In Chapter 2, you graphed linear equations and inequalities.

In Chapter 5, you will:
- Graph quadratic functions.
- Solve quadratic equations.
- Perform operations with complex numbers.
- Graph and solve quadratic inequalities.

MOTION The path that a soccer ball or a firework takes can be modeled by a quadratic function. Quadratic functions can map an object in motion. In this chapter you will look at a pumpkin catapult, an amusement park ride, and a diver in motion.
Factor completely. If the polynomial is not factorable, write prime.

(Lesson 0-3)

9. \(x^2 + 13x + 40\)
10. \(x^2 − 10x + 21\)
11. \(2x^2 + 7x − 4\)
12. \(2x^2 − 7x − 15\)
13. \(x^2 − 11x + 15\)
14. \(x^2 + 12x + 36\)
15. FLOOR PLAN The rectangular room pictured below has an area of \(x^2 + 14x + 48\) square feet. If the width of the room is \((x + 6)\) feet, what is the length?

\[A = (x^2 + 14x + 48) \text{ ft}^2\]

\[(x + 6) \text{ ft}\]
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 5. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

**New Vocabulary**

<table>
<thead>
<tr>
<th>English</th>
<th>Español</th>
</tr>
</thead>
<tbody>
<tr>
<td>quadratic term</td>
<td>término cuadrático</td>
</tr>
<tr>
<td>linear term</td>
<td>término lineal</td>
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<tr>
<td>constant term</td>
<td>término constante</td>
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<tr>
<td>vertex</td>
<td>vértice</td>
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<tr>
<td>maximum value</td>
<td>valor máximo</td>
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<tr>
<td>minimum value</td>
<td>valor mínimo</td>
</tr>
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<td>quadratic equation</td>
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</tr>
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<td>standard form</td>
<td>forma estándar</td>
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<tr>
<td>root</td>
<td>raíz</td>
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<tr>
<td>zero</td>
<td>cero</td>
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<tr>
<td>imaginary unit</td>
<td>unidad imaginaria</td>
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<tr>
<td>pure imaginary number</td>
<td>número imaginario puro</td>
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<tr>
<td>complex number</td>
<td>número complejo</td>
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<td>complex conjugates</td>
<td>conjugados complejos</td>
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<td>completing the square</td>
<td>completar el cuadrado</td>
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<td>Quadratic Formula</td>
<td>fórmula cuadrática</td>
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<td>discriminant</td>
<td>discriminante</td>
</tr>
<tr>
<td>vertex form</td>
<td>forma de vértice</td>
</tr>
<tr>
<td>quadratic inequality</td>
<td>desigualdad cuadrática</td>
</tr>
</tbody>
</table>

**Review Vocabulary**

- **domain** p. P7 dominio the set of all x-coordinates of the ordered pairs of a relation
- **function** p. P7 función a relation in which each x-coordinate is paired with exactly one y-coordinate
- **range** p. P7 rango the set of all y-coordinates of the ordered pairs of a relation

**Foldable Study Organizer**

**Quadratic Functions** Make this Foldable to help you organize your Chapter 5 notes about quadratic functions and relations. Begin with one sheet of 11” by 17” paper.

1. **Fold** in half lengthwise.
   ![Folded paper](image1)

2. **Fold** in fourths crosswise.
   ![Folded paper](image2)

3. **Cut** along the middle fold from the edge to the last crease as shown.
   ![Cut paper](image3)

4. **Refold** along the lengthwise fold and tape the uncut section at the top. Label each section with a lesson number and close to form a booklet.
   ![Refolded paper](image4)
**New Vocabulary**
- quadratic function
- quadratic term
- linear term
- constant term
- parabola
- axis of symmetry
- vertex
- maximum value
- minimum value

**Tennessee Curriculum Standards**
- ✔ 3103.3.2 Determine the domain of a function represented in either symbolic or graphical form.
- ✔ 3103.3.9 Find an equation for a parabola when given its graph or when given its roots.
- SPI 3103.3.5 Describe the domain and range of functions and articulate restrictions imposed either by the operations or by the contextual situations which the functions represent.

**Graphing Quadratic Functions**

1. **Graph Quadratic Functions**
   - In a quadratic function, the greatest exponent is 2. These functions can have a quadratic term, a linear term, and a constant term. The general quadratic function is shown below:
   
   \[ f(x) = ax^2 + bx + c, \text{ where } a \neq 0 \]

   The graph of a quadratic function is called a parabola. To graph a quadratic function, graph ordered pairs that satisfy the function.

   **Example 1** Graph a Quadratic Function by Using a Table

   Graph \( f(x) = 3x^2 - 12x + 6 \) by making a table of values. Choose integer values for \( x \), and evaluate the function for each value. Graph the resulting coordinate pairs, and connect the points with a smooth curve.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 3x^2 - 12x + 6 )</th>
<th>( f(x) )</th>
<th>( (x, f(x)) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( 3(0)^2 - 12(0) + 6 )</td>
<td>6</td>
<td>(0, 6)</td>
</tr>
<tr>
<td>1</td>
<td>( 3(1)^2 - 12(1) + 6 )</td>
<td>-3</td>
<td>(1, -3)</td>
</tr>
<tr>
<td>2</td>
<td>( 3(2)^2 - 12(2) + 6 )</td>
<td>-6</td>
<td>(2, -6)</td>
</tr>
<tr>
<td>3</td>
<td>( 3(3)^2 - 12(3) + 6 )</td>
<td>-3</td>
<td>(3, -3)</td>
</tr>
<tr>
<td>4</td>
<td>( 3(4)^2 - 12(4) + 6 )</td>
<td>6</td>
<td>(4, 6)</td>
</tr>
</tbody>
</table>

**Guided Practice**

1. Graph each function by making a table of values.

   **A.** \( g(x) = -2x^2 + 8x - 3 \)

   **B.** \( h(x) = 4x^2 - 8x + 1 \)
Notice in Example 1 that there seemed to be a pattern in the values for $f(x)$. This is due to the axis of symmetry of parabolas. The **axis of symmetry** is a line through the graph of a parabola that divides the graph into two congruent halves. Each side of the parabola is a reflection of the other side.

The axis of symmetry will intersect a parabola at only one point, called the **vertex**. The vertex of the graph at the right is $A(3, -3)$.

Notice that the $x$-coordinates of points $B$ and $C$ are both 4 units away from the $x$-coordinate of the vertex, and they have the same $y$-coordinate. This is due to the symmetrical nature of the graph.

### Key Concept: Graph of a Quadratic Function—Parabola

Words
- Consider the graph of $y = ax^2 + bx + c$, where $a \neq 0$.
  - The $y$-intercept is $a(0)^2 + b(0) + c$ or $c$.
  - The equation of the axis of symmetry is $x = -\frac{b}{2a}$.
  - The $x$-coordinate of the vertex is $-\frac{b}{2a}$.

Model

**Find the axis of symmetry and the vertex.**

**Find the $y$-intercept and its reflection.**

**Connect the points with a smooth curve.**

Now you can use the axis of symmetry to help plot points and graph a parabola. For $y = x^2 + 6x - 2$ below, the axis of symmetry is $x = -\frac{b}{2a} = -\frac{6}{2(1)}$ or $x = -3$. 

**Study Tip:**

**Plotting Reflections.** The reflection of a point is its mirror image on the other side of the axis of symmetry.
Example 2  Axis of Symmetry, \(y\)-intercept, and Vertex

Consider \(f(x) = x^2 + 4x - 3\).

a. Find the \(y\)-intercept, the equation of the axis of symmetry, and the \(x\)-coordinate of the vertex.

The function is of the form \(f(x) = ax^2 + bx + c\), so we can identify \(a\), \(b\), and \(c\).

\[
\begin{align*}
f(x) &= ax^2 + bx + c \\
f(x) &= 1x^2 + 4x - 3 & \Rightarrow & a = 1, b = 4, \text{ and } c = -3
\end{align*}
\]

The \(y\)-intercept is \(c = -3\).

Use \(a\) and \(b\) to find the equation of the axis of symmetry.

\[
x = -\frac{b}{2a}
\]

Equation of the axis of symmetry

\[
= -\frac{4}{2(1)} \quad a = 1 \text{ and } b = 4
\]

\[
= -2 \quad \text{Simplify.}
\]

The equation of the axis of symmetry is \(x = -2\). Therefore, the \(x\)-coordinate of the vertex is \(-2\).

b. Make a table of values that includes the vertex.

Select five specific points, with the vertex in the middle and two points on either side of the vertex, including the \(y\)-intercept and its reflection. Use symmetry to determine the \(y\)-values of the reflections.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(x^2 + 4x - 3)</th>
<th>(f(x))</th>
<th>((x, f(x)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-6)</td>
<td>((-6)^2 + 4(-6) - 3)</td>
<td>9</td>
<td>((-6, 9))</td>
</tr>
<tr>
<td>(-4)</td>
<td>((-4)^2 + 4(-4) - 3)</td>
<td>(-3)</td>
<td>((-4, -3))</td>
</tr>
<tr>
<td>(-2)</td>
<td>((-2)^2 + 4(-2) - 3)</td>
<td>(-7)</td>
<td>((-2, -7))</td>
</tr>
<tr>
<td>(0)</td>
<td>((0)^2 + 4(0) - 3)</td>
<td>(-3)</td>
<td>((0, -3))</td>
</tr>
<tr>
<td>(2)</td>
<td>((2)^2 + 4(2) - 3)</td>
<td>9</td>
<td>((2, 9))</td>
</tr>
</tbody>
</table>


c. Use this information to graph the function.

Graph the points from the table and connect them with a smooth curve.

Draw the axis of symmetry, \(x = -2\), as a dashed line. The graph should be symmetrical about this line.

Guided Practice

2. Consider \(f(x) = -5x^2 - 10x + 6\).

A. Find the \(y\)-intercept, the equation of the axis of symmetry, and the \(x\)-coordinate of the vertex.

B. Make a table of values that includes the vertex.

C. Use this information to graph the function.
### Maximum and Minimum Values

The $y$-coordinate of the vertex of a quadratic function is the **maximum value** or the **minimum value** of the function. These values represent the greatest or lowest possible value the function can reach.

#### Key Concept: Maximum and Minimum Value

**Words**
The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$, has:
- if $a > 0$, the graph opens up and has a minimum value,
- if $a < 0$, the graph opens down and has a maximum value.

**Model**

- **$a$ is positive.**
  - The $y$-coordinate is the minimum value.
- **$a$ is negative.**
  - The $y$-coordinate is the maximum value.

---

**Example 3** Maximum or Minimum Values

Consider $f(x) = -4x^2 + 12x + 18$.

**a. Determine whether the function has a maximum or minimum value.**

For this function, $a = -4$, so the graph opens down and the function has a maximum value.

**b. State the maximum or minimum value of the function.**

The maximum value of the function is the $y$-coordinate of the vertex.

The $x$-coordinate of the vertex is $x = \frac{-b}{2a}$ or $1.5$.

Find the $y$-coordinate of the vertex by evaluating the function for $x = 1.5$.

$f(x) = -4x^2 + 12x + 18$ (Original function)

$x = 1.5$:

$-4(1.5)^2 + 12(1.5) + 18 = -9 + 18 + 18 = 27$ (The maximum value of the function is 27).

**c. State the domain and range of the function.**

The domain is all real numbers. The range is all real numbers less than or equal to the maximum value, or $\{f(x) | f(x) \leq 27\}$.

### Guided Practice

3. Consider $f(x) = 4x^2 - 24x + 11$.

**A.** Determine whether the function has a maximum or minimum value.

**B.** State the maximum or minimum value of the function.

**C.** State the domain and range of the function.
Real-World Example 4 Quadratic Equations in the Real World

CHARITY Refer to the beginning of the lesson.

a. How much should Eddie charge in order to maximize charity income?

Words Total equals fee times number of entrants.

Variable Let \( x \) = the number of price increases.
Let \( P(x) \) = the total pool as a function of \( x \).

Equation \( P(x) = (20 + 5x) \cdot (80 - 5x) \)

Solve for the \( x \)-value of the vertex.
\[
P(x) = (20 + 5x) \cdot (80 - 5x) \\
= 20(80) + 20(-5x) + 5x(80) + 5x(-5x) \quad \text{Distribute.} \\
= 1600 - 100x + 400x - 25x^2 \quad \text{Multiply.} \\
= 1600 + 300x - 25x^2 \quad \text{Simplify.} \\
= -25x^2 + 300x + 1600 \quad ax^2 + bx + c \text{ form}
\]

Use the formula for the axis of symmetry, \( x = \frac{-b}{2a} \), to find the \( x \)-coordinate.
\[
x = \frac{-300}{2(-25)} \quad \text{or} \quad 6 \\
a = -25 \quad \text{and} \quad b = 300
\]

Eddie needs to have 6 price increases, so he should charge \( 20 + 6(5) \) or $50.

b. What will be the maximum value of the pool?

Find the maximum value of the quadratic function \( P(x) \) by evaluating \( P(6) \).
\[
P(x) = -25x^2 + 300x + 1600 \quad \text{Total pool function} \\
P(6) = -25(6)^2 + 300(6) + 1600 \quad x = 6 \\
= -900 + 1800 + 1600 \quad \text{Simplify.}
\]

Thus, the maximum prize pool is $2500 after 6 price increases.

CHECK Graph the function on a graphing calculator and use the \text{CALC:Maximum} function to confirm the solution.

Select a left bound of 0 and a right bound of 10. The calculator will display the coordinates of the maximum at the bottom of the screen.

The domain is \( \{x \mid x \geq 0\} \) because there can be no negative increases in price. The range is \( \{y \mid 0 \leq y \leq 2500\} \) because the prize pool cannot have a negative monetary value.

Guided Practice

4. Suppose a different tournament that Eddie organizes has 120 players and the entry fee is $40. Each time he increases the fee by $5, he loses 10 players. Determine what the entry fee should be to maximize the value of the pool.
Check Your Understanding

Examples 1–2 Complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.
b. Make a table of values that includes the vertex.
c. Use this information to graph the function.

1. \( f(x) = 3x^2 \)  
2. \( f(x) = -6x^2 \)  
3. \( f(x) = x^2 - 4x \)  
4. \( f(x) = -x^2 - 3x + 4 \)  
5. \( f(x) = 4x^2 - 6x - 3 \)  
6. \( f(x) = 2x^2 - 8x + 5 \)

Example 3 Determine whether each function has a maximum or minimum value, and find that value. Then state the domain and range of the function.

7. \( f(x) = -x^2 + 6x - 1 \)  
8. \( f(x) = x^2 + 3x - 12 \)  
9. \( f(x) = 3x^2 + 8x + 5 \)  
10. \( f(x) = -4x^2 + 10x - 6 \)

Example 4 11. BUSINESS A store rents 1400 videos per week at $2.25 per video. The owner estimates that they will rent 100 fewer videos for each $0.25 increase in price. What price will maximize the income of the store?

Practice and Problem Solving

Examples 1–2 Complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.
b. Make a table of values that includes the vertex.
c. Use this information to graph the function.

12. \( f(x) = 4x^2 \)  
13. \( f(x) = -2x^2 \)  
14. \( f(x) = x^2 - 5 \)  
15. \( f(x) = x^2 + 3 \)  
16. \( f(x) = 4x^2 - 3 \)  
17. \( f(x) = -3x^2 + 5 \)  
18. \( f(x) = x^2 - 6x + 8 \)  
19. \( f(x) = x^2 - 3x - 10 \)  
20. \( f(x) = -x^2 + 4x - 6 \)  
21. \( f(x) = -2x^2 + 3x + 9 \)

Example 3 Determine whether each function has a maximum or minimum value, and find that value. Then state the domain and range of the function.

22. \( f(x) = 5x^2 \)  
23. \( f(x) = -x^2 - 12 \)  
24. \( f(x) = x^2 - 6x + 9 \)  
25. \( f(x) = -x^2 - 7x + 1 \)  
26. \( f(x) = 8x - 3x^2 + 2 \)  
27. \( f(x) = 5 - 4x - 2x^2 \)  
28. \( f(x) = 15 - 5x^2 \)  
29. \( f(x) = x^2 + 12x + 27 \)  
30. \( f(x) = -x^2 + 10x + 30 \)  
31. \( f(x) = 2x^2 - 16x - 42 \)

Example 4 32. PRODUCTION A financial analyst determined the cost in thousands of dollars of producing bicycle frames is \( C = 0.000025f^2 - 0.04f + 40 \), where \( f \) is the number of frames produced.

a. Find the number of frames that minimizes cost.
b. What is the total cost for that number of frames?
Complete parts a–c for each quadratic function.

a. Find the \(y\)-intercept, the equation of the axis of symmetry, and the \(x\)-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

33. \(f(x) = 2x^2 - 6x - 9\)  
34. \(f(x) = -3x^2 - 9x + 2\)

35. \(f(x) = -4x^2 + 5x\)  
36. \(f(x) = 2x^2 + 11x\)

37. \(f(x) = 0.25x^2 + 3x + 4\)  
38. \(f(x) = -0.75x^2 + 4x + 6\)

39. \(f(x) = \frac{3}{2}x^2 + 4x - \frac{5}{2}\)  
40. \(f(x) = \frac{2}{3}x^2 - \frac{7}{3}x + 9\)

41. **FINANCIAL LITERACY**  
A babysitting club sits for 50 different families. They would like to increase their current rate of $9.50 per hour. After surveying the families, the club finds that the number of families will decrease by about 2 for each $0.50 increase in the hourly rate.

a. Write a quadratic function that models this situation.

b. State the domain and range of this function as it applies to the situation.

c. What hourly rate will maximize the club’s income? Is this reasonable?

d. What is the maximum income the club can expect to make?

42. **ACTIVITIES**  
Last year, 300 people attended the Franklin High School Drama Club’s winter play. The ticket price was $8. The advisor estimates that 20 fewer people would attend for each $1 increase in ticket price.

a. What ticket price would give the greatest income for the Drama Club?

b. If the Drama Club raised its tickets to this price, how much income should it expect to bring in?

**GRAPHING CALCULATOR**  
Use a calculator to find the maximum or minimum of each function. Round to the nearest hundredth if necessary.

43. \(f(x) = -9x^2 - 12x + 19\)  
44. \(f(x) = 12x^2 - 21x + 8\)

45. \(f(x) = -8.3x^2 + 14x - 6\)  
46. \(f(x) = 9.7x^2 - 13x - 9\)

47. \(f(x) = 28x - 15 - 18x^2\)  
48. \(f(x) = -16 - 14x - 12x^2\)

Determine whether each function has a **maximum** or **minimum** value, and find that value. Then state the domain and range of the function.

49) \(f(x) = -5x^2 + 4x - 8\)  
50. \(f(x) = -4x^2 - 3x + 2\)

51. \(f(x) = -9 + 3x + 6x^2\)  
52. \(f(x) = 2x - 5 - 4x^2\)

53. \(f(x) = \frac{2}{3}x^2 + 6x - 10\)  
54. \(f(x) = -\frac{3}{5}x^2 + 4x - 8\)

Determine the function represented by each graph.

55. [Graph of a quadratic function]
56. [Graph of a quadratic function]
57. [Graph of a quadratic function]
58. **MULTIPLE REPRESENTATIONS** Consider \( f(x) = x^2 - 4x + 8 \) and \( g(x) = 4x^2 - 4x + 8 \).
   a. **Tabular** Make a table of values for \( f(x) \) and \( g(x) \) if \(-4 \leq x \leq 4\).
   b. **Graphical** Graph \( f(x) \) and \( g(x) \).
   c. **Verbal** Explain the difference in the shapes of the graphs of \( f(x) \) and \( g(x) \). What value was changed to cause this difference?
   d. **Analytical** Predict the appearance of the graph of \( h(x) = 0.25x^2 - 4x + 8 \). Confirm your prediction by graphing all three functions if \(-10 \leq x \leq 10\).

59. **VENDING MACHINES** Omar owns a vending machine in a bowling alley. He currently sells 600 cans of soda per week at $0.65 per can. He estimates that he will lose 100 customers for every $0.05 increase in price and gain 100 customers for every $0.05 decrease in price. (*Hint:* The charge must be a multiple of 5.)
   a. Write and graph the related quadratic equation for a price increase.
   b. If Omar lowers the price, what price should he charge in order to maximize his income?
   c. What will be his income per week from the vending machine?

60. **BASEBALL** Lolita throws a baseball into the air and the height \( h \) of the ball in feet at a given time \( t \) in seconds after she releases the ball is given by the function \( h(t) = -16t^2 + 30t + 5 \).
   a. State the domain and range for this situation.
   b. Find the maximum height the ball will reach.

61. **ERROR ANALYSIS** Trent and Madison are asked to find the maximum of \( f(x) = -4x^2 + 8x - 6 \). Is either of them correct? Explain your reasoning.

   **Madison**
   \[
   x = \frac{-8}{2(-4)} = 1 \\
   f(1) = -4(1)^2 + 8(1) - 6 = 2 \\
   \text{The maximum is 2.}
   \]

   **Trent**
   \[
   x = \frac{-8}{2(-4)} = -1 \\
   f(-1) = -4(-1)^2 + 8(-1) - 6 = -18 \\
   \text{The maximum is -18.}
   \]

62. **REASONING** Determine whether the following is *sometimes*, *always*, or *never* true. Explain your reasoning.

   In a quadratic function, if two \( x \)-coordinates are equidistant from the axis of symmetry, then they will have the same \( y \)-coordinate.

63. **CHALLENGE** The table at the right represents some points on the graph of a quadratic function.
   a. Find the values of \( a \), \( b \), \( c \), and \( d \).
   b. What is the \( x \)-coordinate of the vertex?
   c. Does the function have a maximum or a minimum?

64. **OPEN ENDED** Give an example of a quadratic function with a
   a. maximum of 8. b. minimum of -4. c. vertex of \((-2, 6)\).

65. **WRITING IN MATH** Describe how you determine whether a function is quadratic and if it has a maximum or minimum value.
66. Which expression is equivalent to $\frac{8!}{5!}$?

A $\frac{8}{3}$  
B $8 \cdot 7 \cdot 6$  
C $3!$  
D $8 \cdot 7 \cdot 6 \cdot 5$

67. SAT/ACT  The price of coffee beans is $d$ dollars for 6 ounces, and each ounce makes $c$ cups of coffee. In terms of $c$ and $d$, what is the cost of the coffee beans required to make 1 cup of coffee?

F $\frac{cd}{6}$  
G $\frac{6c}{d}$  
H $\frac{6}{cd}$  
J $6cd$

68. SHORT RESPONSE  Each side of the square base of a pyramid is 20 feet, and the pyramid’s height is 90 feet. What is the volume of the pyramid?

69. Which ordered pair is the solution of the following system of equations?

$3x - 5y = 11$
$3x - 8y = 5$

A (2, 1)  
B (7, -2)  
C (7, 2)  
D $\left(\frac{1}{3}, -2\right)$

Spiral Review

Find the inverse of each matrix, if it exists. (Lesson 4-6)

70. $\begin{bmatrix} 3 & -4 \\ 2 & -1 \end{bmatrix}$  
71. $\begin{bmatrix} -4 & -1 \\ 0 & 6 \end{bmatrix}$  
72. $\begin{bmatrix} 2 & 8 \\ -3 & -5 \end{bmatrix}$

Evaluate each determinant. (Lesson 4-5)

73. $\begin{vmatrix} 6 & -3 \\ -1 & 8 \end{vmatrix}$  
74. $\begin{vmatrix} -3 & -5 \\ -1 & -9 \end{vmatrix}$  
75. $\begin{vmatrix} 8 & 6 \\ 4 & 3 \end{vmatrix}$

76. MANUFACTURING  The Community Service Committee is making canvas tote bags and leather tote bags for a fundraiser. They will line both types of bags with canvas and use leather handles on both. For the canvas bags, they need 4 yards of canvas and 1 yard of leather. For the leather bags, they need 3 yards of leather and 2 yards of canvas. The committee leader purchased 56 yards of leather and 104 yards of canvas. (Lesson 3-4)

a. Let $c$ represent the number of canvas bags, and let $\ell$ represent the number of leather bags. Write a system of inequalities for the number of bags that can be made.

b. Draw the graph showing the feasible region.

c. List the coordinates of the vertices of the feasible region.

d. If the club plans to sell the canvas bags at a profit of $20 each and the leather bags at a profit of $35 each, write a function for the total profit on the bags.

e. How can the club make the maximum profit?

f. What is the maximum profit?

State whether each function is a linear function. Write yes or no. Explain. (Lesson 2-2)

77. $y = 4x^2 - 3x$  
78. $y = -2x - 4$  
79. $y = 4$

Skills Review

Evaluate each function for the given value. (Lesson 2-1)

80. $f(x) = 3x^2 - 4x + 6, x = -2$  
81. $f(x) = -2x^2 + 6x - 5, x = 4$  
82. $f(x) = 6x^2 + 18, x = -5$
You can use a TI-83/84 Plus graphing calculator to model data points for which a curve of best fit is a quadratic function.

**WATER** A bottle is filled with water. The water is allowed to drain from a hole made near the bottom of the bottle. The table shows the level of the water $y$ measured in centimeters from the bottom of the bottle after $x$ seconds.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
<th>160</th>
<th>180</th>
<th>200</th>
<th>220</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water level (cm)</td>
<td>42.6</td>
<td>40.7</td>
<td>38.9</td>
<td>37.2</td>
<td>35.8</td>
<td>34.3</td>
<td>33.3</td>
<td>32.3</td>
<td>31.5</td>
<td>30.8</td>
<td>30.4</td>
<td>30.1</td>
</tr>
</tbody>
</table>

Find and graph a linear regression equation and a quadratic regression equation. Determine which equation is a better fit for the data.

**Activity**

**Step 1** Find and graph a linear regression equation.
- Enter the times in L1 and the water levels in L2. Then find a linear regression equation.

KEYSTROKES: Refer to Lesson 2-5.
- Use STAT PLOT to graph a scatter plot. Copy the equation to the Y= list and graph.

KEYSTROKES: Review statistical plots and graphing a regression equation in Lesson 2-5.

**Step 2** Find and graph a quadratic regression equation.
- Find the quadratic regression equation. Then copy the equation to the Y= list and graph.

KEYSTROKES: STAT 5 ENTER Y=
VARS 5 ENTER GRAPH

Notice that the graph of the linear regression equation appears to pass through just two data points. However, the graph of the quadratic regression equation fits the data very well.

**Exercises**

Refer to the table.

1. Find and graph a linear regression equation and a quadratic regression equation for the data. Determine which equation is a better fit for the data.

2. Estimate the height of the player’s feet after 1 second and 1.5 seconds. Use mental math to check the reasonableness of your estimates.

3. Compare and contrast the estimates you found in Exercise 2.

4. How might choosing a regression equation that does not fit the data well affect predictions made by using the equation?
Solving Quadratic Equations

### Then
- You solved systems of equations by graphing. (Lesson 3-1)

### Now
1. Solve quadratic equations by graphing.
2. Estimate solutions of quadratic equations by graphing.

### Why?
- Arielle works in the marketing department of a major retailer. Her job is to set prices for new products sold in the stores. Arielle determined that for a certain product, the function
  \[ f(p) = -6p^2 + 192p - 1440 \]
  tells the profit \( f(p) \) made at price \( p \).

  Arielle can determine the price range by finding the prices for which the profit is equal to $0. This can be done by finding the solution of the related quadratic equation
  \[ 0 = -6p^2 + 192p - 1440. \]

  The graph of the function indicates that the profit is zero at 12 and 20, so the profitable price range of the item is between $12 and $20.

### New Vocabulary
- **quadratic equation**
- **standard form**
- **root**
- **zero**

### Tennessee Curriculum Standards
- **CLE 3103.3.3** Analyze and apply various methods to solve equations, absolute values, inequalities, and systems of equations over complex numbers.
- **✓ 3103.3.7** Solve quadratic equations by factoring, graphing, completing the square, extracting square roots and using the quadratic formula.
- **SPI 3103.3.2** Solve quadratic equations and systems, and determine roots of a higher order polynomial.

### Solve Quadratic Equations
**Quadratic equations** are quadratic functions that are set equal to a value. The **standard form** of a quadratic equation is \( ax^2 + bx + c = 0 \), where \( a \neq 0 \) and \( a, b, \) and \( c \) are integers.

The solutions of a quadratic equation are called the **roots** of the equation. One method for finding the roots of a quadratic equation is to find the **zeros** of the related quadratic function.

The zeros of the function are the \( x \)-intercepts of its graph.

### Quadratic Function
\[ f(x) = x^2 - x - 6 \]

- \( f(-2) = (-2)^2 - (-2) - 6 = 0 \) or 0
- \( f(3) = 3^2 - 3 = 6 \) or 0

-2 and 3 are zeros of the function.

### Quadratic Equation
\[ x^2 - x - 6 = 0 \]

- \((-2)^2 - (-2) - 6 = 0 \)
- \(3^2 - 3 - 6 = 0 \)

-2 and 3 are roots of the equation.
**Example 1** Two Real Solutions

Solve \( x^2 - 3x - 4 = 0 \) by graphing.

Graph the related function, \( f(x) = x^2 - 3x - 4 \). The equation of the axis of symmetry is \( x = \frac{-3}{2(1)} \) or 1.5. Make a table using \( x \)-values around 1.5. Then graph each point.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>0</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>0</td>
<td>-4</td>
<td>-6.25</td>
<td>-6</td>
<td>-4</td>
<td>0</td>
</tr>
</tbody>
</table>

The zeros of the function are -1 and 4. Therefore, the solutions of the equation are -1 and 4 or \( \{ x | x = -1, 4 \} \).

**Guided Practice**

1. Solve each equation by graphing.
   \( \text{A. } x^2 + 2x - 15 = 0 \)
   \( \text{B. } x^2 - 8x = -12 \)

The graph of the related function in Example 1 has two zeros; therefore, the quadratic equation has two real solutions. This is one of the three possible outcomes when solving a quadratic equation.

**Key Concept** Solutions of a Quadratic Equation

**Words**
A quadratic equation can have one real solution, two real solutions, or no real solutions.

**Models**
- one real solution
- two real solutions
- no real solution

**Example 2** One Real Solution

Solve \( 14 - x^2 = -6x + 23 \) by graphing.

\begin{align*}
14 - x^2 &= -6x + 23 & \text{Original equation} \\
14 &= x^2 - 6x + 23 & \text{Add } x^2 \text{ to each side.} \\
0 &= x^2 - 6x + 9 & \text{Subtract 14.}
\end{align*}

Graph the related function \( f(x) = x^2 - 6x + 9 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

The function has only one zero, 3. Therefore, the solution is 3 or \( \{ x | x = 3 \} \).

**Guided Practice**

2. Solve each equation by graphing.
   \( \text{A. } x^2 + 5 = -8x - 11 \)
   \( \text{B. } 12 - x^2 = 48 - 12x \)
**Example 3** No Real Solution

**NUMBER THEORY** Use a quadratic equation to find two real numbers with a sum of 15 and a product of 63.

**Understand** Let $x$ represent one of the numbers. Then $15 - x$ is the other number.

**Plan**

\[
\begin{align*}
\text{Let } & x(15 - x) = 63 \\
\text{The product of the numbers is } & 63. \\
15x - x^2 & = 63 \\
\text{Distributive Property} \\
-x^2 + 15x & = 63 \\
\text{Subtract 63.}
\end{align*}
\]

**Solve**

Graph the related function.

The graph has no $x$-intercepts. This means the original equation has no real solution.
Thus, it is not possible for two real numbers to have a sum of 15 and a product of 63.

**Check** Try finding the product of several pairs of numbers with sums of 15. Is each product less than 63 as the graph suggests?

**Guided Practice**

3. Find two real numbers with a sum of 6 and a product of $-55$, or show that no such numbers exist.

---

**WatchOut!**

Zeros You will see in later chapters that many zeros can appear within small intervals.

**Estimate Solutions** Often exact roots cannot be found by graphing. You can estimate the solutions by stating the integers between which the roots are located.

When the value of the function is positive for one value and negative for a second value, then there is at least one zero between those two values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-3$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>12</td>
<td>3</td>
<td>$-6$</td>
<td>$-2$</td>
<td>$4$</td>
<td>$8$</td>
<td>$14$</td>
</tr>
</tbody>
</table>

**Example 4** Estimate Roots

Solve $x^2 - 6x + 4 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>4</td>
<td>$-1$</td>
<td>$-4$</td>
<td>$-5$</td>
<td>$-4$</td>
<td>$-1$</td>
<td>$4$</td>
</tr>
</tbody>
</table>

The $x$-intercepts of the graph indicate that one solution is between 0 and 1, and the other solution is between 5 and 6.

**Guided Practice**

4. Solve $x^2 - x - 10 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.
You can also use tables to solve quadratic equations. After entering the equation in your calculator, scroll through the table to locate the solutions.

**Example 5  Solve by Using a Table**

Solve \( x^2 - 6x + 2 = 0 \).

Enter \( y_1 = x^2 - 6x + 2 \) in your graphing calculator. Use the **TABLE** window to find where the sign of \( Y_1 \) changes. Change \( \Delta \text{Tbl} \) to 0.1 and look again for the sign change. Repeat the process with 0.01 and 0.001 to get a more accurate location of the zero.

![Table showing values of \( x \) and \( y_1 = x^2 - 6x + 2 \)]

One solution is approximately 0.354.

**Guided Practice**

5. Locate the second zero in the function above to the nearest thousandth.

Quadratic equations can be solved with a calculator as well. After entering the equation, use the **ZERO** operation in the **CALC** menu.

**Real-World Example 6  Solve by Using a Calculator**

**RELIEF** A package of supplies is tossed from a helicopter at an altitude of 200 feet. The package’s height above the ground is modeled by \( h(t) = -16t^2 + 28t + 200 \), where \( t \) is the time in seconds after it is tossed. How long will it take the package to reach the ground?

We need to find \( t \) when \( h(t) \) is 0. Solve \( 0 = -16t^2 + 28t + 200 \). Then graph the related function \( f(t) = -16t^2 + 28t + 200 \) on a graphing calculator.

- Use the **ZERO** feature in the **CALC** menu to find the positive zero of the function, since time cannot be negative.
- Use the arrow keys to select a left bound and press **ENTER**.
- Locate a right bound and press **ENTER** twice.
- The positive zero of the function is about 4.52. The package would take about 4.52 seconds to reach the ground.

**Guided Practice**

6. How long would it take to reach the ground if the height was modeled by \( h(t) = -16t^2 + 48t + 400 \)?
Check Your Understanding

Example 1
Use the related graph of each equation to determine its solutions.

1. \(x^2 + 2x + 3 = 0\)
2. \(x^2 - 3x - 10 = 0\)
3. \(-x^2 - 8x - 16 = 0\)

Examples 2-5
Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. \(x^2 + 8x = 0\)
5. \(x^2 - 3x - 18 = 0\)
6. \(4x - x^2 + 8 = 0\)
7. \(-12 - 5x + 3x^2 = 0\)
8. \(x^2 - 6x + 4 = -8\)
9. \(9 - x^2 = 12\)
10. \(5x^2 + 10x - 4 = -6\)
11. \(x^2 - 20 = 2 + x\)

12. NUMBER THEORY Use a quadratic equation to find two real numbers with a sum of 2 and a product of -24.

Example 6
13. PHYSICS How long will it take an object to fall from the roof of a building 400 feet above ground? Use the formula \(h(t) = -16t^2 + h_0\), where \(t\) is the time in seconds and the initial height \(h_0\) is in feet.

Practice and Problem Solving

Example 1
Use the related graph of each equation to determine its solutions.

14. \(x^2 + 4x = 0\)
15. \(-2x^2 - 4x - 5 = 0\)
16. \(0.5x^2 - 2x + 2 = 0\)

17. \(-0.25x^2 - x - 1 = 0\)
18. \(x^2 - 6x + 11 = 0\)
19. \(-0.5x^2 + 0.5x + 6 = 0\)
Examples 2–4 Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

20. \( x^2 = 5x \)  
21. \(-2x^2 - 4x = 0\)  
22. \( x^2 - 5x - 14 = 0 \)  
23. \(-x^2 + 2x + 24 = 0\)  
24. \( x^2 - 18x = -81 \)  
25. \( 2x^2 - 8x = -32 \)  
26. \( 2x^2 - 3x - 15 = 4 \)  
27. \(-3x^2 - 7 + 2x = -11\)  
28. \(-0.5x^2 + 3 = -5x - 2\)  
29. \(-2x + 12 = x^2 + 16\)

Example 5 Use the tables to determine the location of the zeros of each quadratic function.

30. \[\begin{array}{c|cccccccc}
 x & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 \\
 \hline
 f(x) & -8 & -1 & 4 & 4 & -1 & -8 & -22 & -48
\end{array}\]

31. \[\begin{array}{c|cccccccc}
 x & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
 \hline
 f(x) & 32 & 14 & 2 & -3 & -3 & 2 & 14 & 32
\end{array}\]

32. \[\begin{array}{c|cccccccc}
 x & -6 & -3 & 0 & 3 & 6 & 9 & 12 & 15 \\
 \hline
 f(x) & -6 & -1 & 3 & 5 & 3 & -1 & -6 & -14
\end{array}\]

Example 6 NUMBER THEORY Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.

33. Their sum is \(-15\), and their product is \(-54\).
34. Their sum is \(4\), and their product is \(-117\).
35. Their sum is \(12\), and their product is \(-84\).
36. Their sum is \(-13\), and their product is \(42\).
37. Their sum is \(-8\) and their product is \(-209\).

For Exercises 38–40, use the formula \(h(t) = v_0 t - 16t^2\), where \(h(t)\) is the height of an object in feet, \(v_0\) is the object’s initial velocity in feet per second, and \(t\) is the time in seconds.

38. BASEBALL A baseball is hit with an initial velocity of 80 feet per second. Ignoring the height of the baseball player, how long does it take for the ball to hit the ground?

39. CANNONS A cannonball is shot with an initial velocity of 55 feet per second. Ignoring the height of the cannon, how long does it take for the cannonball to hit the ground?

40. GOLF A golf ball is hit with an initial velocity of 100 feet per second. How long will it take for it to hit the ground?

Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

41. \(2x^2 + x = 15\)  
42. \(-5x - 12 = -2x^2\)  
43. \(4x^2 - 15 = -4x\)  
44. \(-35 = -3x - 2x^2\)  
45. \(-3x^2 + 11x + 9 = 1\)  
46. \(13 - 4x^2 = -3x\)  
47. \(-0.5x^2 + 18 = -6x + 33\)  
48. \(0.5x^2 + 0.75 = 0.25x\)
49) **WATER BALLOONS** Tony wants to drop a water balloon so that it splashes on his brother. Use the formula 
\[ h(t) = -16t^2 + h_0 \]
where \( t \) is the time in seconds and the initial height \( h_0 \) is in feet, to determine how far his brother should be from the target when Tony lets go of the balloon.

50. **WATER HOSES** A water hose can spray water at an initial velocity of 40 feet per second. Use the formula 
\[ h(t) = v_0t - 16t^2, \]
where \( h(t) \) is the height of the water in feet, \( v_0 \) is the initial velocity in feet per second, and \( t \) is the time in seconds.

   **a.** How long will it take the water to hit the nozzle on the way down?

   **b.** Assuming the nozzle is 5 feet up, what is the maximum height of the water?

51. **SKYDIVING** In 2003, John Fleming and Dan Rossi became the first two blind skydivers to be in free fall together. They jumped from an altitude of 14,000 feet and free fell to an altitude of 4000 feet before their parachutes opened. Ignoring air resistance and using the formula 
\[ h(t) = -16t^2 + h_0 \]
where \( t \) is the time in seconds and the initial height \( h_0 \) is in feet, determine how long they were in free fall.

---

**H.O.T. Problems** Use Higher-Order Thinking Skills

52. **ERROR ANALYSIS** Hakeem and Tanya were asked to find the location of the roots of the quadratic function represented by the table. Is either of them correct? Explain.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>52</td>
<td>26</td>
<td>8</td>
<td>-2</td>
<td>-4</td>
<td>2</td>
<td>16</td>
<td>38</td>
</tr>
</tbody>
</table>

**Hakeem**
The roots are between 4 and 6 because \( f(x) \) stops decreasing and begins to increase between \( x = 4 \) and \( x = 6 \).

**Tanya**
The roots are between -2 and 0 because \( x \) changes signs at that location.

53. **CHALLENGE** Find the value of a positive integer \( k \) such that \( f(x) = x^2 - 2kx + 55 \) has roots at \( k + 3 \) and \( k - 3 \).

54. **REASONING** If a quadratic function has a minimum at \((-6, -14)\) and a root at \( x = -17 \), what is the other root? Explain your reasoning.

55. **OPEN ENDED** Write a quadratic function with a maximum at \((3, 125)\) and roots at -2 and 8.

56. **WRITING IN MATH** Explain how to solve a quadratic equation by graphing its related quadratic function.
57. **SHORT RESPONSE** A bag contains five different colored marbles. The colors of the marbles are black, silver, red, green, and blue. A student randomly chooses a marble. Then, without replacing it, he chooses a second marble. What is the probability that the student chooses the red and then the green marble?

58. Which number would be closest to zero on the number line?

- **A** -0.6
- **B** $\frac{2}{3}
- **C** $\sqrt{2}$
- **D** 0.5

59. **SAT/ACT** A salesman’s monthly gross pay consists of $3500 plus 20 percent of the dollar amount of his sales. If his gross pay for one month was $15,500, what was the dollar amount of his sales for that month?

- **F** $12,000
- **G** $16,000
- **H** $60,000

60. Find the next term in the sequence below.

\[ \frac{2x}{5}, \frac{3x}{5}, \frac{4x}{5}, \ldots \]

- **A** $x$
- **B** $5x$
- **C** $\frac{x}{5}$
- **D** $\frac{5x}{4}$

### Spiral Review

Determine whether each function has a maximum or minimum value, and find that value. Then state the domain and range of the function. (Lesson 5-1)

61. $f(x) = -4x^2 + 8x - 16$
62. $f(x) = 3x^2 + 12x - 18$
63. $f(x) = 4x + 13 - 2x^2$

Determine whether each pair of matrices are inverses of each other. (Lesson 4-6)

64. \[
\begin{bmatrix}
4 & -3 \\
-1 & -6
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
\frac{3}{13} & \frac{1}{18} \\
\frac{1}{26} & \frac{2}{13}
\end{bmatrix}
\]
65. \[
\begin{bmatrix}
6 & -3 \\
4 & 8
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
\frac{1}{10} & \frac{1}{20} \\
\frac{1}{15} & \frac{2}{15}
\end{bmatrix}
\]
66. \[
\begin{bmatrix}
2 & 4 \\
-3 & -2
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
\frac{1}{4} & \frac{1}{2} \\
\frac{3}{8} & \frac{1}{4}
\end{bmatrix}
\]

67. **FOOTPRINTS** The combination of a reflection and a translation is called a glide reflection. An example is a set of footprints. (Lesson 4-4)

- **a.** Describe the reflection and transformation combination shown at the right.
- **b.** Write two matrix operations that can be used to find the coordinates of point C.
- **c.** Does it matter which operation you do first? Explain.
- **d.** What are the coordinates of the next two footprints?

Solve each system of equations. (Lesson 3-2)

68. \[4x - 7y = -9 \\
5x + 2y = -22\]
69. \[8y - 2x = 38 \\
5x - 3y = -27\]
70. \[3x + 8y = 24 \\
-16y - 6x = 48\]

Solve each inequality. (Lesson 1-5)

71. \[3x - 6 \leq -14\]
72. \[6 - 4x \leq 2\]
73. \[-6x + 3 \geq 3x - 16\]

### Skills Review

Find the GCF of each set of numbers.

74. 16, 48, 128
75. 15, 21, 49
76. 12, 28, 36
You can use a TI-83/84 Plus graphing calculator to solve quadratic equations.

**Activity**  
Solving Quadratic Equations

Solve \( x^2 - 8x + 15 = 0 \).

**Step 1**  
Rewrite the equation in the form \( y = ax^2 + bx + c \).  
\[ y = x^2 - 8x + 15 \]

**Step 2**  
Graph \( y = x^2 - 8x + 15 \) in a standard viewing window.

**KEYSTROKES:**  
\[ Y= X,T,\theta,n \ x^2 \ (-) 8 \ x,T,\theta,n \ + \ 15 \ \text{ZOOM} 6 \]

**Step 3**  
Find an \( x \)-intercept.

**KEYSTROKES:**  
\[ \text{2nd} \ \text{[CALC]} \ 2 \]

Use \( \downarrow \) or \( \uparrow \) to position the cursor to the left of the first \( x \)-intercept. Press \( \text{ENTER} \). Then use \( \uparrow \) to position the cursor to the right of the first \( x \)-intercept. Press \( \text{ENTER} \ \text{ENTER} \) to display the \( x \)-intercept.

**Step 4**  
Find the second \( x \)-intercept.

**KEYSTROKES:**  
Use \( \uparrow \) to position the cursor to the left of the second \( x \)-intercept. Press \( \text{ENTER} \). Then use \( \downarrow \) to position the cursor to the right of the second \( x \)-intercept. Press \( \text{ENTER} \ \text{ENTER} \) to display the \( x \)-intercept.

The \( x \)-intercepts are 3 and 5, so \( x = 3 \) and \( x = 5 \).

**Exercises**

Solve each equation. Round to the nearest tenth if necessary.

1. \( x^2 - 7x + 12 = 0 \)
2. \( x^2 + 5x + 6 = 0 \)
3. \( x^2 - 3 = 2x \)
4. \( x^2 + 5x + 6 = 12 \)
5. \( x^2 + 5x = 0 \)
6. \( x^2 - 4 = 0 \)
7. \( x^2 + 8x + 16 = 0 \)
8. \( x^2 - 10x = -25 \)
9. \( 9x^2 + 48x + 64 = 0 \)
10. \( 2x^2 + 3x - 1 = 0 \)
11. \( 5x^2 - 7x = -2 \)
12. \( 6x^2 + 2x + 1 = 0 \)
Lesson 5-3
Solving Quadratic Equations by Factoring

**Then**
- You found the greatest common factors of sets of numbers.

**Now**
1. Write quadratic equations in intercept form.
2. Solve quadratic equations by factoring.

**Why?**
- The factored form of a quadratic equation is $0 = a(x - p)(x - q)$. In the equation, $p$ and $q$ represent the $x$-intercepts of the graph of the equation.
- The $x$-intercepts of the graph at the right are 2 and 6. In this lesson, you will learn how to change a quadratic equation in factored form into standard form and vice versa.

**Factored Form**

New Vocabulary
- factored form
- FOIL method

**Tennessee Curriculum Standards**
CLE 3103.3.3 Analyze and apply various methods to solve equations, absolute values, inequalities, and systems of equations over complex numbers.

3103.3.9 Find an equation for a parabola when given its graph or when given its roots.

SPI 3103.3.2 Solve quadratic equations and systems, and determine roots of a higher order polynomial. Also addresses 3103.3.7.

**Key Concept** FOIL Method for Multiplying Binomials

Words
- To multiply two binomials, find the sum of the products of $F$ the First terms, $O$ the Outer terms, $I$ the Inner terms, and $L$ the Last terms.

<table>
<thead>
<tr>
<th>Examples</th>
<th>Product of First Terms</th>
<th>Product of Outer Terms</th>
<th>Product of Inner Terms</th>
<th>Product of Last Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(x - 6)(x - 2)$</td>
<td>$(x)(x)$</td>
<td>$(x)(-2)$</td>
<td>$(-6)(x)$</td>
<td>$(-6)(-2)$</td>
</tr>
<tr>
<td>$1\cdot L$</td>
<td>$= x^2 - 2x - 6x + 12$</td>
<td>or $x^2 - 8x + 12$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Example 1** Translate Sentences into Equations

Write a quadratic equation in standard form with $-\frac{1}{3}$ and 6 as its roots.

- $(x - p)(x - q) = 0$ Write the pattern.
- $[x - (-\frac{1}{3})](x - 6) = 0$ Replace $p$ with $-\frac{1}{3}$ and $q$ with 6.
- $(x + \frac{1}{3})(x - 6) = 0$ Simplify.
- $x^2 - \frac{17}{3}x - 2 = 0$ Multiply.
- $3x^2 - 17x - 6 = 0$ Multiply each side by 3 so that $b$ and $c$ are integers.

**Guided Practice**

1. Write a quadratic equation in standard form with $\frac{3}{4}$ and $-5$ as its roots.
2 **Solve Equations by Factoring** You have learned various techniques for factoring polynomials. A summary of them is listed below.

### Concept Summary: Factoring Techniques

<table>
<thead>
<tr>
<th>Factoring Technique</th>
<th>General Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greatest Common Factor (GCF)</td>
<td>$ab^2 - nab^2 = ab^2(a^2 - n)$</td>
</tr>
<tr>
<td>General Trinomials</td>
<td>$ax^2 + (ad + bc)x + bd = (ax + b)(cx + d)$</td>
</tr>
<tr>
<td>Difference of Two Squares</td>
<td>$a^2 - b^2 = (a + b)(a - b)$</td>
</tr>
<tr>
<td>Perfect Square Trinomials</td>
<td>$a^2 ± 2ab + b^2 = (a ± b)^2$</td>
</tr>
</tbody>
</table>

---

**Example 2  Factor GCF and by Grouping**

Factor each polynomial.

**a.** $16x^2 + 8x$

$$16x^2 + 8x = 8x(2x) + 8x(1)$$

Factor the GCF.

$$= 8x(2x + 1)$$

Distributive Property

**b.** $7x^2 + 6xy^2 + 14xy + 12y^3$

$$7x^2 + 6xy^2 + 14xy + 12y^3$$

Original expression

$$= (7x^2 + 14xy) + (6xy^2 + 12y^3)$$

Group terms with common factors.

$$= 7x(x + 2y) + 6y^2(x + 2y)$$

Factor the GCF from each group.

$$= (7x + 6y^2)(x + 2y)$$

Distributive Property

---

**Guided Practice**

2A. $20x^2y - 15xy^2$

2B. $4x^2y - 16xy - y^2$

2C. $ab + 3cd + 4a^2b + 12acd$

---

**Review Vocabulary**

**perfect square** a number with a positive square root that is a whole number

---

**Example 3  Perfect Squares and Differences of Squares**

Factor each polynomial.

**a.** $x^2 + 16x + 64$

First and last terms are perfect squares.

$$x^2 = (x)^2; 64 = (8)^2$$

Middle term equals $2ab$.

$$16x = 2(x)(8)$$

$x^2 + 16x + 64$ is a perfect square trinomial.

$$x^2 + 16x + 64 = (x + 8)^2$$

Factor using the pattern.

**b.** $36a^2 - 64y^4$

Factor the GCF.

$$36a^2 - 64y^4 = 4(9a^2 - 16y^4)$$

Write in form $a^2 - b^2$.

$$= 4[(3a)^2 - (4y^2)^2]$$

$$= 4(3a + 4y^2)(3a - 4y^2)$$

Factor the difference of squares.

---

**Guided Practice**

3A. $4x^2 - 12x + 9$

3B. $81x^2 - y^6$

3C. $75x^2y - 27y$
A special pattern is used when factoring trinomials of the form $ax^2 + bx + c$. First, multiply the values of $a$ and $c$. Then, find two values, $m$ and $p$, such that their product equals $ac$ and their sum equals $b$.

Consider $6x^2 + 13x - 5$: $ac = 6(-5) = -30$.

<table>
<thead>
<tr>
<th>Factors of $-30$</th>
<th>Sum</th>
<th>Factors of $-30$</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, $-30$</td>
<td>$-29$</td>
<td>$-1$, $30$</td>
<td>29</td>
</tr>
<tr>
<td>2, $-15$</td>
<td>$-13$</td>
<td>$-2$, $15$</td>
<td>13</td>
</tr>
<tr>
<td>3, $-10$</td>
<td>$-7$</td>
<td>$-3$, $10$</td>
<td>7</td>
</tr>
<tr>
<td>5, $-6$</td>
<td>$-1$</td>
<td>$-5$, $6$</td>
<td>1</td>
</tr>
</tbody>
</table>

Now the middle term, $13x$, can be rewritten as $-2x + 15x$.

This polynomial can now be factored by grouping.

$$6x^2 + 13x - 5 = 6x^2 + mx + px - 5$$

Write the pattern.

$m = -2$ and $p = 15$

Group terms.

$$= (6x^2 - 2x) + (15x - 5)$$

Factor the GCF.

$$= 2x(3x - 1) + 5(3x - 1)$$

Distributive Property

$$= (2x + 5)(3x - 1)$$

**Example 4 Factor Trinomials**

Factor each polynomial.

a. $x^2 + 9x + 20$

First, find two values, $m$ and $p$, such that their product equals $ac$ and their sum equals $b$.

$a = 1, c = 20$

$ac = 20$

<table>
<thead>
<tr>
<th>Factors of 20</th>
<th>Sum</th>
<th>Factors of 20</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 20</td>
<td>21</td>
<td>$-1$, $-20$</td>
<td>$-21$</td>
</tr>
<tr>
<td>2, 10</td>
<td>12</td>
<td>$-2$, $-10$</td>
<td>$-12$</td>
</tr>
<tr>
<td>4, 5</td>
<td>9</td>
<td>$-4$, $-5$</td>
<td>$-9$</td>
</tr>
</tbody>
</table>

$x^2 + 9x + 20$

Write the pattern.

$m = 4, p = 5$

Group terms.

$x(x + 4) + 5(x + 4)$

Factor the GCF from each grouping.

$(x + 5)(x + 4)$

Distributive Property

b. $6y^2 - 23y + 20$

$a = 6, c = 20$

$ac = 120$

$m = -8, p = -15$

$6y^2 - 23y + 20$

Write the pattern.

$m = -8, p = -15$

Group terms.

$2y(3y - 4) - 5(3y - 4)$

Factor the GCF from each grouping.

$(2y - 5)(3y - 4)$

Distributive Property

**Guided Practice**

4A. $x^2 - 11x + 30$

4B. $x^2 - 4x - 21$

4C. $15x^2 - 8x + 1$

4D. $-12x^2 + 8x + 15$
Solving quadratic equations by factoring is an application of the Zero Product Property.

**Key Concept** Zero Product Property

**Words** For any real numbers \(a\) and \(b\), if \(ab = 0\), then either \(a = 0\), \(b = 0\), or both \(a\) and \(b\) equal zero.

**Example** If \((x + 3)(x - 5) = 0\), then \(x + 3 = 0\) or \(x - 5 = 0\).

**Real-World Example 5** Solve Equations by Factoring

**TRACK AND FIELD** The height of a javelin in feet is modeled by \(h(t) = -16t^2 + 79t + 5\), where \(t\) is the time in seconds after the javelin is thrown. How long is it in the air?

To determine how long the javelin is in the air, we need to find when the height equals 0. We can do this by solving \(-16t^2 + 79t + 5 = 0\).

\[
-16t^2 + 79t + 5 = 0 \quad \text{Original equation}
\]

\[
m = 80; p = -1 \quad -16(5) = -80, 80 \cdot (-1) = -80, 80 + (-1) = 79 \quad \text{Write the pattern.}
\]

\[
-16t^2 + 80t - t + 5 = 0 \quad \text{Group terms with common factors.}
\]

\[
(-16t^2 + 80t) + (-t + 5) = 0 \quad \text{Factor GCF from each group.}
\]

\[
16t(-t + 5) + 1(-t + 5) = 0 \quad \text{Distributive Property}
\]

\[
16t + 1 = 0 \quad \text{or} \quad -t + 5 = 0 \quad \text{Zero Product Property}
\]

\[
16t = -1 \quad -t = -5 \quad \text{Solve both equations.}
\]

\[
t = \frac{1}{16} \quad t = 5 \quad \text{Solve.}
\]

**CHECK** We have two solutions.

- The first solution is negative and since time cannot be negative, this solution can be eliminated.
- The second solution of 5 seconds seems reasonable for the time a javelin spends in the air.
- The answer can be confirmed by substituting back into the original equation.

\[
-16t^2 + 79t + 5 = 0 \\
-16(5)^2 + 79(5) + 5 = 0 \\
-400 + 395 + 5 = 0 \\
0 = 0 \quad \checkmark
\]

The javelin is in the air for 5 seconds.

**Guided Practice**

5. **BUNGEE JUMPING** Juan recorded his brother bungee jumping from a height of 1100 feet. At the time the cord lifted his brother back up, he was 76 feet above the ground. If Juan started recording as soon as his brother fell, how much time elapsed when the cord snapped back? Use \(f(t) = -16t^2 + c\), where \(c\) is the height in feet.
Check Your Understanding

Example 1  Write a quadratic equation in standard form with the given root(s).
1. \(-8, 5\)  
2. \(\frac{3}{2}, \frac{1}{4}\)  
3. \(-\frac{2}{3}, \frac{5}{2}\)

Examples 2–4  Factor each polynomial.
4. \(35x^2 - 15x\)  
5. \(18x^2 - 3x + 24x - 4\)  
6. \(x^2 - 12x + 32\)  
7. \(x^2 - 4x - 21\)  
8. \(2x^2 + 7x - 30\)  
9. \(16x^2 - 16x + 3\)  
10. \(x^2 - 36\)  
11. \(12x^2y - 18xy\)  
12. \(12x^2 - 2x - 2\)

Example 5  Solve each equation by factoring.
13. \(x^2 - 9x = 0\)  
14. \(x^2 - 3x - 28 = 0\)  
15. \(2x^2 - 24x = -72\)  
16. **GARDENING** Tamika wants to double the area of her garden by increasing the length and width by the same amount. What will be the dimensions of her garden then?

Practice and Problem Solving

Example 1  Write a quadratic equation in standard form with the given root(s).
17. \(7\)  
18. \(-5, \frac{1}{2}\)  
19. \(\frac{1}{5}, 6\)

Examples 2–4  Factor each polynomial.
20. \(40a^2 - 32a\)  
21. \(51c^3 - 34c\)  
22. \(32xy + 40bx - 12ay - 15ab\)  
23. \(3x^2 - 12\)  
24. \(15y^2 - 240\)  
25. \(48cg + 36cf - 4dg - 3df\)  
26. \(x^2 + 13x + 40\)  
27. \(x^2 - 9x - 22\)  
28. \(3x^2 + 12x - 36\)  
29. \(15x^2 + 7x - 2\)  
30. \(4x^2 + 29x + 30\)  
31. \(18x^2 + 15x - 12\)  
32. \(8x^2 - 4x^2 - 12x^2\)  
33. \(9x^2 - 25\)  
34. \(18x^2y^2 - 24xy^2 + 36y^2\)  
35. \(15x^2 - 84x - 36\)  
36. \(12x^2 + 13x - 14\)  
37. \(12xy^2 - 108x\)

Example 3  Solve each equation by factoring.
38. \(x^2 + 4x - 45 = 0\)  
39. \(x^2 - 5x - 24 = 0\)  
40. \(x^2 = 121\)  
41. \(x^2 + 13 = 17\)  
42. \(-3x^2 - 10x + 8 = 0\)  
43. \(-8x^2 + 46x - 30 = 0\)

44. **GEOMETRY**  The hypotenuse of a right triangle is 1 centimeter longer than one side and 4 centimeters longer than three times the other side. Find the dimensions of the triangle.

45. **NUMBER THEORY**  Find two consecutive even integers with a product of 624.

**GEOMETRY**  Find \(x\) and the dimensions of each rectangle.

46. \(A = 96\ ft^2\)  
47. \(A = 432\ in^2\)  
48. \(A = 448\ ft^2\)
Solve each equation by factoring.

49. $12x^2 - 4x = 5$
50. $5x^2 = 15x$
51. $16x^2 + 36 = -48x$
52. $75x^2 - 60x = -12$
53. $4x^2 - 144 = 0$
54. $-7x + 6 = 20x^2$

55. MOVIE THEATER A company plans to build a large multiplex theater. The financial analyst told her manager that the profit function for their theater was $P(x) = -x^2 + 48x - 512$, where $x$ is the number of movie screens, and $P(x)$ is the profit earned in thousands of dollars. Determine the range of production of movie screens that will guarantee that the company will not lose money.

Write a quadratic equation in standard form with the given root(s).

56. $-\frac{4}{7}, \frac{3}{8}$
57. $3.4, 0.6$
58. $\frac{2}{11}, \frac{5}{9}$

Solve each equation by factoring.

59. $10x^2 + 25x = 15$
60. $27x^2 + 5 = 48x$
61. $x^2 + 0.25x = 1.25$
62. $48x^2 - 15 = -22x$
63. $3x^2 + 2x = 3.75$
64. $-32x^2 + 56x = 12$

65. DESIGN A square is cut out of the figure at the right. Write an expression for the area of the figure that remains, and then factor the expression.

66. FINANCIAL LITERACY After analyzing the market, a company that sells Web sites determined the profitability of their product was modeled by $P(x) = -16x^2 + 368x - 2035$, where $x$ is the price of each Web site and $P(x)$ is the company’s profit. Determine the price range of the Web sites that will be profitable for the company.

67. PAINTINGS Enola wants to add a border to her painting, distributed evenly, that has the same area as the painting itself. What are the dimensions of the painting with the border included?

68. MULTIPLE REPRESENTATIONS In this problem, you will consider $a(x - p)(x - q) = 0$.
   a. Graphical Graph the related function for $a = 1$, $p = 2$, and $q = -3$.
   b. Analytical What are the solutions of the equation?
   c. Graphical Graph the related functions for $a = 4$, $-3$, and $\frac{1}{2}$ on the same graph.
   d. Verbal What are the similarities and differences between the graphs?
   e. Verbal What conclusion can you make about the relationship between the factored form of a quadratic equation and its solutions?

69. GEOMETRY The area of the triangle is 26 square centimeters. Find the length of the base.
70. **SOCCER** When a ball is kicked in the air, its height in meters above the ground can be modeled by \( h(t) = -4.9t^2 + 14.7t \) and the distance it travels can be modeled by \( d(t) = 16t \), where \( t \) is the time in seconds.

a. How long is the ball in the air?

b. How far does it travel before it hits the ground? *(Hint: Ignore air resistance.)*

c. What is the maximum height of the ball?

Factor each polynomial.

71. \( 18a - 24ay + 48b - 64by \)

72. \( 3x^2 + 2xy + 10y + 15x \)

73. \( 6a^2b^2 - 12ab^2 - 18b^3 \)

74. \( 12a^2 - 18ab + 30ab^3 \)

75. \( 32ax + 12bx - 48ay - 18by \)

76. \( 30ac + 80bd + 40ad + 60bc \)

77. \( 5ax^2 - 2by^2 - 5ay^2 + 2bx^2 \)

78. \( 12c^2x + 4d^2y - 3d^2x - 16c^2y \)

**H.O.T. Problems** Use Higher-Order Thinking Skills

79. **ERROR ANALYSIS** Gwen and Morgan are solving \(-12x^2 + 5x + 2 = 0\). Is either of them correct? Explain your reasoning.

Gwen

\[-12x^2 + 5x + 2 = 0\]

\[-12x^2 + 8x - 3x + 2 = 0\]

\[4x(-3x + 2) - (3x + 2) = 0\]

\[(4x - 1)(3x + 2) = 0\]

\[x = \frac{1}{4} \text{ or } \frac{2}{3}\]

Morgan

\[-12x^2 + 5x + 2 = 0\]

\[-12x^2 + 8x - 3x + 2 = 0\]

\[4x(-3x + 2) + (-3x + 2) = 0\]

\[(4x + 1)(-3x + 2) = 0\]

\[x = -\frac{1}{4} \text{ or } \frac{2}{3}\]

80. **CHALLENGE** Solve \(3x^6 - 39x^4 + 108x^2 = 0\) by factoring.

81. **CHALLENGE** The rule for factoring a difference of cubes is shown below. Use this rule to factor \(40x^5 - 135x^2y^3\).

\[a^3 - b^3 = (a - b)(a^2 + ab + b^2)\]

82. **OPEN ENDED** Choose two integers. Then write an equation in standard form with those roots. How would the equation change if the signs of the two roots were switched?

83. **CHALLENGE** For a quadratic equation of the form \((x - p)(x - q) = 0\), show that the axis of symmetry of the related quadratic function is located halfway between the \(x\)-intercepts \(p\) and \(q\).

84. **WRITE A QUESTION** A classmate is using the guess-and-check strategy to factor trinomials of the form \(x^2 + bx + c\). Write a question to help him think of a way to use that strategy for \(ax^2 + bx + c\).

85. **REASONING** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

*In a quadratic equation in standard form where \(a\), \(b\), and \(c\) are integers, if \(b\) is odd, then the quadratic cannot be a perfect square trinomial.*

86. **WRITING IN MATH** Explain how to factor a trinomial in standard form with \(a > 1\).
87. SHORT RESPONSE If 
ABCD is transformed by \((x, y) \rightarrow (3x, 4y)\), determine the area of \(A'B'C'D'\).

88. For \(y = 2|6 - 3x| + 4\), which set describes \(x\) when \(y < 6\)?

\[ \begin{align*}
\text{A} & \quad \left\{ x \mid \frac{5}{3} < x < \frac{7}{3} \right\} \\
\text{B} & \quad \left\{ x \mid x < \frac{5}{3} \text{ or } x > \frac{7}{3} \right\} \\
\text{C} & \quad \left\{ x \mid x < \frac{5}{3} \right\} \\
\text{D} & \quad \left\{ x \mid x > \frac{7}{3} \right\}
\end{align*} \]

89. PROBABILITY A 5-character password can contain the numbers 0 through 9 and 26 letters of the alphabet. None of the characters can be repeated. What is the probability that the password begins with a consonant?

\[ \begin{align*}
\text{A} & \quad \frac{26}{56} \\
\text{B} & \quad \frac{21}{36} \\
\text{C} & \quad \frac{21}{56} \\
\text{D} & \quad \frac{5}{36} \\
\text{E} & \quad \frac{2}{56}
\end{align*} \]

90. SAT/ACT If \(c = \frac{8a^3}{b}\), what happens to the value of \(c\) when both \(a\) and \(b\) are doubled?

\[ \begin{align*}
\text{A} & \quad c \text{ is unchanged.} \\
\text{B} & \quad c \text{ is halved.} \\
\text{C} & \quad c \text{ is doubled.} \\
\text{D} & \quad c \text{ is multiplied by 4.} \\
\text{E} & \quad c \text{ is multiplied by 8.}
\end{align*} \]

Spiral Review

Use the related graph of each equation to determine its solutions. (Lesson 5-2)

91. \(x^2 - 2x - 8 = 0\)

92. \(x^2 + 4x = 12\)

93. \(x^2 + 4x + 4 = 0\)

Graph each function. (Lesson 5-1)

94. \(f(x) = x^2 - 6x + 2\)

95. \(f(x) = -2x^2 + 4x + 1\)

96. \(f(x) = (x - 3)(x + 4)\)

97. FUNDRAISING Lawrence High School sold wrapping paper and boxed cards for their fundraising event. The school gets $1.00 for each roll of wrapping paper sold and $0.50 for each box of cards sold. (Lesson 4-3)

a. Write a matrix that represents the amounts sold for each class and a matrix that represents the amount of money the school earns for each item sold.

b. Write a matrix that shows how much each class earned.

c. Which class earned the most money?

d. What is the total amount of money the school made from the fundraiser?

<table>
<thead>
<tr>
<th>Class</th>
<th>Wrapping Paper</th>
<th>Cards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freshmen</td>
<td>72</td>
<td>49</td>
</tr>
<tr>
<td>Sophomores</td>
<td>68</td>
<td>63</td>
</tr>
<tr>
<td>Juniors</td>
<td>90</td>
<td>56</td>
</tr>
<tr>
<td>Seniors</td>
<td>86</td>
<td>62</td>
</tr>
</tbody>
</table>

Skills Review

Simplify. (Prerequisite Skills 2)

98. \(\sqrt{5} \cdot \sqrt{15}\)

99. \(\sqrt{8} \cdot \sqrt{32}\)

100. \(2\sqrt{3} \cdot \sqrt{27}\)
1 Pure Imaginary Numbers In your math studies so far, you have worked with real numbers. Equations like the one above led mathematicians to define imaginary numbers. The imaginary unit \( i \) is defined to be \( i^2 = -1 \). The number \( i \) is the principal square root of \(-1\); that is, \( i = \sqrt{-1} \).

Numbers of the form \( 6i, -2i, \) and \( i\sqrt{3} \) are called pure imaginary numbers. Pure imaginary numbers are square roots of negative real numbers. For any positive real number \( b \), \( \sqrt{-b^2} = \sqrt{b^2} \cdot \sqrt{-1} \) or \( bi \).

Example 1 Square Roots of Negative Numbers

Simplify.

a. \( \sqrt{-27} \)

\[
\sqrt{-27} = \sqrt{-1 \cdot 3^2 \cdot 3} = \sqrt{-1} \cdot 3 \cdot \sqrt{3} = i \cdot 3 \cdot \sqrt{3} \text{ or } 3i\sqrt{3}
\]

b. \( \sqrt{-216} \)

\[
\sqrt{-216} = \sqrt{-1 \cdot 6^2 \cdot 6} = \sqrt{-1} \cdot \sqrt{6^2} \cdot \sqrt{6} = i \cdot 6 \cdot \sqrt{6} \text{ or } 6i\sqrt{6}
\]

Guided Practice

1A. \( \sqrt{-18} \)  
1B. \( \sqrt{-125} \)

The Commutative and Associative Properties of Multiplication hold true for pure imaginary numbers. The first few powers of \( i \) are shown below.

<table>
<thead>
<tr>
<th>( i )</th>
<th>( i^2 )</th>
<th>( i^3 )</th>
<th>( i^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i = i )</td>
<td>( i = -1 )</td>
<td>( i = i^2 \cdot i ) or ( i = -i )</td>
<td>( i = (i^2)^2 \cdot 1 ) or ( i = 1 )</td>
</tr>
<tr>
<td>( i^5 = i^4 \cdot i )</td>
<td>( i^6 = i^4 \cdot i^2 ) or ( i^6 = -1 )</td>
<td>( i^7 = i^4 \cdot i^3 ) or ( i^7 = -i )</td>
<td>( i^8 = (i^2)^4 ) or ( i^8 = 1 )</td>
</tr>
</tbody>
</table>
Example 2  Products of Pure Imaginary Numbers

Simplify.

a. \(-5i \cdot 3i\)

\[ -5i \cdot 3i = -15i^2 \]

Multiply.

\[ = -15(-1) \]

\[ i^2 = -1 \]

\[ = 15 \]

b. \(\sqrt{-6} \cdot \sqrt{-15}\)

\[ \sqrt{-6} \cdot \sqrt{-15} = i\sqrt{6} \cdot i\sqrt{15} \]

\[ = i^2\sqrt{90} \]

Multiply.

\[ = -1 \cdot \sqrt{9} \cdot \sqrt{10} \]

Simplify.

\[ = -3\sqrt{10} \]

Guided Practice

2A. \(3i \cdot 4i\)  
2B. \(\sqrt{-20} \cdot \sqrt{-12}\)  
2C. \(i^{31}\)

You can solve some quadratic equations by using the Square Root Property.

Example 3  Equation with Pure Imaginary Solutions

Solve \(4x^2 + 256 = 0\).

\[ 4x^2 + 256 = 0 \]

Original equation

\[ 4x^2 = -256 \]

Subtract 256 from each side.

\[ x^2 = -64 \]

Divide each side by 4.

\[ x = \pm\sqrt{-64} \]

Square Root Property

\[ x = \pm 8i \]

\[ \sqrt{-64} = \sqrt{64} \cdot \sqrt{-1} \text{ or } 8i \]

Guided Practice

Solve each equation.

3A. \(4x^2 + 100 = 0\)  
3B. \(x^2 + 4 = 0\)

Operations with Complex Numbers  Consider \(2 + 3i\). Since 2 is a real number and 3i is a pure imaginary number, the terms are not like terms and cannot be combined. This type of expression is called a complex number.

Key Concept  Complex Numbers

Words  A complex number is any number that can be written in the form \(a + bi\), where \(a\) and \(b\) are real numbers and \(i\) is the imaginary unit. \(a\) is called the real part, and \(b\) is called the imaginary part.

Examples  \(5 + 2i\)  
\(1 - 3i = 1 + (-3)i\)
The Venn diagram shows the set of complex numbers.
- If $b = 0$, the complex number is a real number.
- If $b \neq 0$, the complex number is imaginary.
- If $a = 0$, the complex number is a pure imaginary number.

Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal. That is, $a + bi = c + di$ if and only if $a = c$ and $b = d$.

**Example 4  Equate Complex Numbers**

Find the values of $x$ and $y$ that make $3x - 5 + (y - 3)i = 7 + 6i$ true.

Set the real parts equal to each other and the imaginary parts equal to each other.

\[
\begin{align*}
3x - 5 &= 7 & \text{Real parts} \\
3x &= 12 & \text{Add 5 to each side.} \\
x &= 4 & \text{Divide each side by 3.}
\end{align*}
\]

\[
\begin{align*}
y - 3 &= 6 & \text{Imaginary parts} \\
y &= 9 & \text{Add 3 to each side.}
\end{align*}
\]

**Guided Practice**

4. Find the values of $x$ and $y$ that make $5x + 1 + (3 + 2y)i = 2x - 2 + (y - 6)i$ true.

The Commutative, Associative, and Distributive Properties of Multiplication and Addition hold true for complex numbers. To add or subtract complex numbers, combine like terms. That is, combine the real parts, and combine the imaginary parts.

**Example 5  Add and Subtract Complex Numbers**

Simplify.

a. $(5 - 7i) + (2 + 4i)$

\[
(5 - 7i) + (2 + 4i) = (5 + 2) + (-7 + 4)i \quad \text{Commutative and Associative Properties}
\]

\[
= 7 - 3i \quad \text{Simplify.}
\]

b. $(4 - 8i) - (3 - 6i)$

\[
(4 - 8i) - (3 - 6i) = (4 - 3) + (-8 + 6)i \quad \text{Commutative and Associative Properties}
\]

\[
= 1 - 2i \quad \text{Simplify.}
\]

**Guided Practice**

5A. $(-2 + 5i) + (1 - 7i)$

5B. $(4 + 6i) - (-1 + 2i)$

Complex numbers are used with electricity. In these problems, $j$ usually represents the imaginary unit. In a circuit with alternating current, the voltage, current, and impedance, or hindrance to current, can be represented by complex numbers. To multiply these numbers, use the FOIL method.
**Real-World Example 6  Multiply Complex Numbers**

**ELECTRICITY** In an AC circuit, the voltage \( V \), current \( C \), and impedance \( I \) are related by the formula \( V = C \cdot I \). Find the voltage in a circuit with current \( 2 + 4j \) amps and impedance \( 9 - 3j \) ohms.

\[
V = C \cdot I \\
= (2 + 4j) \cdot (9 - 3j) \\
= 2(9) + 2(-3j) + 4j(9) + 4j(-3j) \\
= 18 - 6j + 36j - 12j^2 \\
= 18 + 30j - 12(-1) \\
= 30 + 30j
\]

The voltage is \( 30 + 30j \) volts.

**Guided Practice**

6. Find the voltage in a circuit with current \( 2 - 4j \) amps and impedance \( 3 - 2j \) ohms.

Two complex numbers of the form \( a + bi \) and \( a - bi \) are called **complex conjugates**. The product of complex conjugates is always a real number. You can use this fact to simplify the quotient of two complex numbers.

**Example 7  Divide Complex Numbers**

**Simplify.**

a. \[
\frac{2i}{3 + 6i} = \frac{2i}{3 + 6i} \cdot \frac{3 - 6i}{3 - 6i} \\
= \frac{6i - 12i^2}{9 - 36i^2} \\
= \frac{6i - 12(-1)}{9 - 36(-1)} \\
= \frac{6i + 12}{45} \\
= \frac{4}{15} + \frac{2}{15}i \\
\]

\( a + bi \) form

b. \[
\frac{4 + i}{5i} = \frac{4 + i}{5i} \cdot \frac{i}{i} \\
= \frac{4i + i^2}{5i^2} \\
= \frac{4i - 1}{-5} \\
= \frac{1}{5} - \frac{4}{5}i \\
\]

\( a + bi \) form

**Guided Practice**

7A. \[
\frac{-2i}{3 + 5i} \\
\]

7B. \[
\frac{2 + i}{1 - i} \\
\]
Check Your Understanding

Examples 1–2 Simplify.
1. \(\sqrt{-81}\)
2. \(\sqrt{-32}\)
3. \((4i)(-3i)\)
4. \(3\sqrt{-24} \cdot 2\sqrt{-18}\)
5. \(i^{40}\)

Example 3 Solve each equation.
7. \(4x^2 + 32 = 0\)
8. \(2x^2 + 24 = 0\)

Example 4 Find the values of \(a\) and \(b\) that make each equation true.
9. \(3a + (4b + 2)i = 9 - 6i\)
10. \(4b - 5 + (-a - 3)i = 7 - 8i\)

Examples 5 and 7 Simplify.
11. \((-1 + 5i) + (-2 - 3i)\)
12. \((7 + 4i) - (1 + 2i)\)
13. \((6 - 8i)(9 + 2i)\)
14. \((3 + 2i)(-2 + 4i)\)
15. \(\frac{3 - i}{4 + 2i}\)
16. \(\frac{2 + i}{5 + 6i}\)

Example 6 17. ELECTRICITY The current in one part of a series circuit is \(5 - 3j\) amps. The current in another part of the circuit is \(7 + 9j\) amps. Add these complex numbers to find the total current in the circuit.

Practice and Problem Solving

Examples 1–2 Simplify.
18. \(\sqrt{-121}\)
19. \(\sqrt{-169}\)
20. \(\sqrt{-100}\)
21. \(\sqrt{-81}\)
22. \((-3)(-7i)(2i)\)
23. \(4i(-6i)^2\)
24. \(i^{11}\)
25. \(i^{25}\)
26. \((10 - 7i) + (6 + 9i)\)
27. \((-3 + i) + (-4 - i)\)
28. \((12 + 5i) - (9 - 2i)\)
29. \((11 - 8i) - (2 - 8i)\)
30. \((1 + 2i)(1 - 2i)\)
31. \((3 + 5i)(5 - 3i)\)
32. \((4 - i)(6 - 6i)\)
33. \(\frac{2i}{1+i}\)
34. \(\frac{5}{2+4i}\)
35. \(\frac{5 + i}{3i}\)

Example 3 Solve each equation.
36. \(4x^2 + 4 = 0\)
37. \(3x^2 + 48 = 0\)
38. \(2x^2 + 50 = 0\)
39. \(2x^2 + 10 = 0\)
40. \(6x^2 + 108 = 0\)
41. \(8x^2 + 128 = 0\)

Example 4 Find the values of \(x\) and \(y\) that make each equation true.
42. \(9 + 12i = 3x + 4yi\)
43. \(x + 1 + 2yi = 3 - 6i\)
44. \(2x + 7 + (3 - y)i = -4 + 6i\)
45. \(5 + y + (3x - 7)i = 9 - 3i\)
46. \(a + 3b + (3a - b)i = 6 + 6i\)
47. \((2a - 4b)i + a + 5b = 15 + 58i\)
Simplify.

48. \(\sqrt{-10} \cdot \sqrt{-24}\)

49. \(4i \left(\frac{1}{2}i\right)^2 (-2i)^2\)

50. \(i^{41}\)

51. \((4 - 6i) + (4 + 6i)\)

52. \((8 - 5i) - (7 + i)\)

53. \((-6 - i)(3 - 3i)\)

54. \(\frac{(5 + i)^2}{3 - i}\)

55. \(\frac{6 - i}{2 - 3i}\)

56. \((-4 + 6i)(2 - i)(3 + 7i)\)

57. \((1 + i)(2 + 3i)(4 - 3i)\)

58. \(\frac{4 - i\sqrt{2}}{4 + i\sqrt{2}}\)

59. \(\frac{2 - i\sqrt{3}}{2 + i\sqrt{3}}\)

Example 6

60. **ELECTRICITY** The impedance in one part of a series circuit is 7 + 8j ohms, and the impedance in another part of the circuit is 13 - 4j ohms. Add these complex numbers to find the total impedance in the circuit.

**ELECTRICITY** Use the formula \(V = C \cdot I\).

61. The current in a circuit is 3 + 6j amps, and the impedance is 5 - j ohms. What is the voltage?

62. The voltage in a circuit is 20 - 12j volts, and the impedance is 6 - 4j ohms. What is the current?

63. Find the sum of \(ix^2 - (4 + 5i)x + 7\) and \(3x^2 + (2 + 6i)x - 8i\).

64. Simplify \([(2 + i)x^2 - ix + 5 + i] - [(-3 + 4i)x^2 + (5 - 5i)x - 6]\).

65. **MULTIPLE REPRESENTATIONS** In this problem, you will explore quadratic equations that have complex roots.

   a. **Algebraic** Write a quadratic equation in standard form with 3i and -3i as its roots.

   b. **Graphical** Graph the quadratic equation found in part a by graphing its related function.

   c. **Algebraic** Write a quadratic equation in standard form with 2 + i and 2 - i as its roots.

   d. **Graphical** Graph the quadratic equation found in part c by graphing its related function.

   e. **Analytical** How do you know when a quadratic equation will have only complex solutions?

**H.O.T. Problems** Use Higher-Order Thinking Skills

66. **ERROR ANALYSIS** Joe and Sue are simplifying \((2i)(3i)(4i)\). Is either of them correct? Explain your reasoning.

   **Joe**
   \[24i^3 = -24\]

   **Sue**
   \[24i^3 = -24i\]

67. **CHALLENGE** Simplify \((1 + 2i)^3\).

68. **REASONING** Determine whether the following statement is always, sometimes, or never true. Explain your reasoning.

   *Every complex number has both a real part and an imaginary part.*

69. **OPEN ENDED** Write two complex numbers with a product of 20.

70. **WRITING IN MATH** Explain how complex numbers are related to quadratic equations.
71. **EXTENDED RESPONSE** Refer to the figure to answer the following.

![Diagram]

- **a.** Name two congruent triangles with vertices in correct order.
- **b.** Explain why the triangles are congruent.
- **c.** What is the length of \( EC \)? Explain your procedure.

72. \((3 + 6)^2 = \)

- **A** \( 2 \times 3 + 2 \times 6 \)
- **B** \( 9^2 \)
- **C** \( 3^2 + 6^2 \)
- **D** \( 3^2 \times 6^2 \)

73. **SAT/ACT** A store charges $49 for a pair of pants. This price is 40% more than the amount it costs the store to buy the pants. After a sale, any employee is allowed to purchase any remaining pairs of pants at 30% off the store’s cost. How much would it cost an employee to purchase the pants after the sale?

- **F** $10.50
- **J** $24.50
- **G** $12.50
- **K** $35.00
- **H** $13.72

74. What are the values of \( x \) and \( y \) when \((5 + 4i) - (x + yi) = (-1 - 3i)\)?

- **A** \( x = 6, y = 7 \)
- **B** \( x = 4, y = i \)
- **C** \( x = 6, y = i \)
- **D** \( x = 4, y = 7 \)

75. \( 2x^2 + 7x = 15 \)

76. \( 4x^2 - 12 = 22x \)

77. \( 6x^2 = 5x + 4 \)

78. Their sum is \(-3\), and their product is \(-40\).

80. Their sum is \(-15\), and their product is \(56\).

82. **RECREATION** Refer to the table. (Lesson 4-2)

- **a.** Write a matrix that represents the cost of admission for residents and a matrix that represents the cost of admission for nonresidents.
- **b.** Write the matrix that represents the additional cost for nonresidents.
- **c.** Write a matrix that represents the difference in cost if a child or adult goes after 6:00 P.M. instead of before 6:00 P.M.

<table>
<thead>
<tr>
<th>Daily Admission Fees</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Residents</strong></td>
</tr>
<tr>
<td>Time of day</td>
</tr>
<tr>
<td>Before 6:00 p.m.</td>
</tr>
<tr>
<td>After 6:00 p.m.</td>
</tr>
<tr>
<td><strong>Nonresidents</strong></td>
</tr>
<tr>
<td>Time of day</td>
</tr>
<tr>
<td>Before 6:00 p.m.</td>
</tr>
<tr>
<td>After 6:00 p.m.</td>
</tr>
</tbody>
</table>

83. **PART-TIME JOBS** Terrell makes $10 an hour cutting grass and $12 an hour for raking leaves. He cannot work more than 15 hours per week. Graph two inequalities that Terrell can use to determine how many hours he needs to work at each job if he wants to earn at least $120 per week. (Lesson 3-3)

---

**Skills Review**

Determine whether each trinomial is a perfect square trinomial. Write yes or no. (Lesson 5-3)

- **84.** \( x^2 + 16x + 64 \) yes
- **85.** \( x^2 - 12x + 36 \) yes
- **86.** \( x^2 + 8x - 16 \) no
- **87.** \( x^2 - 14x - 49 \) no
- **88.** \( x^2 + x + 0.25 \) yes
- **89.** \( x^2 + 5x + 6.25 \) yes
OBJECTIVE Graph complex numbers in the complex plane and determine the absolute values of complex numbers.

A complex number $a + bi$ can be graphed in the complex plane by representing it with the point $(a, b)$. Similar to a coordinate plane, the complex plane is comprised of two axes. The real component is plotted on the real axis, which is horizontal. The imaginary component is plotted on the imaginary axis, which is vertical. The complex plane may also be referred to as the Argand (ar GON) plane.

Consider the complex number $a + 0i$. Notice that $b = 0$ and this complex number simplifies to the real number $a$. We can graph $a$ using just a real number line or using just the real axis in the complex plane. When $b \neq 0$, the complex number $a + bi$ has an imaginary component. The imaginary axis is now needed to represent the imaginary component.

Example 1 Graph in the Complex Plane

Graph $z = 3 + 4i$ in the complex plane.

**Step 1** Represent $z$ with the point $(a, b)$.
- The real component $a$ of $z$ is 3.
- The imaginary component $bi$ of $z$ is $4i$.
- $z$ can be represented by the point $(a, b)$ or $(3, 4)$.

**Step 2** Graph $z$ in the complex plane.
- Construct the complex plane and plot the point $(3, 4)$.
Recall that for a real number, the absolute value is its distance from zero on the number line. Similarly, the **absolute value of a complex number** is its distance from the origin in the complex plane. When \(a + bi\) is graphed in the complex plane, the absolute value of \(a + bi\) is the distance from \((a, b)\) to the origin. This can be found by using the Distance Formula.

\[
\sqrt{(a - 0)^2 + (b - 0)^2} \text{ or } \sqrt{a^2 + b^2}
\]

**Key Concept** Absolute Value of a Complex Number

The absolute value of the complex number \(z = a + bi\) is

\[
|z| = |a + bi| = \sqrt{a^2 + b^2}.
\]

**Example 2** Absolute Value of a Complex Number

Find the absolute value of \(z = -5 + 12i\).

**Step 1** Determine values for \(a\) and \(b\).

- The real component \(a\) of \(z\) is \(-5\).
- The imaginary component \(bi\) of \(z\) is \(12i\).

Thus, \(a = -5\) and \(b = 12\).

**Step 2** Find the absolute value of \(z\).

\[
|z| = \sqrt{a^2 + b^2} \quad \text{Absolute value of a complex number}
\]

\[
= \sqrt{(-5)^2 + 12^2} \quad a = -5 \text{ and } b = 12
\]

\[
= \sqrt{25 + 144} \quad \text{Simplify.}
\]

\[
= \sqrt{169} \quad = 13
\]

The absolute value of \(z = -5 + 12i\) is 13.

**Exercises**

Graph each number in the complex plane.

1. \(z = 3 + i\)  
2. \(z = -4 - 2i\)  
3. \(z = -1 + i\)  
4. \(z = -3 - 4i\)  
5. \(z = 2 - 2i\)  
6. \(z = 1 + 2i\)

Find the absolute value of each complex number.

7. \(z = -4 - 3i\)  
8. \(z = 8 + 15i\)  
9. \(z = -24 + 7i\)  
10. \(z = 7 - 2i\)  
11. \(z = -6 - i\)  
12. \(z = 10 + 5i\)
1. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex for \( f(x) = 2x^2 + 8x - 3 \). Then graph the function by making a table of values. (Lesson 5-1)

2. **MULTIPLE CHOICE** For which equation is the axis of symmetry \( x = 5 \)? (Lesson 5-1)
   - A \( f(x) = x^2 - 5x + 3 \)
   - B \( f(x) = x^2 - 10x + 7 \)
   - C \( f(x) = x^2 + 10x - 3 \)
   - D \( f(x) = x^2 + 5x + 2 \)

3. Determine whether \( f(x) = 5 - x^2 + 2x \) has a maximum or a minimum value. Then find this maximum or minimum value and state the domain and range of the function. (Lesson 5-1)

4. **PHYSICAL SCIENCE** From 4 feet above the ground, Maya throws a ball upward with a velocity of 18 feet per second. The height \( h(t) \) of the ball \( t \) seconds after Maya throws the ball is given by \( h(t) = -16t^2 + 18t + 4 \). Find the maximum height reached by the ball and the time that this height is reached. (Lesson 5-1)

5. Solve \( 3x^2 - 17x + 5 = 0 \) by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. (Lesson 5-2)

Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist. (Lesson 5-2)

6. Their sum is 15, and their product is 36.
7. Their sum is 7, and their product is 15.

8. **MULTIPLE CHOICE** Using the graph of the function \( f(x) = x^2 + 6x - 7 \), what are the solutions to the equation \( x^2 + 6x - 7 = 0? \) (Lesson 5-2)

9. **BASEBALL** A baseball is hit upward with a velocity of 40 feet per second. Ignoring the height of the baseball player, how long does it take for the ball to fall to the ground? Use the formula \( h(t) = v_0t - \frac{1}{2}gt^2 \) where \( h(t) \) is the height of an object in feet, \( v_0 \) is the object's initial velocity in feet per second, and \( t \) is the time in seconds. (Lesson 5-2)

Solve each equation by factoring. (Lesson 5-3)

10. \( x^2 - x - 12 = 0 \)
11. \( 3x^2 + 7x + 2 = 0 \)
12. \( x^2 - 2x - 15 = 0 \)
13. \( 2x^2 + 5x - 3 = 0 \)
14. Write a quadratic equation in standard form with roots \(-6\) and \(\frac{1}{4}\). (Lesson 5-3)

15. **TRIANGLES** Find the dimensions of a triangle if the base is \( \frac{2}{3} \) the measure of the height and the area is 12 square centimeters. (Lesson 5-3)

16. **PATIO** Eli is putting a cement slab in his backyard. The original slab was going to have dimensions of 8 feet by 6 feet. He decided to make the slab larger by adding \( x \) feet to each side. The area of the new slab is 120 square feet. (Lesson 5-3)

   a. Write a quadratic equation that represents the area of the new slab.
   b. Find the new dimensions of the slab.

Simplify. (Lesson 5-4)

17. \( \sqrt{-81} \)
18. \( \sqrt{-25x^4y^5} \)
19. \( (15 - 3i) - (4 - 12i) \)
20. \( i^{37} \)
21. \( (5 - 3i)(5 + 3i) \)
22. \( \frac{3 - i}{2 + 5i} \)
23. The impedance in one part of a series circuit is \( 3 + 4i \) ohms and the impedance in another part of the circuit is \( 6 - 7i \) ohms. Add these complex numbers to find the total impedance in the circuit. (Lesson 5-4)
Completing the Square

Lesson 5-5

Then

You factored perfect square trinomials.

(Lesson 5-3)

Now

1. Solve quadratic equations by using the Square Root Property.

2. Solve quadratic equations by completing the square.

Why?

When going through a school zone, drivers must slow to a speed of 20 miles per hour. Once they are out of the school zone, the drivers can increase their speed.

Suppose Arturo is leaving school to go home for lunch, and he lives 5000 feet from the school zone. If Arturo accelerates at a constant rate of 8 feet per second squared, the equation $t^2 + 2t + 8 = 16$ represents the time it takes him to reach home.

To solve this equation, you can use the Square Root Property.

New Vocabulary

completing the square

Example 1  Equation with Rational Roots

You have solved equations like $x^2 - 25 = 0$ by factoring. You have also used the Square Root Property to solve such an equation. This method can be useful with equations like the one above that describes the car’s speed.

**Square Root Property** You have solved equations like $x^2 - 25 = 0$ by factoring. You have also used the Square Root Property to solve such an equation. This method can be useful with equations like the one above that describes the car’s speed.

**Example 1  Equation with Rational Roots**

Solve $x^2 + 6x + 9 = 36$ by using the Square Root Property.

$x^2 + 6x + 9 = 36$  
Original equation

$(x + 3)^2 = 36$  
Factor the perfect square trinomial.

$x + 3 = \pm \sqrt{36}$  
Square Root Property

$x + 3 = \pm 6$  
$\sqrt{36} = 6$

$x = -3 \pm 6$  
Subtract 3 from each side.

$x = -3 + 6$  
$x = -3 - 6$

Write as two equations.

$x = 3$  
$x = -9$

Simplify.

The solution set is \( \{3, -9\} \) or \( \{x|x = -9, 3\} \).

**CHECK** Substitute both values into the original equation.

$x^2 + 6x + 9 = 36$  
Original equation

$x^2 + 6x + 9 = 36$

$3^2 + 6(3) + 9 = 36$  
Substitute 3 and $-9$.

$(-9)^2 + 6(-9) + 9 = 36$

$9 + 18 + 9 = 36$  
Simplify.

$81 - 54 + 9 = 36$

$36 = 36$  
Both solutions are correct.

$36 = 36$

**Guided Practice**

1. Solve each equation by using the Square Root Property.

A. $x^2 - 12x + 36 = 25$

B. $x^2 - 16x + 64 = 49$
Roots that are irrational numbers may be written as exact answers in radical form or as approximate answers in decimal form when a calculator is used.

**Example 2**  Equation with Irrational Roots

Solve \( x^2 - 10x + 25 = 27 \) by using the Square Root Property.

\[
x^2 - 10x + 25 = 27
\]
\[
(x - 5)^2 = 27
\]
\[
x - 5 = \pm \sqrt{27}
\]
\[
x = 5 \pm 3\sqrt{3}
\]
\[
x = 5 + 3\sqrt{3} \quad \text{or} \quad x = 5 - 3\sqrt{3}
\]
\[
\approx 10.2 \quad \text{or} \quad \approx -0.2
\]

The exact solutions of this equation are \( 5 + 3\sqrt{3} \) and \( 5 - 3\sqrt{3} \). The approximate solutions are \(-0.2\) and \(10.2\). Check these results by finding and graphing the related quadratic function.

\[
x^2 - 10x + 25 = 27
\]
\[
x^2 - 10x - 2 = 0
\]
\[
y = x^2 - 10x - 2
\]

**CHECK** Use the ZERO function of a graphing calculator. The approximate zeros of the related function are \(-0.2\) and \(10.2\).

![Graph of quadratic function]

**Guided Practice**

2. Solve each equation by using the Square Root Property.

   A. \( x^2 + 8x + 16 = 20 \)
   
   B. \( x^2 - 6x + 9 = 32 \)

**2 Complete the Square** All quadratic equations can be solved using the Square Root Property by manipulating the equation until one side is a perfect square. This method is called completing the square.

Consider \( x^2 + 16x = 9 \). Remember to perform each operation on each side of the equation.

\[
x^2 + 16x + \blacksquare = 9
\]
\[
x^2 + 16x + 64 = 9 + 64
\]
\[
x^2 + 16x + 64 = 73
\]
\[
(x + 8)^2 = 73
\]

Use this pattern of coefficients to complete the square of a quadratic expression.
Watch Out!
Each Side When solving equations by completing the square, don’t forget to add \( \left( \frac{b}{2} \right)^2 \) to each side of the equation.

Key Concept Completing the Square

Words
To complete the square for any quadratic expression of the form \( x^2 + bx \), follow the steps below.

**Step 1** Find one half of \( b \), the coefficient of \( x \).

**Step 2** Square the result in Step 1.

**Step 3** Add the result of Step 2 to \( x^2 + bx \).

Symbols
\[ x^2 + bx + \left( \frac{b}{2} \right)^2 = \left( x + \frac{b}{2} \right)^2 \]

Example 3 Complete the Square

Find the value of \( c \) that makes \( x^2 + 16x + c \) a perfect square. Then write the trinomial as a perfect square.

**Step 1** Find one half of 16.
\[ \frac{16}{2} = 8 \]

**Step 2** Square the result in Step 1.
\[ 8^2 = 64 \]

**Step 3** Add the result of Step 2 to \( x^2 + 16x \).
\[ x^2 + 16x + 64 \]

The trinomial \( x^2 + 16x + 64 \) can be written as \( (x + 8)^2 \).

Guided Practice

3. Find the value of \( c \) that makes \( x^2 - 14x + c \) a perfect square. Then write the trinomial as a perfect square.

You can solve any quadratic equation by completing the square. Because you are solving an equation, add the value you use to complete the square to each side.

Example 4 Solve an Equation by Completing the Square

Solve \( x^2 + 10x - 11 = 0 \) by completing the square.

\[
\begin{align*}
x^2 + 10x - 11 &= 0 & \text{Notice that} \ x^2 + 10x - 11 &= 0 \ \text{is not a perfect square.} \\
x^2 + 10x &= 11 & \text{Rewrite so the left side is of the form} \ x^2 + bx. \\
x^2 + 10x + 25 &= 11 + 25 & \text{Since} \ \left( \frac{10}{2} \right)^2 = 25, \ \text{add} \ 25 \ \text{to each side.} \\
(x + 5)^2 &= 36 & \text{Write the left side as a perfect square.} \\
x + 5 &= \pm 6 & \text{Square Root Property} \\
x &= -5 \pm 6 & \text{Subtract} \ 5 \ \text{from each side.} \\
x &= -5 + 6 \ \text{or} \ x &= -5 - 6 & \text{Write as two equations.} \\
&= 1 & \text{Simplify.} \\
&= -11 & \\
\end{align*}
\]

The solution set is \( \{-11, 1\} \) or \( \{x | x = -11, 1\} \). Check the result by using factoring.

Guided Practice

4. Solve each equation by completing the square.

A. \( x^2 - 10x + 24 = 0 \) 
B. \( x^2 + 10x + 9 = 0 \)
When the coefficient of the quadratic term is not 1, you must divide the equation by that coefficient before completing the square.

**Example 5  Equation with \( a \neq 1 \)**

Solve \( 2x^2 - 7x + 5 = 0 \) by completing the square.

\[
2x^2 - 7x + 5 = 0 \quad \text{Notice that } 2x^2 - 7x + 5 = 0 \text{ is not a perfect square.}
\]

\[
x^2 - \frac{7}{2}x + \frac{5}{2} = 0 \quad \text{Divide by the coefficient of the quadratic term, 2.}
\]

\[
x^2 - \frac{7}{2}x = -\frac{5}{2} \quad \text{Subtract } \frac{5}{2} \text{ from each side.}
\]

\[
x^2 - \frac{7}{2}x + \frac{49}{16} = -\frac{5}{2} + \frac{49}{16} \quad \text{Since } \left(-\frac{7}{2} \div 2\right)^2 = \frac{49}{16} \text{, add } \frac{49}{16} \text{ to each side.}
\]

\[
\left(x - \frac{7}{4}\right)^2 = \frac{9}{16} \quad \text{Write the left side as a perfect square by factoring.}
\]

\[
x - \frac{7}{4} = \pm \frac{3}{4} \quad \text{Simplify the right side.}
\]

\[
x = \frac{7}{4} \pm \frac{3}{4} \quad \text{Square Root Property}
\]

\[
x = \frac{7}{4} + \frac{3}{4} \quad \text{Add } \frac{3}{4} \text{ to each side.}
\]

\[
x = \frac{5}{2} \quad \text{Write as two equations.}
\]

\[
x = \frac{5}{2} \quad \text{or } x = \frac{5}{2} \quad \text{The solution set is } \left\{1, \frac{5}{2}\right\} \text{ or } \left\{x | x = 1, \frac{5}{2}\right\}.
\]

**Guided Practice**

5. Solve each equation by completing the square.

A. \( 3x^2 + 10x - 8 = 0 \)  
B. \( 3x^2 + 14x - 16 = 0 \)

Not all solutions of quadratic equations are real numbers. In some cases, the solutions are complex numbers of the form \( a + bi \), where \( b \neq 0 \).

**Example 6  Equation with Imaginary Solutions**

Solve \( x^2 + 8x + 22 = 0 \) by completing the square.

\[
x^2 + 8x + 22 = 0 \quad \text{Notice that } x^2 + 8x + 22 \text{ is not a perfect square.}
\]

\[
x^2 + 8x = -22 \quad \text{Rewrite so the left side is of the form } x^2 + bx.
\]

\[
x^2 + 8x + 16 = -22 + 16 \quad \text{Since } \left(\frac{8}{2}\right)^2 = 16, \text{ add 16 to each side.}
\]

\[
(x + 4)^2 = -6 \quad \text{Write the left side as a perfect square.}
\]

\[
x + 4 = \pm \sqrt{-6} \quad \text{Square Root Property}
\]

\[
x + 4 = \pm i\sqrt{6} \quad \text{Subtract 4 from each side.}
\]

\[
x = -4 \pm i\sqrt{6} \quad \text{The solution set is } \{-4 + i\sqrt{6}, -4 - i\sqrt{6}\} \text{ or } \{x | x = -4 + i\sqrt{6}, -4 - i\sqrt{6}\}.
\]

**Guided Practice**

6. Solve each equation by completing the square.

A. \( x^2 + 2x + 2 = 0 \)  
B. \( x^2 - 6x + 25 = 0 \)
Check Your Understanding

Examples 1–2 Solve each equation by using the Square Root Property. Round to the nearest hundredth if necessary.

1. \(x^2 + 12x + 36 = 6\)  
2. \(x^2 - 8x + 16 = 13\)  
3. \(x^2 + 18x + 81 = 15\)  
4. \(9x^2 + 30x + 25 = 11\)

5. LASER LIGHT SHOW The area \(A\) in square feet of a projected laser light show is given by \(A = 0.16d^2\), where \(d\) is the distance from the laser to the screen in feet. At what distance will the projected laser light show have an area of 100 square feet?

Example 3 Find the value of \(c\) that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

6. \(x^2 - 10x + c\)  
7. \(x^2 - 5x + c\)

Examples 4–6 Solve each equation by completing the square.

8. \(x^2 + 2x - 8 = 0\)  
9. \(x^2 - 4x + 9 = 0\)  
10. \(2x^2 - 3x - 3 = 0\)  
11. \(2x^2 + 6x - 12 = 0\)  
12. \(x^2 + 4x + 6 = 0\)  
13. \(x^2 + 8x + 10 = 0\)

Practice and Problem Solving

Examples 1–2 Solve each equation by using the Square Root Property. Round to the nearest hundredth if necessary.

14. \(x^2 + 4x + 4 = 10\)  
15. \(x^2 - 6x + 9 = 20\)  
16. \(x^2 + 8x + 16 = 18\)  
17. \(x^2 + 10x + 25 = 7\)  
18. \(x^2 + 12x + 36 = 5\)  
19. \(x^2 - 2x + 1 = 4\)  
20. \(x^2 - 5x + 6.25 = 0\)  
21. \(x^2 - 15x + 56.25 = 8\)  
22. \(x^2 + 32x + 256 = 1\)  
23. \(x^2 - 3x + \frac{9}{4} = 6\)  
24. \(x^2 + 7x + \frac{49}{4} = 4\)  
25. \(x^2 - 9x + \frac{81}{4} = \frac{1}{4}\)

Example 3 Find the value of \(c\) that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

26. \(x^2 + 8x + c\)  
27. \(x^2 + 16x + c\)  
28. \(x^2 - 11x + c\)  
29. \(x^2 + 9x + c\)

Examples 4–6 Solve each equation by completing the square.

30. \(x^2 - 4x + 12 = 0\)  
31. \(x^2 + 2x - 12 = 0\)  
32. \(x^2 + 6x + 8 = 0\)  
33. \(x^2 - 4x + 3 = 0\)  
34. \(2x^2 - x - 3 = 0\)  
35. \(2x^2 - 3x + 5 = 0\)  
36. \(2x^2 + 5x + 7 = 0\)  
37. \(3x^2 - 6x - 9 = 0\)  
38. \(x^2 - 2x + 3 = 0\)  
39. \(x^2 + 4x + 11 = 0\)  
40. \(x^2 - 6x + 18 = 0\)  
41. \(x^2 - 10x + 29 = 0\)  
42. \(3x^2 - 4x = 2\)  
43. \(2x^2 - 7x = -12\)  
44. \(x^2 - 2.4x = 2.2\)  
45. \(x^2 - 5.3x = -8.6\)  
46. \(x^2 - \frac{1}{5}x - \frac{11}{5} = 0\)  
47. \(x^2 - \frac{9}{2}x - \frac{24}{5} = 0\)

48. ARCHITECTURE An architect’s blueprints call for a dining room measuring 13 feet by 13 feet. The customer would like the dining room to be a square, but with an area of 250 square feet. How much will this add to the dimensions of the room?

Solve each equation. Round to the nearest hundredth if necessary.

49. \(4x^2 - 28x + 49 = 5\)  
50. \(9x^2 + 30x + 25 = 11\)  
51. \(x^2 + x + \frac{1}{3} = \frac{2}{3}\)  
52. \(x^2 + 1.2x + 0.56 = 0.91\)
53. **FIREWORKS** A firework’s distance \( d \) meters from the ground is given by \( d = -1.5t^2 + 25t \), where \( t \) is the number of seconds after the firework has been lit.

a. How many seconds have passed since the firework was lit when the firework explodes if it explodes at the maximum height of its path?

b. What is the height of the firework when it explodes?

Find the value of \( c \) that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

54. \( x^2 + 0.7x + c \)  
55. \( x^2 - 3.2x + c \)  
56. \( x^2 - 1.8x + c \)

57. **MULTIPLE REPRESENTATIONS** In this problem, you will use quadratic equations to investigate golden rectangles and the golden ratio.

a. **Geometric**
   - Draw square \( ABCD \).
   - Locate the midpoint of \( CD \). Label the midpoint \( P \). Draw \( PB \).
   - Construct an arc with a radius of \( PB \) from \( B \) clockwise past the bottom of the square.
   - Extend \( CD \) until it intersects the arc. Label this point \( Q \).
   - Construct rectangle \( ARQD \).

b. **Algebraic** Let \( AD = x \) and \( CQ = 1 \). Use completing the square to solve \( \frac{DQ}{AD} = \frac{QR}{CQ} \) for \( x \).

c. **Tabular** Make a table of \( x \) and value for \( CQ = 2, 3, \) and \( 4 \).

d. **Verbal** What do you notice about the \( x \)-values? Write an equation you could use to determine \( x \) for \( CQ = n \), where \( n \) is a nonzero real number.

---

**H.O.T. Problems**  
*Use Higher-Order Thinking Skills*

58. **ERROR ANALYSIS** Alonso and Aida are solving \( x^2 + 8x - 20 = 0 \) by completing the square. Is either of them correct? Explain your reasoning.

- **Alonso**
  \[
  x^2 + 8x - 20 = 0 \\
  x^2 + 8x = 20 \\
  x^2 + 8x + 16 = 20 + 16 \\
  (x + 4)^2 = 36 \\
  x + 4 = \pm 6 \\
  x = -4 \pm 6 
  \]

- **Aida**
  \[
  x^2 + 8x - 20 = 0 \\
  x^2 + 8x = 20 \\
  x^2 + 8x + 16 = 20 \\
  (x + 4)^2 = 20 \\
  x + 4 = \pm \sqrt{20} \\
  x = -4 \pm \sqrt{20} 
  \]

59. **CHALLENGE** Solve \( x^2 + bx + c = 0 \) by completing the square. Your answer will be an expression for \( x \) in terms of \( b \) and \( c \).

60. **REASONING** Without solving, determine how many unique solutions there are for each equation. Are they rational, real, or complex? Justify your reasoning.

   a. \( (x + 2)^2 = 16 \)  
   b. \( (x - 2)^2 = 16 \)  
   c. \( -(x - 2)^2 = 16 \)  
   d. \( 36 - (x - 2)^2 = 16 \)  
   e. \( 16(x+2)^2 = 0 \)  
   f. \( (x + 4)^2 = (x + 6)^2 \)

61. **OPEN ENDED** Write a perfect square trinomial equation in which the linear coefficient is negative and the constant term is a fraction. Then solve the equation.

62. **WRITING IN MATH** Explain what it means to complete the square. Describe each step.
63. SAT/ACT If \(x^2 + y^2 = 2xy\), then \(y\) must equal
A -1  C 1  E \(x\)
B 0  D \(-x\)

64. GEOMETRY Find the area of the shaded region.

65. SHORT RESPONSE What value of \(c\) should be used to solve the following equation by completing the square?
\[5x^2 - 50x + c = 12 + c\]

66. If \(5 - 3i\) is a solution for \(x^2 + ax + b = 0\), where \(a\) and \(b\) are real numbers, what is the value of \(b\)?
A 10  C 34
B 14  D 40

Simplify. (Lesson 5-4)

67. \((8 + 5i)^2\)

68. \(4(3 - i) + 6(2 - 5i)\)

69. \(\frac{5 - 2i}{6 + 9i}\)

Write a quadratic equation in standard form with the given root(s). (Lesson 5-3)

70. \(\frac{4}{5}, \frac{3}{4}\)

71. \(-\frac{2}{5}, 6\)

72. \(-\frac{1}{4} - \frac{6}{7}\)

73. MOVIES Refer to the table. (Lesson 4-1)
a. Write a matrix for the prices of movie tickets for adults, children, and seniors.
b. What are the dimensions of the matrix?

74. TRAVEL Yoko is going with the Spanish Club to Costa Rica. She buys 10 traveler’s checks in denominations of $20, $50, and $100, totaling $370. She has twice as many $20 checks as $50 checks. How many of each denomination of traveler’s checks does she have? (Lesson 3-5)

Graph each inequality. (Lesson 2-8)

75. \(y \geq 4x - 3\)

76. \(2x - 3y < 6\)

77. \(5x + 2y + 3 \leq 0\)

Write the piecewise function shown in each graph. (Lesson 2-6)

78. 

79. 

80. 

Skills Review

Evaluate \(b^2 - 4ac\) for the given values of \(a, b,\) and \(c\). (Lesson 1-2)

81. \(a = 5, b = 6, c = 2\)

82. \(a = -2, b = -7, c = 3\)

83. \(a = -5, b = -8, c = -10\)
You can use a TI-Nspire™ CAS to solve quadratic equations.

### Activity

**Finding Roots**

Solve each equation.

**a.** \(3x^2 - 4x + 1 = 0\)

- **Step 1** From the Home screen, select **New Document**. Then select **Add Calculator**.
- **Step 2** Under menu, select **Algebra**, then select **Solve**.
- **Step 3** Enter the equation.
  
  **KEYSTROKES:** \(3 \times x^2 - 4 \times x + 1 = 0 \Rightarrow \)  
  
  The solutions are \(x = \frac{1}{3}\) or \(x = 1\).

**b.** \(6x^2 + 4x - 3 = 0\)

- **Step 1** Under menu, select **Algebra**, then select **Solve**.
- **Step 2** Enter the equation.
  
  **KEYSTROKES:** \(6 \times x^2 + 4 \times x - 3 = 0 \Rightarrow \)  
  
  The solutions are \(x = \frac{-2 \pm \sqrt{22}}{6}\).

**c.** \(x^2 - 6x + 10 = 0\)

- **Step 1** Under menu, select **Algebra**, then select **Solve**.
- **Step 2** Enter the equation.
  
  **KEYSTROKES:** \(x^2 - 6 \times x + 10 = 0 \Rightarrow \)  
  
  The calculator returns a value of **false**, meaning that there are no real solutions.

- **Step 3** Under menu, select **Algebra, Complex**, then **Solve**. Reenter the equation.
  
  The solutions are \(x = 3 \pm i\).

### Exercises

Solve each equation.

1. \(x^2 - 2x - 24 = 0\)
2. \(-x^2 + 4x - 1 = 0\)
3. \(0 = -3x^2 - 6x + 9\)
4. \(x^2 - 2x + 5 = 0\)
5. \(0 = 4x^2 - 8\)
6. \(0 = 2x^2 - 4x + 1\)
7. \(x^2 + 3x + 8 = 5\)
8. \(25 + 4x^2 = -20x\)
9. \(x^2 - x = -6\)
LESSON 5-6  The Quadratic Formula and the Discriminant

Then  Now  Why?

- You solved equations by completing the square.  (Lesson 5-5)
- Solve quadratic equations by using the Quadratic Formula.
- Pumpkin catapult is an event in which a contestant builds a catapult and launches a pumpkin at a target.

1. Solve quadratic equations by using the Quadratic Formula.

2. Use the discriminant to determine the number and type of roots of a quadratic equation.

Pumpkin catapult is an event in which a contestant builds a catapult and launches a pumpkin at a target.

The path of the pumpkin can be modeled by the quadratic function

\[ h = -4.9t^2 + 117t + 42, \]

where \( h \) is the height of the pumpkin and \( t \) is the number of seconds.

To predict when the pumpkin will hit the target, you can solve the equation

\[ 0 = -4.9t^2 + 117t + 42. \]

This equation would be difficult to solve using factoring, graphing, or completing the square.

<table>
<thead>
<tr>
<th>New Vocabulary</th>
</tr>
</thead>
</table>

**Quadratic Formula** You have found solutions of some quadratic equations by graphing, by factoring, and by using the Square Root Property. There is also a formula that can be used to solve any quadratic equation. This formula can be derived by solving the standard form of a quadratic equation.

### General Case

\[ ax^2 + bx + c = 0 \]

### Specific Case

\[ 2x^2 + 8x + 1 = 0 \]

Divide each side by \( a \).

\[ x^2 + 4x + \frac{1}{2} = 0 \]

Subtract \( \frac{c}{a} \) from each side.

\[ x^2 + 4x = -\frac{1}{2} \]

Complete the square.

\[ x^2 + 4x + \left(\frac{4}{2}\right)^2 = -\frac{1}{2} + \left(\frac{4}{2}\right)^2 \]

Factor the left side.

\[ (x + 2)^2 = -\frac{1}{2} + \left(\frac{4}{2}\right)^2 \]

Simplify the right side.

\[ (x + 2)^2 = \frac{7}{2} \]

Square Root Property

\[ x + 2 = \pm \sqrt{\frac{7}{2}} \]

Subtract \( \frac{b}{2a} \) from each side.

\[ x = -2 \pm \sqrt{\frac{7}{2}} \]

Simplify.

\[ x = -4 \pm \sqrt{14} \]

The equation \( x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \) is known as the **Quadratic Formula**.
Study Tip

Quadratic Formula  Although factoring may be an easier method to solve some of the equations, the Quadratic Formula can be used to solve any quadratic equation.

Key Concept  Quadratic Formula

Words  The solutions of a quadratic equation of the form \( ax^2 + bx + c = 0 \), where \( a \neq 0 \), are given by the following formula.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Example  \( x^2 + 5x + 6 = 0 \) → \( x = \frac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2(1)} \)

Example 1  Two Rational Roots

Solve \( x^2 - 10x = 11 \) by using the Quadratic Formula.

First, write the equation in the form \( ax^2 + bx + c = 0 \) and identify \( a \), \( b \), and \( c \).

\[
\begin{align*}
ax^2 + bx + c &= 0 \\
\downarrow & \quad \downarrow \\
x^2 - 10x &= 11 \\
1x^2 - 10x - 11 &= 0
\end{align*}
\]

Then, substitute these values into the Quadratic Formula.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
= \frac{10 \pm \sqrt{100 + 44}}{2}
\]

\[
= \frac{10 \pm \sqrt{144}}{2}
\]

\[
x = \frac{10 + 12}{2} \quad \text{or} \quad x = \frac{10 - 12}{2}
\]

\[
= 11 \quad \quad = -1
\]

The solutions are \(-1\) and \(11\).

CHECK  Substitute both values into the original equation.

\[
\begin{align*}
x^2 - 10x &= 11 \\
(-1)^2 - 10(-1) &= 11 \\
1 + 10 &= 11 \checkmark
\end{align*}
\]

\[
\begin{align*}
x^2 - 10x &= 11 \\
(11)^2 - 10(11) &= 11 \\
121 - 110 &= 11 \checkmark
\end{align*}
\]

Guided Practice

1. Solve each equation by using the Quadratic Formula.

   A.  \( x^2 + 6x = 16 \)  
   B.  \( 2x^2 + 25x + 33 = 0 \)

When the value of the radicand in the Quadratic Formula is 0, the quadratic equation has exactly one rational root.
Example 2 One Rational Root

Solve \( x^2 + 8x + 16 = 0 \) by using the Quadratic Formula. Identify \( a, b, \) and \( c \). Then, substitute these values into the Quadratic Formula.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Quadratic Formula

\[
x = \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(16)}}{2(1)}
\]

Replace \( a \) with 1, \( b \) with 8, and \( c \) with 16.

\[
x = \frac{-8 \pm \sqrt{0}}{2}
\]

Simplify.

\[
x = \frac{-8}{2} \text{ or } -4
\]

\( \sqrt{0} = 0 \)

The solution is \(-4\).

CHECK A graph of the related function shows that there is one solution at \( x = -4 \).

Guided Practice

2. Solve each equation by using the Quadratic Formula.

A. \( x^2 - 16x + 64 = 0 \)

B. \( x^2 + 34x + 289 = 0 \)

You can express irrational roots exactly by writing them in radical form.

Example 3 Irrational Roots

Solve \( 2x^2 + 6x - 7 = 0 \) by using the Quadratic Formula.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Quadratic Formula

\[
x = \frac{-6 \pm \sqrt{(6)^2 - 4(2)(-7)}}{2(2)}
\]

Replace \( a \) with 2, \( b \) with 6, and \( c \) with \(-7 \).

\[
x = \frac{-6 \pm \sqrt{92}}{4}
\]

Simplify.

\[
x = \frac{-6 \pm 2\sqrt{23}}{4} \text{ or } \frac{-3 \pm \sqrt{23}}{2}
\]

\( \sqrt{92} = \sqrt{4 \cdot 23} \) or \( 2\sqrt{23} \)

The approximate solutions are \(-3.9 \) and \( 0.9 \).

CHECK Check these results by graphing the related quadratic function, \( y = 2x^2 + 6x - 7 \). Using the ZERO function of a graphing calculator, the approximate zeros of the related function are \(-3.9 \) and \( 0.9 \).

Guided Practice

3. Solve each equation by using the Quadratic Formula.

A. \( 3x^2 + 5x + 1 = 0 \)

B. \( x^2 - 8x + 9 = 0 \)
When using the Quadratic Formula, if the value of the radicand is negative, the solutions will be complex. Complex solutions always appear in conjugate pairs.

**Example 4 Complex Roots**

Solve $x^2 - 6x = -10$ by using the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$  
**Quadratic Formula**

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$$  
Replace $a$ with 1, $b$ with $-6$, and $c$ with 10.

$$= \frac{6 \pm \sqrt{-4}}{2}$$  
Simplify.

$$= \frac{6 \pm 2i}{2}$$  
Simplify.

$$= 3 \pm i$$

The solutions are the complex numbers $3 + i$ and $3 - i$.

**CHECK** A graph of the related function shows that the solutions are complex, but it cannot help you find them. To check complex solutions, substitute them into the original equation.

$$x^2 - 6x = -10$$  
**Original equation**

$$x = 3 + i$$

$$9 + 6i + i^2 - 18 - 6i \neq -10$$  
Square of a sum; Distributive Property

$$-9 + 1 \neq -10$$  
Simplify.

$$-9 - 1 = -10$$  
✓

$$i^2 = -1$$

$$x^2 - 6x = -10$$  
**Original equation**

$$x = 3 - i$$

$$9 - 6i + i^2 - 18 + 6i \neq -10$$  
Square of a sum; Distributive Property

$$-9 + 1 \neq -10$$  
Simplify.

$$-9 - 1 = -10$$  
✓

$$i^2 = -1$$

**Guided Practice**

4. Solve each equation by using the Quadratic Formula.

A. $3x^2 + 5x + 4 = 0$  

B. $x^2 - 4x = -13$

2 **Roots and the Discriminant** In the previous examples, observe the relationship between the value of the expression under the radical and the roots of the quadratic equation. The expression $b^2 - 4ac$ is called the **discriminant**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \leftrightarrow \text{discriminant}$$

The value of the discriminant can be used to determine the number and type of roots of a quadratic equation. The table on the following page summarizes the possible types of roots.

The discriminant can also be used to confirm the number and type of solutions after you solve the quadratic equation.
**Study Tip**

**Roots** Remember that the solutions of an equation are called *roots* or *zeros* and are the value(s) where the graph crosses the *x*-axis.

### Key Concept: Discriminant

Consider $ax^2 + bx + c = 0$, where $a$, $b$, and $c$ are rational numbers and $a \neq 0$.

<table>
<thead>
<tr>
<th>Value of Discriminant</th>
<th>Type and Number of Roots</th>
<th>Example of Graph of Related Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b^2 - 4ac &gt; 0$; $b^2 - 4ac$ is a perfect square.</td>
<td>2 real, rational roots</td>
<td><img src="image1.png" alt="Graph" /></td>
</tr>
<tr>
<td>$b^2 - 4ac &gt; 0$; $b^2 - 4ac$ is not a perfect square.</td>
<td>2 real, irrational roots</td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
<tr>
<td>$b^2 - 4ac = 0$</td>
<td>1 real rational root</td>
<td><img src="image3.png" alt="Graph" /></td>
</tr>
<tr>
<td>$b^2 - 4ac &lt; 0$</td>
<td>2 complex roots</td>
<td><img src="image4.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

### Example 5: Describe Roots

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.

**a.** $7x^2 - 11x + 5 = 0$

- $a = 7$, $b = -11$, $c = 5$
- $b^2 - 4ac = (-11)^2 - 4(7)(5)$
- $= 121 - 140$
- $= -19$

The discriminant is negative, so there are two complex roots.

**b.** $x^2 + 22x + 121 = 0$

- $a = 1$, $b = 22$, $c = 121$
- $b^2 - 4ac = (22)^2 - 4(1)(121)$
- $= 484 - 484$
- $= 0$

The discriminant is 0, so there is one rational root.

### Guided Practice

5A. $-5x^2 + 8x - 1 = 0$

5B. $-7x + 15x^2 - 4 = 0$
You have studied a variety of methods for solving quadratic equations. The table below summarizes these methods.

**Concept Summary: Solving Quadratic Equations**

<table>
<thead>
<tr>
<th>Method</th>
<th>Can be Used</th>
<th>When to Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>graphing</td>
<td>sometimes</td>
<td>Use only if an exact answer is not required. Best used to check the reasonableness of solutions found algebraically.</td>
</tr>
<tr>
<td>factoring</td>
<td>sometimes</td>
<td>Use if the constant term is 0 or if the factors are easily determined.</td>
</tr>
<tr>
<td>Square Root Property</td>
<td>sometimes</td>
<td>Use for equations in which a perfect square is equal to a constant.</td>
</tr>
<tr>
<td>completing the square</td>
<td>always</td>
<td>Useful for equations of the form (x^2 + bx + c = 0), where (b) is even.</td>
</tr>
<tr>
<td>Quadratic Formula</td>
<td>always</td>
<td>Useful when other methods fail or are too tedious.</td>
</tr>
</tbody>
</table>

**Check Your Understanding**

**Examples 1–4** Solve each equation by using the Quadratic Formula.

1. \(x^2 + 12x - 9 = 0\)
2. \(x^2 + 8x + 5 = 0\)
3. \(4x^2 - 5x - 2 = 0\)
4. \(9x^2 + 6x - 4 = 0\)
5. \(10x^2 - 3 = 13x\)
6. \(22x = 12x^2 + 6\)
7. \(-3x^2 + 4x = -8\)
8. \(x^2 + 3 = -6x + 8\)

**Examples 3–4**

9. **AMUSEMENT PARK** An amusement park ride takes riders to the top of a tower and drops them at speeds reaching 80 feet per second. A function that models this ride is \(h = -16t^2 - 64t + 60\), where \(h\) is the height in feet and \(t\) is the time in seconds. About how many seconds does it take for riders to drop from 60 feet to 0 feet?

**Example 5**

Complete parts a and b for each quadratic equation.

a. Find the value of the discriminant.
b. Describe the number and type of roots.

10. \(3x^2 + 8x + 2 = 0\)
11. \(2x^2 - 6x + 9 = 0\)
12. \(-16x^2 + 8x - 1 = 0\)
13. \(5x^2 + 2x + 4 = 0\)
Practice and Problem Solving

Examples 1–4 Solve each equation by using the Quadratic Formula.

14. \( x^2 + 45x = -200 \)
15. \( 4x^2 - 6 = -12x \)
16. \( 3x^2 - 4x - 8 = -6 \)
17. \( 4x^2 - 9 = -7x - 4 \)
18. \( 5x^2 - 9 = 11x \)
19. \( 12x^2 + 9x - 2 = -17 \)

20. **DIVING** Competitors in the 10-meter platform diving competition jump upward and outward before diving into the pool below. The height \( h \) of a diver in meters above the pool after \( t \) seconds can be approximated by the equation \( h = -4.9t^2 + 3t + 10 \).
   a. Determine a domain and range for which this function makes sense.
   b. When will the diver hit the water?

Example 5 Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>( 2x^2 + 3x - 3 = 0 )</td>
<td>( 22. \ 4x^2 - 6x + 2 = 0 )</td>
</tr>
<tr>
<td>24</td>
<td>( 6x^2 - x - 5 = 0 )</td>
<td>( 25. \ 3x^2 - 3x + 8 = 0 )</td>
</tr>
<tr>
<td>27</td>
<td>( -5x^2 + 4x + 1 = 0 )</td>
<td>( 28. \ x^2 - 6x = -9 )</td>
</tr>
<tr>
<td>30</td>
<td>( -8x^2 + 5 = -4x )</td>
<td>( 31. \ x^2 + 2x - 4 = -9 )</td>
</tr>
</tbody>
</table>

33. **VIDEO GAMES** While Darnell is grounded his friend Jack brings him a video game. Darnell stands at his bedroom window, and Jack stands directly below the window. If Jack tosses a game cartridge to Darnell with an initial velocity of 35 feet per second, an equation for the height \( h \) feet of the cartridge after \( t \) seconds is \( h = -16t^2 + 35t + 5 \).
   a. If the window is 25 feet above the ground, will Darnell have 0, 1, or 2 chances to catch the video game cartridge?
   b. If Darnell is unable to catch the video game cartridge, when will it hit the ground?

34. **ENGINEERING** Civil engineers are designing a section of road that is going to dip below sea level. The road’s curve can be modeled by the equation \( y = 0.00005x^2 - 0.06x \), where \( x \) is the horizontal distance in feet between the points where the road is at sea level and \( y \) is the elevation. The engineers want to put stop signs at the locations where the elevation of the road is equal to sea level. At what horizontal distances will they place the stop signs?

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>( 5x^2 + 8x = 0 )</td>
<td>( 36. \ 8x^2 = -2x + 1 )</td>
</tr>
<tr>
<td>38</td>
<td>( 0.8x^2 + 2.6x = -3.2 )</td>
<td>( 39. \ 0.6x^2 + 1.4x = 4.8 )</td>
</tr>
</tbody>
</table>
**SMOKING** A decrease in smoking in the United States has resulted in lower death rates caused by lung cancer. The number of deaths per 100,000 people \( y \) can be approximated by \( y = -0.26x^2 - 0.55x + 91.81 \), where \( x \) represents the number of years after 2000.

<table>
<thead>
<tr>
<th>Year</th>
<th>Deaths per 100,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>91.8</td>
</tr>
<tr>
<td>2002</td>
<td>89.7</td>
</tr>
<tr>
<td>2004</td>
<td>85.5</td>
</tr>
<tr>
<td>2010</td>
<td>60.3</td>
</tr>
<tr>
<td>2015</td>
<td>?</td>
</tr>
<tr>
<td>2017</td>
<td>?</td>
</tr>
</tbody>
</table>

**a.** Calculate the number of deaths per 100,000 people for 2015 and 2017.

**b.** Use the Quadratic Formula to solve for \( x \) when \( y = 50 \).

**c.** According to the quadratic function, when will the death rate be 0 per 100,000? Do you think that this prediction is reasonable? Why or why not?

**42. NUMBER THEORY** The sum \( S \) of consecutive integers 1, 2, 3, ..., \( n \) is given by the formula \( S = \frac{1}{2}n(n + 1) \). How many consecutive integers, starting with 1, must be added to get a sum of 666?

**H.O.T. Problems** Use Higher-Order Thinking Skills

**43. ERROR ANALYSIS** Tama and Jonathan are determining the number of solutions of \( 3x^2 - 5x = 7 \). Is either of them correct? Explain your reasoning.

**Tama**

\[
3x^2 - 5x = 7
\]

\[
\begin{align*}
\text{b}^2 - 4ac &= (-5)^2 - 4(3)(7) \\
&= -59
\end{align*}
\]

Since the discriminant is negative, there are no real solutions.

**Jonathan**

\[
3x^2 - 5x = 7
\]

\[
\begin{align*}
\text{b}^2 - 4ac &= (-5)^2 - 4(3)(-7) \\
&= 109
\end{align*}
\]

Since the discriminant is positive, there are two real roots.

**44. CHALLENGE** Find the solutions of \( 4ix^2 - 4ix + 5i = 0 \) by using the Quadratic Formula.

**45. REASONING** Determine whether each statement is sometimes, always, or never true. Explain your reasoning.

a. In a quadratic equation in standard form, if \( a \) and \( c \) are different signs, then the solutions will be real.

b. If the discriminant of a quadratic equation is greater than 1, the two roots are real irrational numbers.

**46. OPEN ENDED** Sketch the corresponding graph and state the number and type of roots for each of the following.

a. \( b^2 - 4ac = 0 \)

b. A quadratic function in which \( f(x) \) never equals zero.

c. A quadratic function in which \( f(a) = 0 \) and \( f(b) = 0; a \neq b \).

d. The discriminant is less than zero.

e. \( a \) and \( b \) are both solutions and can be represented as fractions.

**47. CHALLENGE** Find the value(s) of \( m \) in the quadratic equation \( x^2 + x + m + 1 = 0 \) such that it has one solution.

**48. WRITING IN MATH** Describe three different ways to solve \( x^2 - 2x - 15 = 0 \). Which method do you prefer, and why?
49. A company determined that its monthly profit \( P \) is given by \( P = -8x^2 + 165x - 100 \), where \( x \) is the selling price for each unit of product. Which of the following is the best estimate of the maximum price per unit that the company can charge without losing money?

A $10  
B $20  
C $30  
D $40

50. SAT/ACT For which of the following sets of numbers is the mean greater than the median?

F \( \{4, 5, 6, 7, 8\} \)  
J \( \{3, 5, 6, 7, 8\} \)  
G \( \{4, 6, 6, 6, 8\} \)  
K \( \{2, 6, 6, 6, 6\} \)  
H \( \{4, 5, 6, 7, 9\} \)

51. SHORT RESPONSE In the figure below, \( P \) is the center of the circle with radius 15 inches. What is the area of \( \triangle APB \)?

52. 75\% of 88 is the same as 60\% of what number?

A 100  
B 101  
C 108  
D 110

Spiral Review

Find the value of \( c \) that makes each trinomial a perfect square. Then write the trinomial as a perfect square. (Lesson 5-5)

53. \( x^2 + 13x + c \)  
54. \( x^2 + 2.4x + c \)  
55. \( x^2 + \frac{4}{5}x + c \)

Simplify. (Lesson 5-4)

56. \( i^{26} \)  
57. \( \sqrt{-16} \)  
58. \( 4\sqrt{-9} \cdot 2\sqrt{-25} \)

59. PILOT TRAINING Evita is training for her pilot’s license. Flight instruction costs $105 per hour, and the simulator costs $45 per hour. She spent 4 more hours in airplane training than in the simulator. If Evita spent $3870, how much time did she spend training in an airplane and in a simulator? (Lesson 4-6)

60. BUSINESS Ms. Larson owns three fruit farms on which she grows apples, peaches, and apricots. She sells apples for $22 a case, peaches for $25 a case, and apricots for $18 a case. (Lesson 4-3)

a. Write an inventory matrix for the number of cases for each type of fruit for each farm and a cost matrix for the price per case for each type of fruit.

b. Find the total income of the three fruit farms expressed as a matrix.

c. What is the total income from all three fruit farms?

Skills Review

Write an equation for each graph. (Lesson 2-7)

61.  
62.  
63.
OBJECTIVE Use a graphing calculator to investigate changes to parabolas.

The general form of a quadratic function is \( y = a(x - h)^2 + k \). Changing the values of \( a, h, \) and \( k \) results in a different parabola in the family of quadratic functions. You can use a TI-83/84 Plus graphing calculator to analyze the effects that result from changing each of these parameters.

Activity 1

Graph each set of functions on the same screen in the standard viewing window. Describe any similarities and differences among the graphs.

\[
\begin{align*}
y &= x^2, \quad y = x^2 + 4, \quad y = x^2 - 3 \\
\end{align*}
\]

The graphs have the same shape, and all open up. The vertex of each graph is on the y-axis. However, the graphs have different vertical positions.

Example 1 shows how changing the value of \( k \) in the function \( y = a(x - h)^2 + k \) translates the parabola along the y-axis. If \( k > 0 \), the parabola is translated \( k \) units up, and if \( k < 0 \), it is translated \( k \) units down.

How do you think changing the value of \( h \) will affect the graph of \( y = x^2 \)?

Activity 2

Graph each set of functions on the same screen in the standard viewing window. Describe any similarities and differences among the graphs.

\[
\begin{align*}
y &= x^2, \quad y = (x + 4)^2, \quad y = (x - 3)^2 \\
\end{align*}
\]

These three graphs all open up and have the same shape. The vertex of each graph is on the x-axis. However, the graphs have different horizontal positions.

Example 2 shows how changing the value of \( h \) in the equation \( y = a(x - h)^2 + k \) translates the graph horizontally. If \( h > 0 \), the graph translates to the right \( h \) units. If \( h < 0 \), the graph translates to the left \( h \) units.

Activity 3

Graph each set of functions on the same screen in the standard viewing window. Describe any similarities and differences among the graphs.

\[
\begin{align*}
y &= x^2, \quad y = (x + 6)^2 - 5, \quad y = (x - 4)^2 + 6 \\
\end{align*}
\]

These three graphs all open up and have the same shape. However, the graphs have different horizontal and vertical positions.

(continued on the next page)

Tennessee Curriculum Standards

SPI 3103.3.10 Identify and/or graph a variety of functions and their translations.

Also addresses 3103.3.4 and 3103.3.11.
Activity 4

Graph each set of functions on the same screen in the standard viewing window. Describe any similarities and differences among the graphs.

a. \( y = x^2, y = -x^2 \)

The graphs have the same vertex and the same shape. However, the graph of \( y = x^2 \) opens up and the graph of \( y = -x^2 \) opens down.

b. \( y = x^2, y = 5x^2, y = \frac{1}{5}x^2 \)

The graphs have the same vertex, \((0, 0)\), but each has a different shape. The graph of \( y = 5x^2 \) is narrower than the graph of \( y = x^2 \). The graph of \( y = \frac{1}{5}x^2 \) is wider than the graph of \( y = x^2 \).

Changing the value of \( a \) in the function \( y = a(x - h)^2 + k \) can affect the direction of the opening and the shape of the graph. If \( a > 0 \), the graph opens up, and if \( a < 0 \), the graph opens down or is reflected over the \( x \)-axis. If \( |a| > 1 \), the graph is expanded vertically and is narrower than the graph of \( y = x^2 \). If \( |a| < 1 \), the graph is compressed vertically and is wider than the graph of \( y = x^2 \). Thus, a change in the absolute value of \( a \) results in a dilation of the graph of \( y = x^2 \).

Analyze the Results

1. How does changing the value of \( h \) in \( y = a(x - h)^2 + k \) affect the graph? Give an example.
2. How does changing the value of \( k \) in \( y = a(x - h)^2 + k \) affect the graph? Give an example.
3. How does using \(-a\) instead of \( a \) in \( y = a(x - h)^2 + k \) affect the graph? Give an example.

Examine each pair of functions and predict the similarities and differences in their graphs. Use a graphing calculator to confirm your predictions. Write a sentence or two comparing the two graphs.

4. \( y = x^2, y = x^2 + 3.5 \)
5. \( y = -x^2, y = x^2 - 7 \)
6. \( y = x^2, y = 4x^2 \)
7. \( y = x^2, y = -8x^2 \)
8. \( y = x^2, y = (x + 2)^2 \)
9. \( y = -\frac{1}{6}x^2, y = -\frac{1}{6}x^2 + 2 \)
10. \( y = x^2, y = (x - 5)^2 \)
11. \( y = x^2, y = 2(x + 3)^2 - 6 \)
12. \( y = x^2, y = -\frac{1}{2}x^2 + 1 \)
13. \( y = (x + 5)^2 - 4, y = (x + 5)^2 + 7 \)
14. \( y = 2(x + 1)^2 - 4, y = 5(x + 3)^2 - 1 \)
15. \( y = 5(x - 2)^2 - 3, y = \frac{1}{4}(x - 5)^2 - 6 \)
Transformations with Quadratic Functions

1 Write Quadratic Functions in Vertex Form

Each function above is written in vertex form, \( y = a(x - h)^2 + k \), where \((h, k)\) is the vertex of the parabola, \(x = h\) is the axis of symmetry, and \(a\) determines the shape of the parabola and the direction in which it opens.

When a quadratic function is in the form \( y = ax^2 + bx + c \), you can complete the square to write the function in vertex form. If the coefficient of the quadratic term is not 1, then factor the coefficient from the quadratic and linear terms before completing the square.

Example 1 Write Functions in Vertex Form

Write each function in vertex form.

a. \( y = x^2 + 6x - 5 \)

\[
y = x^2 + 6x - 5
\]

\[
y = (x^2 + 6x + 9) - 5 - 9
\]

\[
y = (x + 3)^2 - 14
\]

Original function

Group \( ax^2 + bx \) and factor, dividing by \(a\).

Complete the square by adding 4 inside the parentheses. This is an overall addition of \(-2(4)\). Balance the equation by subtracting \(-2(4)\).

Write \( x^2 - 4x + 4 \) as a perfect square.

b. \( y = -2x^2 + 8x - 3 \)

\[
y = -2x^2 + 8x - 3
\]

\[
y = -2(x^2 - 4x) - 3
\]

\[
y = -2(x^2 - 4x + 4) - 3 - (-2)(4)
\]

\[
y = -2(x - 2)^2 + 5
\]

\[
y = -2x^2 + 8x - 3
\]

\[
y = -2(x^2 - 4x + 4) - 3 - (-2)(4)
\]

\[
y = -2(x - 2)^2 + 5
\]

\[
y = -2x^2 + 8x - 3
\]

\[
y = -2(x^2 - 4x + 4) - 3 - (-2)(4)
\]

\[
y = -2(x - 2)^2 + 5
\]

Guided Practice

1A. \( y = x^2 + 4x + 6 \)

1B. \( y = 2x^2 - 12x + 17 \)
If the vertex and one additional point on the graph of a parabola are known, you can write the equation of the parabola in vertex form.

Test Example 2

Which is an equation of the function shown in the graph?

A  \( y = -4(x - 3)^2 + 2 \)

B  \( y = -\frac{1}{4}(x - 3)^2 + 2 \)

C  \( y = \frac{1}{4}(x + 3)^2 - 2 \)

D  \( y = 4(x + 3)^2 - 2 \)

Read the Test Item

You are given a graph of a parabola with the vertex and a point on the graph labeled. You need to find an equation of the parabola.

Solve the Test Item

The vertex of the parabola is at (3, 2), so \( h = 3 \) and \( k = 2 \). Since \((-1, -2)\) is a point on the graph, let \( x = -1 \) and \( y = -2 \). Substitute these values into the vertex form of the equation and solve for \( a \).

\[
\begin{align*}
  y &= a(x - h)^2 + k \\
  -2 &= a(-1 - 3)^2 + 2 \\
  -2 &= a(16) + 2 \\
  -4 &= 16a \\
  -\frac{1}{4} &= a
\end{align*}
\]

The equation of the parabola in vertex form is \( y = -\frac{1}{4}(x - 3)^2 + 2 \).

The answer is B.

Guided Practice

2. Which is an equation of the function shown in the graph?

F  \( y = \frac{9}{25}(x - 1)^2 + 2 \)

G  \( y = \frac{3}{5}(x + 1)^2 - 2 \)

H  \( y = \frac{5}{3}(x + 1)^2 - 2 \)

J  \( y = \frac{25}{9}(x - 1)^2 + 2 \)

Transformations of Quadratic Functions In Lesson 2-7, you learned how different transformations affect the graphs of parent functions. The following summarizes these transformations for quadratic functions.
**Study Tip**

**Absolute Value**

$0 < |a| < 1$ means that $a$ is a rational number between 0 and 1, such as $\frac{3}{4}$, or a rational number between $-1$ and 0, such as $-0.3$.

**Concept Summary**  
Transformations of Quadratic Functions

$$f(x) = a(x - h)^2 + k$$

- **$h$, Horizontal Translation**
  - $h$ units to the right if $h$ is positive
  - $|h|$ units to the left if $h$ is negative
  - $h = 0$
  - $h < 0$
  - $h > 0$

- **$k$, Vertical Translation**
  - $k$ units up if $k$ is positive
  - $|k|$ units down if $k$ is negative
  - $k = 0$
  - $k > 0$
  - $k < 0$

- **$a$, Reflection**
  - If $a > 0$, the graph opens up.
  - If $a < 0$, the graph opens down.

- **$a$, Dilation**
  - If $|a| > 1$, the graph is stretched vertically. If $0 < |a| < 1$, the graph is compressed vertically.
  - $|a| = 1$

**Example 3**  
Graph Equations in Vertex Form

Graph $y = 4x^2 - 16x - 40$.

**Step 1**  
Rewrite the equation in vertex form.

Original equation:

$$y = 4x^2 - 16x - 40$$

Distributive Property:

$$y = 4(x^2 - 4x) - 40$$

Complete the square:

$$y = 4(x - 4x + 4) - 40 - 4(4)$$

Simplify:

$$y = 4(x - 2)^2 - 56$$

**Step 2**  
The vertex is at $(2, -56)$. The axis of symmetry is $x = 2$. Because $a = 4$, the graph is narrower than the graph of $y = x^2$.

**Step 3**  
Plot additional points to help you complete the graph.

**Guided Practice**

3A. $y = (x - 3)^2 - 2$

3B. $y = 0.25(x + 1)^2$
Check Your Understanding

Example 1
Write each function in vertex form.
1. \( y = x^2 + 6x + 2 \)  
2. \( y = -2x^2 + 8x - 5 \)  
3. \( y = 4x^2 + 24x + 24 \)

Example 2
4. MULTIPLE CHOICE Which function is shown in the graph?
A \( y = -(x + 3)^2 + 6 \)  
B \( y = -(x - 3)^2 - 6 \)  
C \( y = -2(x + 3)^2 + 6 \)  
D \( y = -2(x - 3)^2 - 6 \)

Example 3
Graph each function.
5. \( y = (x - 3)^2 - 4 \)  
6. \( y = -2x^2 + 5 \)  
7. \( y = \frac{1}{2}(x + 6)^2 - 8 \)

Practice and Problem Solving

Example 1
Write each function in vertex form.
8. \( y = x^2 + 9x + 8 \)  
9. \( y = x^2 - 6x + 3 \)  
10. \( y = -2x^2 + 5x \)
11. \( y = x^2 + 2x + 7 \)  
12. \( y = -3x^2 + 12x - 10 \)  
13. \( y = x^2 + 8x + 16 \)
14. \( y = 2x^2 - 4x - 3 \)  
15. \( y = 3x^2 + 10x \)  
16. \( y = x^2 - 4x + 9 \)
17. \( y = -4x^2 - 24x - 15 \)  
18. \( y = x^2 - 12x + 36 \)  
19. \( y = -x^2 - 4x - 1 \)

Example 2
20. FIREWORKS During an Independence Day fireworks show, the height \( h \) in meters of a specific rocket after \( t \) seconds can be modeled by \( h = -4.9(t - 4)^2 + 80 \). Graph the function.

21. FINANCIAL LITERACY A bicycle rental shop rents an average of 120 bicycles per week and charges $25 per day. The manager estimates that there will be 15 additional bicycles rented for each $1 reduction in the rental price. The maximum income the manager can expect can be modeled by \( y = -15x^2 + 255x + 3000 \), where \( y \) is the weekly income and \( x \) is the number of bicycles rented. Write this function in vertex form. Then graph.

Example 3
Graph each function.
22. \( y = (x - 5)^2 + 3 \)  
23. \( y = 9x^2 - 8 \)  
24. \( y = -2(x - 5)^2 \)
25. \( y = \frac{1}{10}(x + 6)^2 + 6 \)  
26. \( y = -3(x - 5)^2 - 2 \)  
27. \( y = -\frac{1}{4}x^2 - 5 \)
28. \( y = 2x^2 + 10 \)  
29. \( y = -(x + 3)^2 \)  
30. \( y = \frac{1}{6}(x - 3)^2 - 10 \)
31. \( y = (x - 9)^2 - 7 \)  
32. \( y = -\frac{5}{8}x^2 - 8 \)  
33. \( y = -4(x - 10)^2 - 10 \)

34. SAILBOARDING A sailboard manufacturer uses an automated process to manufacture the masts for its sailboards. The function \( f(x) = \frac{1}{250}x^2 + \frac{3}{5}x \) is programmed into a computer to make one such mast.
   a. Write the quadratic function in vertex form. Then graph the function.
   b. Describe how the manufacturer can adjust the function to make its masts with a greater or smaller curve.
Write an equation in vertex form for each parabola.

35. \[ y = x^2 - 4x + 2 \]
36. \[ y = x^2 + 7x - 12 \]
37. \[ y = x^2 - 4.7x + 2.8 \]

38. \[ y = x^2 + 1.4x - 1.2 \]
39. \[ y = x^2 - \frac{2}{3}x - \frac{26}{9} \]
40. \[ y = x^2 + 7x + \frac{49}{4} \]

Write each function in vertex form. Then identify the vertex, axis of symmetry, and direction of opening.

41. \[ 3x^2 - 4x = 2 + y \]
42. \[ -2x^2 + 7x = y - 12 \]
43. \[ -x^2 - 4.7x = y - 2.8 \]
44. \[ x^2 + 1.4x - 1.2 = y \]
45. \[ x^2 - \frac{2}{3}x - \frac{26}{9} = y \]
46. \[ x^2 + 7x + \frac{49}{4} = y \]

47. **CARS** The formula \( S(t) = \frac{1}{2}at^2 + v_0t \) can be used to determine the position \( S(t) \) of an object after \( t \) seconds at a rate of acceleration \( a \) with initial velocity \( v_0 \). Valerie’s car can accelerate 0.002 miles per second squared.

a. Express \( S(t) \) in vertex form as she accelerates from 35 miles per hour to enter highway traffic.

b. How long will it take Valerie to match the average speed of highway traffic of 68 miles per hour? (Hint: Use acceleration \( \times \) time = velocity.)

c. If the entrance ramp is \( \frac{1}{8} \)-mile long, will Valerie have sufficient time to match the average highway speed? Explain.

**H.O.T. Problems** Use Higher-Order Thinking Skills

48. **OPEN ENDED** Write an equation for a parabola that has been translated, compressed, and reflected in the \( x \)-axis.

49. **CHALLENGE** Explain how you can find an equation of a parabola using the coordinates of three points on the graph.

50. **CHALLENGE** Write the standard form of a quadratic function \( ax^2 + bx + c = y \) in vertex form. Identify the vertex and the axis of symmetry.

51. **REASONING** Describe the graph of \( f(x) = a(x - h)^2 + k \) when \( a = 0 \). Is the graph the same as that of \( g(x) = ax^2 + bx + c \) when \( a = 0 \)? Explain.

52. **WRITING IN MATH** Explain how the graph of \( y = x^2 \) can be used to graph any quadratic function. Include a description of the effects produced by changing \( a, h, \) and \( k \) in the equation \( y = a(x - h)^2 + k \), and a comparison of the graph of \( y = x^2 \) and the graph of \( y = a(x - h)^2 + k \) using values you choose for \( a, h, \) and \( k \).
53. Flowering bushes need a mixture of 70% soil and 30% vermiculite. About how many buckets of vermiculite should you add to 20 buckets of soil?
   A 6.0 C 14.0
   B 8.0 D 24.0

54. SAT/ACT The sum of the integers \( x \) and \( y \) is 495. The units digit of \( x \) is 0. If \( x \) is divided by 10, the result is equal to \( y \). What is the value of \( x \)?
   F 40 J 250
   G 45 K 450
   H 245

55. What is the solution set of the inequality \(|4x – 1| < 9|?
   A \(|x | 2.5 < x \) or \( x < -2|
   B \(|x | x < 2.5|
   C \(|x | x > -2|
   D \(|x | -2 < x < 2.5|

56. SHORT RESPONSE At your store, you buy wrenches for $30.00 a dozen and sell them for $3.50 each. What is the percent markup for the wrenches?

Spiral Review

Solve each equation by using the method of your choice. Find exact solutions. (Lesson 5-6)
57. \( 4x^2 + 15x = 21 \) 58. \( -3x^2 + 19 = 5x \) 59. \( 6x – 5x^2 + 9 = 3 \)

Find the value of \( c \) that makes each trinomial a perfect square. (Lesson 5-5)
60. \( x^2 – 12x + c \) 61. \( x^2 + 0.1x + c \) 62. \( x^2 – 0.45x + c \)

Determine whether each function has a maximum or minimum value, and find that value. (Lesson 5-1)
63. \( f(x) = 6x^2 – 8x + 12 \) 64. \( f(x) = -4x^2 + x – 18 \) 65. \( f(x) = 3x^2 – 9 + 6x \)

66. ARCHAEOLOGY A coordinate grid is laid over an archeology dig to identify the location of artifacts. Three corners of a building have been partially unearthed at \((1, 6), (4, 5), \) and \((-1, -2)\). If each square on the grid measures one square foot, estimate the area of the floor of the building. (Lesson 4-5)

67. HOTELS Use the costs for an overnight stay at a hotel provided at the right. (Lesson 4-1)
   a. Write a \( 3 \times 2 \) matrix that represents the cost of each room.
   b. Write a \( 2 \times 3 \) matrix that represents the cost of each room.

Solve each system of equations by graphing. (Lesson 3-1)
68. \( y = 3x - 4 \) \( y = -2x + 16 \) 69. \( 2x + 5y = 1 \) \( 6y - 5x = 16 \) 70. \( 4x + 3y = -30 \) \( 3x - 2y = 3 \)

Evaluate each function. (Lesson 2-1)
71. \( f(3) \) if \( f(x) = x^2 - 4x + 12 \) 72. \( f(-2) \) if \( f(x) = -4x^2 + x - 8 \) 73. \( f(4) \) if \( f(x) = 3x^2 + x \)

Skills Review

Determine whether the given value satisfies the inequality. (Lesson 1-6)
74. \( 3x^2 - 5 > 6; x = 2 \) 75. \( -2x^2 + x - 1 < 4; x = -2 \) 76. \( 4x^2 + x - 3 \leq 36; x = 3 \)
You have learned that a linear function has a constant rate of change. You will investigate the rate of change for quadratic functions.

### Activity: Determine Rate of Change

Consider \( f(x) = 0.1875x^2 - 3x + 12 \).

**Step 1** Make a table like the one below. Use values from 0 through 16 for \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>...</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>12</td>
<td>9.1875</td>
<td>6.75</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>First-Order Differences</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Second-Order Differences</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Step 2** Find each \( y \)-value. For example, when \( x = 1 \), \( y = 0.1875(1)^2 - 3(1) + 12 \) or 9.1875.

**Step 3** Graph the ordered pairs \((x, y)\). Then connect the points with a smooth curve. Notice that the function decreases when \( 0 < x < 8 \) and increases when \( 8 < x < 16 \).

**Step 4** The rate of change from one point to the next can be found by using the slope formula. From \((0, 12)\) to \((1, 9.1875)\), the slope is \( \frac{9.1875 - 12}{1 - 0} \) or \(-2.8125\).

This is the first-order difference at \( x = 1 \). Complete the table for all the first-order differences. Describe any patterns in the differences.

**Step 5** The second-order differences can be found by subtracting consecutive first-order differences. For example, the second-order difference at \( x = 2 \) is found by subtracting the first-order difference at \( x = 1 \) from the first-order difference at \( x = 2 \). Describe any patterns in the differences.

### Exercises

For each function make a table of values for the given \( x \)-values. Graph the function. Then determine the first-order and second-order differences.

1. \( y = -x^2 + 2x - 1 \) for \( x = -3, -2, -1, 0, 1, 2, 3 \)
2. \( y = 0.5x^2 + 2x - 2 \) for \( x = -5, -4, -3, -2, -1, 0, 1 \)
3. \( y = -3x^2 - 18x - 26 \) for \( x = -6, -5, -4, -3, -2, -1, 0 \)

4. **MAKE A CONJECTURE** Repeat the activity for a cubic function. At what order difference would you expect \( g(x) = x^4 \) to be constant? \( h(x) = x^n \)?
Graph Quadratic Inequalities

**Why?** A water balloon launched from a slingshot can be represented by several different quadratic equations and inequalities.

Suppose the height of a water balloon \( h(t) \) in meters above the ground \( t \) seconds after being launched is modeled by the quadratic function \( h(t) = -4.9t^2 + 32t + 1.2 \). You can solve a quadratic inequality to determine how long the balloon will be a certain distance above the ground.

**New Vocabulary**

- **quadratic inequality**

**Graph Quadratic Inequalities** You can graph quadratic inequalities in two variables by using the same techniques used to graph linear inequalities in two variables.

**Step 1** Graph the related function.

**Step 2** Test a point not on the parabola.

\[ y_1 \geq a(x_1)^2 + b(x_1) + c \]

Is \((x_1, y_1)\) a solution?

**Step 3** Shade accordingly.

\((x_1, y_1)\) is a solution.

\((x_1, y_1)\) is not a solution.

**Example 1** Graph a Quadratic Inequality

Graph \( y > x^2 + 2x + 1 \).

**Step 1** Graph the related function, \( y = x^2 + 2x + 1 \). The parabola should be dashed.

**Step 2** Test a point not on the graph of the parabola.

\[
egin{align*}
y &> x^2 + 2x + 1 \\
-1 &> 0^2 + 2(0) + 1 \\
-1 &< 1
\end{align*}
\]

So, \((0, -1)\) is not a solution of the inequality.

**Step 3** Shade the region that does not contain the point \((0, -1)\).

**Guided Practice**

1. Graph each inequality.
   
   A. \( y \leq x^2 + 2x + 4 \)
   
   B. \( y < -2x^2 + 3x + 5 \)
2 Solve Quadratic Inequalities

Quadratic inequalities in one variable can be solved using the graphs of the related quadratic functions.

\(ax^2 + bx + c < 0\)

Graph \(y = ax^2 + bx + c\) and identify the \(x\)-values for which the graph lies below the \(x\)-axis.

For \(\leq\), include the \(x\)-intercepts in the solution.

\(ax^2 + bx + c > 0\)

Graph \(y = ax^2 + bx + c\) and identify the \(x\)-values for which the graph lies above the \(x\)-axis.

For \(\geq\), include the \(x\)-intercepts in the solution.

**Example 2** Solve \(ax^2 + bx + c < 0\) by Graphing

Solve \(x^2 + 2x - 8 < 0\) by graphing.

The solution consists of \(x\)-values for which the graph of the related function lies below the \(x\)-axis. Begin by finding the roots of the related function.

\[x^2 + 2x - 8 = 0\]

Related equation

\((x - 2)(x + 4) = 0\)

Factor.

\[x - 2 = 0 \quad \text{or} \quad x + 4 = 0\]

Zero Product Property

\[x = 2 \quad \quad x = -4\]

Solve each equation.

Sketch the graph of a parabola that has \(x\)-intercepts at \(-4\) and \(2\). The graph should open up because \(a > 0\).

The graph lies below the \(x\)-axis between \(x = -4\) and \(x = 2\). Thus, the solution set is \(\{x \mid -4 < x < 2\}\) or \((-4, 2)\).

**CHECK** Test one value of \(x\) less than \(-4\), one between \(-4\) and \(2\), and one greater than \(2\) in the original inequality.

\[
\begin{align*}
\text{Test } x = -6. & \quad x^2 + 2x - 8 < 0 \quad \Rightarrow \quad (-6)^2 + 2(-6) - 8 \leq 0 \\
& \quad 16 < 0 \ \text{X} \\
\text{Test } x = 0. & \quad x^2 + 2x - 8 < 0 \quad \Rightarrow \quad 0^2 + 2(0) - 8 \leq 0 \\
& \quad -8 < 0 \ \text{✓} \\
\text{Test } x = 5. & \quad x^2 + 2x - 8 < 0 \quad \Rightarrow \quad 5^2 + 2(5) - 8 \leq 0 \\
& \quad 27 < 0 \ \text{X}
\end{align*}
\]

**Guided Practice**

2. Solve each inequality by graphing.

A. \(0 > x^2 + 5x - 6\)

B. \(-x^2 + 3x + 10 \leq 0\)
Example 3 Solve \( ax^2 + bx + c \geq 0 \) by Graphing

Solve \( 2x^2 + 4x - 5 \geq 0 \) by graphing.

The solution consists of \( x \)-values for which the graph of the related function lies on and above the \( x \)-axis. Begin by finding the roots of the related function.

\[
2x^2 + 4x - 5 = 0
\]

**Related equation**

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Use the Quadratic Formula**

\[
x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-5)}}{2(2)}
\]

Replace \( a \) with 4, \( b \) with 2, and \( c \) with -5.

\[
x = \frac{-4 + \sqrt{56}}{4} \quad \text{or} \quad x = \frac{-4 - \sqrt{56}}{4}
\]

Simplify and write as two equations.

\[
x \approx 0.87 \quad \text{and} \quad x \approx -2.87
\]

Sketch the graph of a parabola with \( x \)-intercepts at -2.87 and 0.87. The graph opens up since \( a > 0 \). The graph lies on and above the \( x \)-axis at about \( x \leq -2.87 \) and \( x \geq 0.87 \). Therefore, the solution is approximately \( \{x \mid x \leq -2.87 \text{ or } x \geq 0.87\} \text{ or } (-\infty, -2.87) \cup [0.87, \infty) \).

Guided Practice

3. Solve each inequality by graphing.
   
   A. \( x^2 - 6x + 2 > 0 \)  
   B. \(-4x^2 + 5x + 7 \geq 0\)

Real-world problems can be solved by graphing quadratic inequalities.

Real-World Example 4 Solve a Quadratic Inequality

**WATER BALLOONS** Refer to the application at the beginning of the lesson. At what time will a water balloon be within 3 meters of the ground after it has been launched?

The function \( h(t) = -4.9t^2 + 32t + 1.2 \) describes the height of the water balloon. Therefore, you want to find the values of \( t \) for which \( h(t) \leq 3 \).

\[
\begin{align*}
h(t) &\leq 3 \\
-4.9t^2 + 32t + 1.2 &\leq 3 \\
-4.9t^2 + 32t - 1.8 &\leq 0
\end{align*}
\]

Graph the related function \( y = -4.9x^2 + 32x - 1.8 \) using a graphing calculator. The zeros of the function are about 0.06 and 6.47, and the graph lies below the \( x \)-axis when \( x < 0.06 \) and \( x > 6.47 \).

So, the water balloon is within 3 meters of the ground during the first 0.06 second after being launched and again after about 6.47 seconds until it hits the ground.

Guided Practice

4. **ROCKETS** The height \( h(t) \) of a model rocket in feet \( t \) seconds after its launch can be represented by the function \( h(t) = -16t^2 + 82t + 0.25 \). During what interval is the rocket at least 100 feet above the ground?
Example 5  Solve a Quadratic Inequality Algebraically

Solve \( x^2 - 3x \leq 18 \) algebraically.

**Step 1** Solve the related quadratic equation \( x^2 - 3x = 18 \).

\[
\begin{align*}
x^2 - 3x &= 18 \\
x^2 - 3x - 18 &= 0 \\
(x + 3)(x - 6) &= 0 \\
x + 3 &= 0 & \text{or} & & x - 6 &= 0 \\
x &= -3 & & x &= 6
\end{align*}
\]

**Related quadratic equation**

**Subtract 18 from each side.**

**Factor.**

**Zero Product Property**

**Solve each equation.**

**Step 2** Plot -3 and 6 on a number line. Use dots since these values are solutions of the original inequality. Notice that the number line is divided into three intervals.

\[
\begin{align*}
x &
\end{align*}
\]

**Step 3** Test a value from each interval to see if it satisfies the original inequality.

\[
\begin{align*}
x \leq -3 & & -3 \leq x \leq 6 & & x \geq 6 \\
\text{Test } x = -5. & & \text{Test } x = 0. & & \text{Test } x = 8. \\
x^2 - 3x &\leq 18 & x^2 - 3x &\leq 18 & x^2 - 3x &\leq 18 \\
(-5)^2 - 3(-5) &\leq 18 & (0)^2 - 3(0) &\leq 18 & (8)^2 - 3(8) &\leq 18 \\
40 &\leq 18 & 0 &\leq 18 & 40 &\not\leq 18
\end{align*}
\]

The solution set is \( \{ x \mid -3 \leq x \leq 6 \} \) or \([-3, 6]\).

Guided Practice

5. Solve each inequality algebraically.
   A. \( x^2 + 5x < -6 \)  
   B. \( x^2 + 11x + 30 \geq 0 \)

Check Your Understanding

Example 1  Graph each inequality.

1. \( y \leq x^2 - 8x + 2 \)  
2. \( y > x^2 + 6x - 2 \)  
3. \( y \geq -x^2 + 4x + 1 \)

Examples 2–3  Solve each inequality by graphing.

4. \( 0 < x^2 - 5x + 4 \)  
5. \( x^2 + 8x + 15 < 0 \)  
6. \( -2x^2 - 2x + 12 \geq 0 \)  
7. \( 0 \geq 2x^2 - 4x + 1 \)

Example 4  SOCCER  A midfielder kicks a ball toward the goal during a match. The height of the ball in feet above the ground \( h(t) \) at time \( t \) can be represented by \( h(t) = -0.1t^2 + 2.4t + 1.5 \). If the height of the goal is 8 feet, at what time during the kick will the ball be able to enter the goal?

Example 5  Solve each inequality algebraically.

9. \( x^2 + 6x - 16 < 0 \)  
10. \( x^2 - 14x > -49 \)  
11. \( -x^2 + 12x \geq 28 \)  
12. \( x^2 - 4x \leq 21 \)
Example 1  Graph each inequality.

13. \( y \geq x^2 + 5x + 6 \)  
14. \( x^2 - 2x - 8 < y \)  
15. \( y \leq -x^2 - 7x + 8 \)

16. \( -x^2 + 12x - 36 > y \)  
17. \( y > 2x^2 - 2x - 3 \)  
18. \( y \geq -4x^2 + 12x - 7 \)

Examples 2–3  Solve each inequality by graphing.

19. \( x^2 - 9x + 9 < 0 \)  
20. \( x^2 - 2x - 24 \leq 0 \)  
21. \( x^2 + 8x + 16 \geq 0 \)

22. \( x^2 + 6x + 3 > 0 \)  
23. \( 0 > -x^2 + 7x + 12 \)  
24. \( -x^2 + 2x - 15 < 0 \)

25. \( 4x^2 + 12x + 10 \leq 0 \)  
26. \( -3x^2 - 3x + 9 > 0 \)  
27. \( 0 > -2x^2 + 4x + 4 \)

28. \( 3x^2 + 12x + 36 \leq 0 \)  
29. \( 0 \leq -4x^2 + 8x + 5 \)  
30. \( -2x^2 + 3x + 3 \leq 0 \)

Example 4  ARCHITECTURE  An arched entry of a room is shaped like a parabola that can be represented by the equation \( f(x) = -x^2 + 6x + 1 \). How far from the sides of the arch is its height at least 7 feet?

32. MANUFACTURING  A box is formed by cutting 4-inch squares from each corner of a square piece of cardboard and then folding the sides. If \( V(x) = 4x^2 - 64x + 256 \) represents the volume of the box, what should the dimensions of the original piece of cardboard be if the volume of the box cannot exceed 750 cubic inches?

Example 5  Solve each inequality algebraically.

33. \( x^2 - 9x < -20 \)  
34. \( x^2 + 7x \geq -10 \)  
35. \( 2 > x^2 - x \)

36. \( -3 \leq -x^2 - 4x \)  
37. \( -x^2 + 2x \leq -10 \)  
38. \( -6 > x^2 + 4x \)

39. \( 2x^2 + 4 \geq 9 \)  
40. \( 3x^2 + x \geq -3 \)  
41. \( -4x^2 + 2x < 3 \)

42. \( -11 \geq -2x^2 - 5x \)  
43. \( -12 < -5x^2 - 10x \)  
44. \( -3x^2 - 10x > -1 \)

45. SWIMMING POOLS  The Sanchez family is adding a deck along two sides of their swimming pool. The deck width will be the same on both sides and the total area of the pool and deck cannot exceed 750 square feet.

a. Graph the quadratic inequality.

b. Determine the possible widths of the deck.

Write a quadratic inequality for each graph.

46.  
47.  
48.
Solve each quadratic inequality by using a graph, a table, or algebraically.

49. \(-2x^2 + 12x < -15\)  
50. \(5x^2 + x + 3 \geq 0\)  
51. \(11 \leq 4x^2 + 7x\)  
52. \(x^2 - 4x \leq -7\)  
53. \(-3x^2 + 10x < 5\)  
54. \(-1 \geq -x^2 - 5x\)

55. **BUSINESS** An electronics manufacturer uses the function \(P(x) = x(-27.5x + 3520) + 20,000\) to model their monthly profits when selling \(x\) thousand digital audio players.

   a. Graph the quadratic inequality for a monthly profit of at least $100,000.
   
   b. How many digital audio players must the manufacturer sell to earn a profit of at least $100,000 in a month?
   
   c. Suppose the manufacturer has an additional monthly expense of $25,000. Explain how this affects the graph of the profit function. Then determine how many digital audio players the manufacturer needs to sell to have at least $100,000 in profits.

56. **UTILITIES** A contractor is installing drain pipes for a shopping center’s parking lot. The outer diameter of the pipe is to be 10 inches. The cross sectional area of the pipe must be at least 35 square inches and should not be more than 42 square inches.

   a. Graph the quadratic inequalities.
   
   b. What thickness of drain pipe can the contractor use?

---

**H.O.T. Problems** Use Higher-Order Thinking Skills

57. **OPEN ENDED** Write a quadratic inequality for each condition.

   a. The solution set is all real numbers.
   
   b. The solution set is the empty set.

58. **ERROR ANALYSIS** Don and Diego used a graph to solve the quadratic inequality \(x^2 - 2x - 8 > 0\). Is either of them correct? Explain.

   ![Don and Diego graphs]

59. **REASONING** Are the boundaries of the solution set of \(x^2 + 4x - 12 \leq 0\) twice the value of the boundaries of \(\frac{1}{2}x^2 + 2x - 6 \leq 0\)? Explain.

60. **REASONING** Determine if the following statement is sometimes, always, or never true. Explain your reasoning.

   \[The \ intersection \ of \ y \leq -ax^2 + c \ and \ y \geq ax^2 - c \ is \ the \ empty \ set.\]

61. **CHALLENGE** Graph the intersection of the graphs of \(y \leq -x^2 + 4\) and \(y \geq x^2 - 4\).

62. **WRITING IN MATH** Compare and contrast graphing linear and quadratic inequalities.
63. GRIDDED RESPONSE You need to seed an area that is 80 feet by 40 feet. Each bag of seed can cover 25 square yards of land. How many bags of seed will you need?

64. SAT/ACT The product of two integers is between 107 and 116. Which of the following cannot be one of the integers?

A 5  
B 10  
C 12  
D 15  
E 23

65 PROBABILITY Five students are to be arranged side by side with the tallest student in the center and the two shortest students on the ends. If no two students are the same height, how many different arrangements are possible?

F 2  
G 4  
H 5  
J 6

66. SHORT RESPONSE Simplify \(\frac{5 + i}{6 - 3i}\).

Spiral Review

Write an equation in vertex form for each parabola. (Lesson 5-7)

67.  
68.  
69.

Complete parts a and b for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots. (Lesson 5-6)

70. \(4x^2 + 7x - 3 = 0\)  
71. \(-3x^2 + 2x - 4 = 9\)  
72. \(6x^2 + x - 4 = 12\)

73. GYMNASTICS Suppose the drawing is placed on a coordinate grid with the hand grips at H(0, 0) and the toe of the figure in the upper right corner at T(7, 8). Find the coordinates of the toes at the other three positions, if each successive position has been rotated 90° counterclockwise about the origin. (Lesson 4-4)

Perform the indicated operation. If the matrix does not exist, write impossible. (Lesson 4-2)

74. \[4 \begin{bmatrix} 3 & -6 \\ -5 & 2 \end{bmatrix} - 3 \begin{bmatrix} 4 & -1 \\ -2 & 8 \end{bmatrix}\]
75. \[-2 \begin{bmatrix} 5 & -9 \\ 5 & 11 \end{bmatrix} - 6 \begin{bmatrix} 3 & -7 \\ -5 & 8 \end{bmatrix}\]
76. \[2 \begin{bmatrix} -6 & 2 \\ -4 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 1 \\ 6 & 4 \end{bmatrix}\]

Skills Review

Use the Distributive Property to find each product. (Lesson 1-2)

77. \(-6(x - 4)\)  
78. \(8(w + 3x)\)  
79. \(-4(-2y + 3z)\)

80. \(-1(c - d)\)  
81. \(0.5(5x + 6y)\)  
82. \(-3(-6y - 4z)\)
Set Up the Lab

- Place a board on a stack of books to create a ramp.
- Connect the data collection device to the graphing calculator. Place at the top of the ramp so that the data collection device can read the motion of the car on the ramp.
- Hold the car still about 6 inches up from the bottom of the ramp and zero the collection device.

Activity

**Step 1** One group member should press the button to start collecting data.

**Step 2** Another group member places the car at the bottom of the ramp. After data collection begins, gently but quickly push the car so it travels up the ramp toward the motion detector.

**Step 3** Stop collecting data when the car returns to the bottom of the ramp. Save the data as Trial 1.

**Step 4** Remove one book from the stack. Then repeat the experiment. Save the data as Trial 2. For Trial 3, create a steeper ramp and repeat the experiment.

Analyze the Results

1. What type of function could be used to represent the data? Justify your answer.

2. Use the **CALC** menu to find the vertex of the graph. Record the coordinates in a table like the one at the right.

3. Use the **TRACE** feature of the calculator to find the coordinates of another point on the graph. Then use the coordinates of the vertex and the point to find an equation of the graph.

4. Find an equation for each of the graphs of Trials 2 and 3.

5. How do the equations for Trials 1, 2, and 3 compare? Which graph is widest and which is most narrow? Explain what this represents in the context of the situation. How is this represented in the equations?

6. What do the x-intercepts and vertex of each graph represent?

7. Why were the values of \( h \) and \( k \) different in each trial?
Study Guide

**Key Concepts**

Graphing Quadratic Functions (Lesson 5-1)
- The graph of \( y = ax^2 + bx + c, a \neq 0 \), opens up, and the function has a minimum value when \( a > 0 \). The graph opens down, and the function has a maximum value when \( a < 0 \).

Solving Quadratic Equations (Lessons 5-2 and 5-3)
- Roots of a quadratic equation are the zeros of the related quadratic function. You can find the zeros of a quadratic function by finding the x-intercepts of the graph.

Complex Numbers (Lesson 5-4)
- \( i \) is the imaginary unit; \( i^2 = -1 \).

Solving Quadratic Equations (Lessons 5-5 and 5-6)
- Completing the square: **Step 1** Find one half of \( b \), the coefficient of \( x \). **Step 2** Square the result in Step 1. **Step 3** Add the result of Step 2 to \( x^2 + bx \).
- Quadratic Formula: \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Transformations with Quadratic Functions (Lesson 5-7)
- The graph of \( y = (x - h)^2 + k \) is the graph of \( y = x^2 \) translated \( |h| \) units left if \( h \) is negative or \( |h| \) units right if \( h \) is positive and \( |k| \) units up if \( k \) is positive or \( |k| \) units down if \( k \) is negative.
- Consider \( y = a(x - h)^2 + k, a \neq 0 \). If \( a > 0 \), the graph opens up; if \( a < 0 \) the graph opens down. If \( |a| > 1 \), the graph is narrower than the graph of \( y = x^2 \). If \( |a| < 1 \), the graph is wider than the graph of \( y = x^2 \).

Quadratic Inequalities (Lesson 5-8)
- Graph the related function, test a point on the parabola and determine if it is a solution, and shade accordingly.

**Key Vocabulary**

- axis of symmetry (p. 250)
- complex conjugates (p. 279)
- complex number (p. 277)
- completing the square (p. 287)
- constant term (p. 249)
- discriminant (p. 297)
- factored form (p. 268)
- FOIL method (p. 268)
- imaginary unit (p. 276)
- linear term (p. 249)
- maximum value (p. 252)
- minimum value (p. 252)
- parabola (p. 249)
- pure imaginary number (p. 276)
- quadratic equation (p. 259)
- Quadratic Formula (p. 294)
- quadratic function (p. 249)
- quadratic inequality (p. 312)
- quadratic term (p. 249)
- root (p. 259)
- Square Root Property (p. 277)
- standard form (p. 259)
- vertex (p. 250)
- vertex form (p. 305)
- zero (p. 259)

**Vocabulary Check**

State whether each sentence is **true** or **false**. If **false**, replace the underlined term to make a true sentence.

1. The factored form of a quadratic equation is \( ax^2 + bx + c = 0 \), where \( a \neq 0 \) and \( a, b, \) and \( c \) are integers.

2. The graph of a quadratic function is called a **parabola**.

3. The **vertex** form of a quadratic function is \( y = a(x - p)(x - q) \).

4. The axis of symmetry will intersect a parabola in one point called the **vertex**.

5. A method called **FOIL method** is used to make a quadratic expression a perfect square in order to solve the related equation.

6. The equation \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \) is known as the **discriminant**.

7. The number \( 6i \) is called a **pure imaginary number**.

8. The two numbers \( 2 + 3i \) and \( 2 - 3i \) are called **complex conjugates**.
Lesson-by-Lesson Review

5-1 Graphing Quadratic Functions (pp. 249–257)

5-2 Solving Quadratic Functions by Graphing (pp. 259–266)

5-1 Graphing Quadratic Functions (pp. 249–257)

Complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

9. \( f(x) = x^2 + 5x + 12 \)  
10. \( f(x) = x^2 - 7x + 15 \)  
11. \( f(x) = -2x^2 + 9x - 5 \)  
12. \( f(x) = -3x^2 + 12x - 1 \)

Determine whether each function has a maximum or minimum value and find the maximum or minimum value. Then state the domain and range of the function.

13. \( f(x) = -x^2 + 3x - 1 \)  
14. \( f(x) = -3x^2 - 4x + 5 \)

15. BUSINESS Sal’s Shirt Store sells 100 T-shirts per week at a rate of $10 per shirt. Sal estimates that he will sell 5 less shirts for each $1 increase in price. What price will maximize Sal’s T-shirt income?

Example 1

Consider the quadratic function \( f(x) = x^2 - 4x + 11 \). Find the y-intercept, the equation for the axis of symmetry, and the x-coordinate of the vertex.

In the function, \( a = 1 \), \( b = -4 \), and \( c = 11 \). The y-intercept is \( c = 11 \).

Use \( a \) and \( b \) to find the equation of the axis of symmetry.

\[
\begin{align*}
\frac{x}{2} &= \frac{-b}{2a} \\
&= \frac{4}{2(1)} \\
&= 2
\end{align*}
\]

The equation of the axis of symmetry is \( x = 2 \). Therefore, the x-coordinate of the vertex is 2.

Example 2

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

16. \( x^2 - x - 20 = 0 \)  
17. \( 2x^2 - x - 3 = 0 \)  
18. \( 4x^2 - 6x - 15 = 0 \)

19. BASEBALL A baseball is hit upward at 120 feet per second. Use the formula \( h(t) = v_0t - \frac{1}{2}gt^2 \), where \( h(t) \) is the height of an object in feet, \( v_0 \) is the object’s initial velocity in feet per second, and \( t \) is the time in seconds. Ignoring the height of the ball when it was hit, how long does it take for the ball to hit the ground?

Solve \( 2x^2 - 7x + 3 = 0 \) by graphing.

The equation of the axis of symmetry is \( x = \frac{-7}{2} \) or \( x = \frac{7}{4} \).

The zeros of the related function are \( \frac{1}{2} \) and 3. Therefore, the solutions of the equation are \( \frac{1}{2} \) and 3.
5–3 Solving Quadratic Equations by Factoring (pp. 268–275)

Write a quadratic equation in standard form with the given roots.

20. 5, 6  
21. -3, -7  
22. -4, 2  
23. -2/3, 1  
24. 1/6, 5  
25. -1/4, -1

Solve each equation by factoring.

26. 2x^2 - 2x - 24 = 0
27. 2x^2 - 5x - 3 = 0
28. 3x^2 - 16x + 5 = 0
29. Find x and the dimensions of the rectangle below.

\[
A = 126 \text{ ft}^2
\]

\[
x + 2
\]

\[
x - 3
\]

Example 3

Write a quadratic equation in standard form with \(-\frac{1}{2}\) and 4 as its roots.

\[
(x - p)(x - q) = 0
\]

Replace \(p\) with \(-\frac{1}{2}\) and \(q\) with 4.

\[
\left[x - \left(-\frac{1}{2}\right)\right](x - 4) = 0
\]

Simplify.

\[
(x + \frac{1}{2})(x - 4) = 0
\]

Multiply.

\[
x^2 - \frac{7}{2}x - 2 = 0
\]

\[
2x^2 - 7x - 4 = 0
\]

Multiply each side by 2 so that \(b\) and \(c\) are integers.

Example 4

Solve \(2x^2 - 3x - 5 = 0\) by factoring.

\[
2x^2 - 3x - 5 = 0
\]

Original equation

\[
(2x - 5)(x + 1) = 0
\]

Factor the trinomial.

\[
2x - 5 = 0 \quad \text{or} \quad x + 1 = 0
\]

Zero Product Property

\[
x = \frac{5}{2} \quad \text{or} \quad x = -1
\]

The solution set is \(\left\{-1, \frac{5}{2}\right\}\) or \(\left\{x \mid x = -1, \frac{5}{2}\right\}\).

5–4 Complex Numbers (pp. 276–282)

Simplify.

30. \(\sqrt{-8}\)  
31. \((2 - i) + (13 + 4i)\)  
32. \((6 + 2i) - (4 - 3i)\)  
33. \((6 + 5i)(3 - 2i)\)

34. ELECTRICITY The impedance in one part of a series circuit is \(3 + 2i\) ohms, and the impedance in the other part of the circuit is \(4 - 3i\) ohms. Add these complex numbers to find the total impedance in the circuit.

Solve each equation.

35. \(2x^2 + 50 = 0\)
36. \(4x^2 + 16 = 0\)
37. \(3x^2 + 15 = 0\)
38. \(8x^2 + 16 = 0\)
39. \(4x^2 + 1 = 0\)

Example 5

Simplify \((12 + 3i) - (-5 + 2i)\).

\[
(12 + 3i) - (-5 + 2i)
\]

Group the real and imaginary parts.

\[
= [12 - (-5)] + (3 - 2i)
\]

Add.

\[
= 17 + i
\]

Example 6

Solve \(3x^2 + 12 = 0\).

\[
3x^2 + 12 = 0
\]

Original equation

\[
3x^2 = -12
\]

Subtract 12 from each side.

\[
x^2 = -4
\]

Divide each side by 3.

\[
x = \pm \sqrt{-4}
\]

Square Root Property

\[
x = \pm 2i
\]

\[
\sqrt{-4} = \sqrt{4} \cdot \sqrt{-1}
\]
5-5 Completing the Square (pp. 286–292)

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

40. \(x^2 + 18x + c\)  
41. \(x^2 - 4x + c\)  
42. \(x^2 - 7x + c\)  
43. \(x^2 + 2.4x + c\)  
44. \(x^2 - \frac{1}{2}x + c\)  
45. \(x^2 + \frac{6}{5}x + c\)

Solve each equation by completing the square.

46. \(x^2 - 6x - 7 = 0\)
47. \(x^2 - 2x + 8 = 0\)
48. \(2x^2 + 4x - 3 = 0\)
49. \(2x^2 + 3x - 5 = 0\)
50. **FLOOR PLAN** Mario’s living room has a length 6 feet wider than the width. The area of the living room is 280 square feet. What are the dimensions of his living room?

Example 7

Find the value of c that makes \(x^2 + 14x + c\) a perfect square. Then write the trinomial as a perfect square.

**Step 1** Find one half of 14.

**Step 2** Square the result of Step 1.

**Step 3** Add the result of Step 2 to \(x^2 + 14x\).

The trinomial \(x^2 + 14x + 49\) can be written as \((x + 7)^2\).

Example 8

Solve \(x^2 + 12x - 13 = 0\) by completing the square.

\[x^2 + 12x - 13 = 0\]
\[x^2 + 12x = 13\]
\[(x + 6)^2 = 49\]
\[x + 6 = \pm 7\]
\[x + 6 = 7 \quad \text{or} \quad x + 6 = -7\]
\[x = 1 \quad \text{or} \quad x = -13\]

The solution set is \{-13, 1\} or \(\{x \mid x = -13, 1\}\).

5-6 The Quadratic Formula and the Discriminant (pp. 294–302)

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

51. \(x^2 - 10x + 25 = 0\)
52. \(x^2 + 4x - 32 = 0\)
53. \(2x^2 + 3x - 18 = 0\)
54. \(2x^2 + 19x - 33 = 0\)
55. \(x^2 - 2x + 9 = 0\)
56. \(4x^2 - 4x + 1 = 0\)
57. \(2x^2 + 5x + 9 = 0\)
58. **PHYSICAL SCIENCE** Lauren throws a ball with an initial velocity of 40 feet per second. The equation for the height of the ball is \(h = -16t^2 + 40t + 5\), where \(h\) represents the height in feet and \(t\) represents the time in seconds. When will the ball hit the ground?

Example 9

Solve \(x^2 - 4x - 45 = 0\) by using the Quadratic Formula.

In \(x^2 - 4x - 45 = 0\), \(a = 1\), \(b = -4\), and \(c = -45\).

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]
\[= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-45)}}{2(1)}\]
\[= \frac{4 \pm 14}{2}\]

Write as two equations.

\[x = \frac{4 + 14}{2} \quad \text{or} \quad x = \frac{4 - 14}{2}\]
\[= 9 \quad \text{or} \quad x = -5\]

The solution set is \{-5, 9\} or \(\{x \mid x = -5, 9\}\).
**5-7 Transformations with Quadratic Functions** (pp. 305–310)

Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening. Then graph the function.

69. \( y = -3(x - 1)^2 + 5 \)  
70. \( y = 2x^2 + 12x - 8 \)
71. \( y = \frac{1}{2}x^2 - 2x + 12 \)  
72. \( y = 3x^2 + 36x + 25 \)
73. The graph at the right shows a product of two numbers with a sum of 10. Find a function that models this product and use it to determine the two numbers that would give a maximum product.

**Example 10**

Write the quadratic function \( y = 3x^2 + 24x + 15 \) in vertex form. Then identify the vertex, axis of symmetry, and direction of opening.

- Original equation: \( y = 3x^2 + 24x + 15 \)
- Group and factor: \( y = 3(x^2 + 8x) + 15 \)
- Complete the square: \( y = 3(x + 4)^2 - 33 \)

So, \( a = 3 \), \( h = -4 \), and \( k = -33 \). The vertex is at \((-4, -33)\) and the axis of symmetry is \( x = -4 \). Since \( a \) is positive, the graph opens up.

**5-8 Quadratic Inequalities** (pp. 312–318)

Graph each quadratic inequality.

64. \( y \geq x^2 + 5x + 4 \)  
65. \( y < -x^2 + 5x - 6 \)
66. \( y > x^2 - 6x + 8 \)  
67. \( y \leq x^2 + 10x - 4 \)
68. Solomon wants to put a deck along two sides of his garden. The deck width will be the same on both sides and the total area of the garden and deck cannot exceed 500 square feet. How wide can the deck be?

Solve each inequality using a graph or algebraically.

69. \( x^2 + 8x + 12 > 0 \)
70. \( 6x + x^2 \geq -9 \)
71. \( 2x^2 + 3x - 20 > 0 \)
72. \( 4x^2 - 3 < -5x \)
73. \( 3x^2 + 4 > 8x \)

**Example 11**

Graph \( y > x^2 + 3x + 2 \).

**Step 1** Graph the related function, \( y > x^2 + 3x + 2 \). Because the inequality symbol \( > \) is used, the parabola should be dashed.

**Step 2** Test a point not on the graph of the parabola such as \((0, 0)\).

- \( y > x^2 + 3x + 2 \)
- \((0) > (0)^2 + 3(0) + 2 \)
- \(0 \geq 2 \)

So, \((0, 0)\) is not a solution of the inequality.

**Step 3** Shade the region that does not contain the point \((0, 0)\).
Complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

1. \( f(x) = x^2 + 4x - 7 \)
2. \( f(x) = -2x^2 + 5x \)
3. \( f(x) = -x^2 - 6x - 9 \)

Determine whether each function has a maximum or minimum value. State the maximum or minimum value of each function.

4. \( f(x) = x^2 + 10x + 25 \)
5. \( f(x) = -x^2 + 6x \)

Solve each equation using the method of your choice. Find exact solutions.

6. \( x^2 - 8x - 9 = 0 \)
7. \( -4.8x^2 + 1.6x + 24 = 0 \)
8. \( 12x^2 + 15x - 4 = 0 \)
9. \( x^2 - 7x - \frac{17}{4} = 0 \)
10. \( 4x^2 + x = 3 \)
11. \( -9x^2 + 40x + 84 = 0 \)

12. **PHYSICAL SCIENCE** Parker throws a ball off the top of a building. The building is 350 feet high and the initial velocity of the ball is 30 feet per second. Find out how long it will take the ball to hit the ground by solving the equation \(-16t^2 - 30t + 350 = 0\).

13. **MULTIPLE CHOICE** Which equation below has roots at \(-6\) and \(\frac{5}{7}\)?
   - A 0 = \(5x^2 - 29x - 6\)
   - B 0 = \(5x^2 + 31x + 6\)
   - C 0 = \(5x^2 + 29x - 6\)
   - D 0 = \(5x^2 - 31x + 6\)

14. **PHYSICS** A ball is thrown into the air vertically with a velocity of 112 feet per second. The ball was released 6 feet above the ground. The height above the ground \(t\) seconds after release is modeled by \(h(t) = -16t^2 + 112t + 6\).
   a. When will the ball reach 130 feet?
   b. Will the ball ever reach 250 feet? Explain.
   c. In how many seconds after its release will the ball hit the ground?

15. The rectangle below has an area of 104 square inches. Find the value of \(x\) and the dimensions of the rectangle.

![Rectangle](image)

16. \((3 - 4i) - (9 - 5i)\)
17. \(\frac{-4i}{4 - i}\)

18. **MULTIPLE CHOICE** Which value of \(c\) makes the trinomial \(x^2 - 12x + c\) a perfect square trinomial?
   - F 6
   - G 12
   - H 36
   - J 144

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

c. Find the exact solution by using the Quadratic Formula.

19. \(6x^2 + 7x = 0\)
20. \(5x^2 = -6x + 1\)
21. \(2x^2 + 5x - 8 = -13\)

Write each quadratic function in vertex form. Then identify the vertex, axis of symmetry, and direction of opening.

22. \(3x^2 + 6x = 2 + y\)
23. \(x^2 + 9x + \frac{81}{4} = y\)
24. Graph the quadratic inequality \(0 < -3x^2 + 4x + 10\).

Solve each inequality by using a graph or algebraically.

25. \(x^2 + 6x > -5\)
26. \(4x^2 - 19x \leq -12\)
Use a Graph

Using a graph can help you solve many different kinds of problems on standardized tests. Graphs can help you solve equations, evaluate functions, and interpret solutions to real-world problems.

Strategies for Using a Graph

**Step 1**
Read the problem statement carefully.

Ask yourself:
- What am I being asked to solve?
- What information is given in the problem?
- How could a graph help me solve the problem?

**Step 2**
Create your graph.

- Sketch your graph on scrap paper if appropriate.
- If allowed, you can also use a graphing calculator to create the graph.

**Step 3**
Solve the problem.

- Use your graph to help you model and solve the problem.
- Check to be sure your answer makes sense.

Test Practice Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

The students in Mr. Himebaugh’s physics class built a model rocket. The rocket is launched in a large field with an initial upward velocity of 128 feet per second. The function \( h(t) = -16t^2 + 128t \) models the height of the rocket above the ground (in feet) \( t \) seconds after it is launched. How long will it take for the rocket to reach its maximum height?

A 4 seconds  
B 5 seconds  
C 6 seconds  
D 8 seconds
Graphing the quadratic function will allow you to determine the peak height of the rocket and when it occurs. A graphing calculator can help you quickly graph the function and analyze it.

**KEYSTROKES:** \[ Y = (-) 16 x, t, \theta , n x^2 + 128 x, t, \theta , n \text{ GRAPH} \]

After graphing the equation, use maximum under the **CALC** menu.

Press \[ 2 \text{nd} [\text{CALC}] \text{ 4} \]. Then use \[ \leftarrow \text{ to place the cursor to the left of the maximum point and press [ENTER]. Use } \rightarrow \text{ to place the cursor to the right of the maximum point and press [ENTER ENTER].} \]

The graph shows that the rocket takes 4 seconds to reach its maximum height of 256 feet. The correct answer is A.

### Exercises

**Read each problem. Identify what you need to know. Then use the information in the problem to solve.**

1. What are the roots of \( y = 2x^2 + 10x - 48 \)?
   - A -5, 4
   - B -6, 1
   - C -8, 3
   - D 2, 3

2. How many times does the graph of \( f(x) = 2x^2 - 3x + 2 \) cross the \( x \)-axis?
   - F 0
   - G 1
   - H 2
   - J 3

3. Which statement best describes the graphs of the two equations?
   \[
   \begin{align*}
   16x - 2y &= 24 \\
   12x &= 3y - 36 \\
   \end{align*}
   \]
   - A The lines are parallel.
   - B The lines are the same.
   - C The lines intersect in only one point.
   - D The lines intersect in more than one point, but are not the same.

4. Adrian is using 120 feet of fencing to enclose a rectangular area for her puppy. One side of the enclosure will be her house.

   \[
   \begin{array}{c}
   \text{House} \\
   \hline
   \text{(120 - 2x) ft} \\
   \hline
   \text{x ft} \\
   \text{x ft}
   \end{array}
   \]

   The function \( f(x) = x(120 - 2x) \) represents the area of the enclosure. What is the greatest area Adrian can enclose with the fencing?
   - F 1650 ft²
   - G 1800 ft²
   - H 1980 ft²
   - J 2140 ft²

5. For which equation is the \( x \)-coordinate of the vertex at 4?
   - A \( f(x) = x^2 - 8x + 15 \)
   - B \( f(x) = -x^2 - 4x + 12 \)
   - C \( f(x) = x^2 + 6x + 8 \)
   - D \( f(x) = -x^2 - 2x + 2 \)

6. For what value of \( x \) does \( f(x) = x^2 + 5x + 6 \) reach its minimum value?
   - F -5
   - G -3
   - H \(-\frac{5}{2}\)
   - J -2
Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. A rental store charges $32 per day to rent a bicycle. At this rate, the store rents about 200 bikes per month. The owner of the store estimates that they will rent 5 fewer bikes per month for each $2 increase in the rental price. What price will maximize the income of the store?
   
   A $46   C $55
   B $51   D $56

2. What are the solutions of the quadratic equation graphed below?

   \[ y = f(x) \]

   F $-4, -1$   H $-1, 4$
   G $1, 4$   J $-4, 1$

3. Peyton works as a nanny. She charges at least $10 to drive to a home and $10.50 per hour. Which inequality best represents the relationship between the number of hours working \( n \) and the total charge \( c \)?
   
   A \( c \geq 10 + 10.50n \)
   B \( c \geq 10.50 + 10n \)
   C \( c \leq 10.50 + 10 \)
   D \( c \leq 10n + 10.50n \)

4. Leo sells T-shirts at a local swim meet. It costs him $250 to set up the stand and rent the machine. It costs him an additional $5 to make each T-shirt. If he sells each T-shirt for $15, how many T-shirts does he have to sell before he can make a profit?
   
   F 10   H 25
   G 15   J 50

5. Solve \( x^2 - 2x = 15 \) by completing the square.
   
   A $-4, -1$   C $-2, 3$
   B $-3, 5$   D $5, 7$

6. The graph of \( g(x) = \frac{2}{3}x^2 - 4x + 2 \) is translated down 5 units to produce the graph of the function \( h(x) \). Which of the following could be \( h(x) \)?
   
   F \( h(x) = \frac{2}{3}x^2 - 4x + 7 \)
   G \( h(x) = \frac{2}{3}x^2 - 4x - 3 \)
   H \( h(x) = \frac{2}{3}x^2 - 9x + 2 \)
   J \( h(x) = \frac{2}{3}x^2 + x + 2 \)

7. Triangle \( ABC \) has vertices with coordinates \( A(-4, 2), B(-4, -3), \) and \( C(3, -2) \). After a dilation, triangle \( A'B'C' \) has coordinates \( A'(-12, 6), B'(-12, -9), \) and \( C'(9, -6) \). How many times as great is the perimeter of \( \triangle A'B'C' \) as that of \( \triangle ABC \)?
   
   A 3   C 12
   B 6   D \( \frac{1}{3} \)

8. The function \( P(t) = -0.068t^2 + 7.85t + 56 \) can be used to approximate the population, in thousands, of Clarksville between 1960 and 2000. The domain \( t \) of the function is the number of years since 1960. According to the model, in what year did the population of Clarksville reach 200,000 people?
   
   F 1974   H 1981
   G 1977   J 1983
Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. GRATDED RESPONSE What is the $y$-coordinate of the solution of the system of equations below?

$$
y = 4x - 7
$$
$$
y = -\frac{1}{2}x + 2
$$

10. GRииDED RESPONSE Simplify $-2i \cdot 5i$.

11. Use the quadratic function below to answer each question.

$$
y = x^2 + kx - 12
$$

a. How many real roots does the quadratic function have?

b. How many complex roots does the function have?

c. What do you know about the discriminant of the quadratic equation? Explain.

12. If one of the roots of $x^2 + kx - 12 = 0$ is 4, what is the value of $k$?

13. Describe the translation of the graph of $y = (x + 5)^2 - 1$ to the graph of $y = (x - 1)^2 + 3$.

Extended Response

Record your answers on a sheet of paper. Show your work.

14. For a given quadratic equation $y = ax^2 + bx + c$, describe what the discriminant $b^2 - 4ac$ tells you about the roots of the equation.

15. Craig is checking in a shipment of ink jet printers that cost $200 each and laser printers that cost $500 each. There are 40 boxes in the shipment, and the invoice total is $11,600.

a. Write a system of equations to model the situation. Let $x$ represent the number of ink jet printers, and let $y$ represent the number of laser printers.

b. Write a matrix equation that can be used to solve the system of equations you wrote in part a.

c. Find the inverse of the coefficient matrix and solve the matrix equation. How many ink jet printers and laser printers were included in the shipment?

16. If Robert kicks a football straight up into the air with an initial velocity of 100 feet per second, the function $h(t) = -16t^2 + 100t$ gives the height, in feet, of the ball after $t$ seconds.

a. Graph the quadratic function on a coordinate grid.

b. What is the maximum height reached by the football? Round to the nearest foot.

c. How long is the football in the air before it hits the ground?