short Response/Gridded Response**

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. Write an equation for the piecewise function shown in the graph below.

![Graph](image)

9. **GRIDDED RESPONSE** Evaluate the piecewise function shown in Exercise 8 for \(x = -3\).

10. The graph of the absolute value parent function is shown below.

![Graph](image)

a. What is the equation of this parent function?

b. What equation would result in the graph of the parent function being reflected over the \(x\)-axis and shifted up 2 units?

c. What equation would result in the graph of the parent function being shifted left 3 units and down 1 unit?

**Extended Response**

Record your answers on a sheet of paper. Show your work.

11. The soccer team is having a bake sale this week to raise money for the program. For each cookie sold, the profit is $0.45, and for each brownie sold, the profit is $0.50.

a. The team hopes to earn $150 in profits from the bake sale. Let \(x\) represent the number of cookies sold and \(y\) the number of brownies sold. Write an inequality to model the situation.

b. Graph the inequality.

c. If the team sells 180 cookies and 160 brownies this week, will they meet their goal? Explain.

12. Use the scatter plot to answer each question.

![Scatter Plot](image)

a. What type of correlation is shown by the data in the plot?

b. Determine a regression line for the data.

c. Use your regression line to predict the value of \(y\) when \(x = 12\).

**Need Extra Help?**

<table>
<thead>
<tr>
<th>If you missed Question...</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Go to Lesson...</td>
<td>2-1</td>
<td>2-3</td>
<td>2-3</td>
<td>2-4</td>
<td>1-4</td>
<td>1-3</td>
<td>2-7</td>
<td>2-6</td>
<td>2-7</td>
<td>2-6</td>
<td>2-7</td>
<td>2-8</td>
</tr>
</tbody>
</table>

For help with TN SPI...

3102.3.7 3102.1.6 3102.1.6 3102.3.5 3102.3.5 3102.3.5 3102.1.5 3103.3.10 3103.3.7 3103.3.10 3103.3.5 3103.5.6