In Chapter 6, you simplified polynomial expressions.

In Chapter 7, you will:
- Find compositions and inverses of functions.
- Graph and analyze square root functions and inequalities.
- Simplify and solve equations involving roots, radicals, and rational exponents.

FINANCE Connecting finances to mathematics is a skill that, once mastered, you will use your entire life. Learning to manage your finances entails creating a budget and living within that budget. In this chapter, you will explore financial topics such as saving for college, income, profit, inflation, and converting money when traveling.
Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

Use the related graph of each equation to determine its roots. If exact roots cannot be found, state the consecutive integers between which the roots are located. (Lesson 5-2)

1. \( x^2 - 4x + 1 = 0 \)
2. \( 2x^2 + x - 6 = 0 \)
3. PHYSICS Allie drops a ball from the top of a 30-foot building. How long does it take for the ball to reach the ground, assuming there is no air resistance? Use the formula \( h(t) = -16t^2 + h_0 \), where \( t \) is the time in seconds and the initial height \( h_0 \) is in feet.

Example 1

Use the related graph of \( 0 = 3x^2 - 4x + 1 \) to determine its roots. If exact roots cannot be found, state the consecutive integers between which the roots are located.

The roots are the \( x \)-coordinates where the graph crosses the \( x \)-axis.

The graph crosses the \( x \)-axis between 0 and 1 and at 1.

Example 2

Simplify \((3x^4 + 4x^3 + x^2 + 9x - 6) \div (x + 2)\) by using synthetic division.

\[ x - r = x + 2, \text{ so } r = -2. \]

\[ \begin{array}{c|ccccc}
  -2 & 3 & 4 & 1 & 9 & -6 \\
  \hline
      & -6 & 4 & -10 & 2 \\
  \hline
      & 3  & -2 & 5  & -1 & -4
\end{array} \]

The result is \(3x^3 - 2x^2 + 5x - 1 - \frac{4}{x+2}\).

2 Online Option Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.
You will learn several new concepts, skills, and vocabulary terms as you study Chapter 7. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

**New Vocabulary**

<table>
<thead>
<tr>
<th>English</th>
<th>Español</th>
</tr>
</thead>
<tbody>
<tr>
<td>composition of functions</td>
<td>composición de funciones</td>
</tr>
<tr>
<td>inverse relations</td>
<td>relaciones inversas</td>
</tr>
<tr>
<td>inverse function</td>
<td>función inversa</td>
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<tr>
<td>square root function</td>
<td>función raíz cuadrada</td>
</tr>
<tr>
<td>radical function</td>
<td>función radical</td>
</tr>
<tr>
<td>square root inequality</td>
<td>desigualdad raíz cuadrada</td>
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<tr>
<td>( n )th root</td>
<td>raíz enésima</td>
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<tr>
<td>principal root</td>
<td>raíz principal</td>
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<td>radical sign</td>
<td>signo radical</td>
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<td>index</td>
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<td>rationalizing the</td>
<td>racionalizar el</td>
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<td>denominator</td>
<td>denominador</td>
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<td>conjugates</td>
<td>conjugados</td>
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<td>radical equation</td>
<td>ecuación radical</td>
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<tr>
<td>extraneous solution</td>
<td>solución extraña</td>
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<tr>
<td>radical inequality</td>
<td>desigualdad radical</td>
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</tbody>
</table>

**Review Vocabulary**

<table>
<thead>
<tr>
<th>English</th>
<th>Español</th>
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</thead>
<tbody>
<tr>
<td>absolute value</td>
<td>valor absoluto</td>
</tr>
<tr>
<td>rational number</td>
<td>número racional</td>
</tr>
<tr>
<td>relation</td>
<td>relación</td>
</tr>
</tbody>
</table>

**English**

**Spanish**

- A number's distance from zero on the number line, represented by \(| x | \).
- Any number \( \frac{m}{n} \), where \( m \) and \( n \) are integers and \( n \) is not zero; the decimal form is either a terminating or repeating decimal.
**Operations on Functions**

**Then**
- You performed operations on polynomials. (Lesson 6-1)

**Now**
1. Find the sum, difference, product, and quotient of functions.
2. Find the composition of functions.

**Why?**
- The graphs model the income for the Brooks family since 2000, where \( m(x) \) represents Mr. Brooks’ income and \( f(x) \) represents Mrs. Brooks’ income. The total household income for the Brooks household can be represented by \( f(x) + m(x) \).

**Arithmetic Operations** In Chapter 6, you performed arithmetic operations with polynomials. You can also use addition, subtraction, multiplication, and division with functions.

You can perform arithmetic operations according to the following rules.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>((f + g)(x) = f(x) + g(x))</td>
<td>(2x + (\text{(-x + 5)}) = \text{x + 5})</td>
</tr>
<tr>
<td>Subtraction</td>
<td>((f - g)(x) = f(x) - g(x))</td>
<td>(2x - (\text{(-x + 5)}) = 3x - 5)</td>
</tr>
<tr>
<td>Multiplication</td>
<td>((f \cdot g)(x) = f(x) \cdot g(x))</td>
<td>(2x(\text{(-x + 5)}) = -2x^2 + 10x)</td>
</tr>
<tr>
<td>Division</td>
<td>((\frac{f}{g})(x) = \frac{f(x)}{g(x)}, g(x) \neq 0)</td>
<td>(\frac{2x}{\text{-x + 5}}, \text{x} \neq 5)</td>
</tr>
</tbody>
</table>

**Example 1 Add and Subtract Functions**

Given \(f(x) = x^2 - 4\) and \(g(x) = 2x + 1\), find each function.

a. \((f + g)(x)\)

\[(f + g)(x) = f(x) + g(x)\]

\[= (x^2 - 4) + (2x + 1)\]

\[= x^2 + 2x - 3\]

Addition of functions

\(f(x) = x^2 - 4\) and \(g(x) = 2x + 1\)

Simplify.

b. \((f - g)(x)\)

\[(f - g)(x) = f(x) - g(x)\]

\[= (x^2 - 4) - (2x + 1)\]

\[= x^2 - 2x - 5\]

Subtraction of functions

\(f(x) = x^2 - 4\) and \(g(x) = 2x + 1\)

Simplify.

**Guided Practice**

Given \(f(x) = x^2 + 5x - 2\) and \(g(x) = 3x - 2\), find each function.

1A. \((f + g)(x)\)

1B. \((f - g)(x)\)
You can graph sum and difference functions by graphing each function involved separately, then adding their corresponding functional values. Let \( f(x) = x^2 \) and \( g(x) = x \). Examine the graphs of \( f(x) \), \( g(x) \), and their sum and difference.

Find \((f + g)(x)\).

\[
\begin{array}{|c|c|c|c|}
\hline
x & f(x) = x^2 & g(x) = x & (f + g)(x) = x^2 + x \\
\hline
-3 & 9 & -3 & 9 + (-3) = 6 \\
-2 & 4 & -2 & 4 + (-2) = 2 \\
-1 & 1 & -1 & 1 + (-1) = 0 \\
0 & 0 & 0 & 0 + 0 = 0 \\
1 & 1 & 1 & 1 + 1 = 2 \\
2 & 4 & 2 & 4 + 2 = 6 \\
3 & 9 & 3 & 9 + 3 = 12 \\
\hline
\end{array}
\]

Find \((f - g)(x)\).

\[
\begin{array}{|c|c|c|c|}
\hline
x & f(x) = x^2 & g(x) = x & (f - g)(x) = x^2 - x \\
\hline
-3 & 9 & -3 & 9 - (-3) = 12 \\
-2 & 4 & -2 & 4 - (-2) = 6 \\
-1 & 1 & -1 & 1 - (-1) = 2 \\
0 & 0 & 0 & 0 - 0 = 0 \\
1 & 1 & 1 & 1 - 1 = 0 \\
2 & 4 & 2 & 4 - 2 = 2 \\
3 & 9 & 3 & 9 - 3 = 6 \\
\hline
\end{array}
\]

In Example 1, the functions \( f(x) \) and \( g(x) \) have the same domain of all real numbers. The functions \((f + g)(x)\) and \((f - g)(x)\) also have domains that include all real numbers. For each new function, the domain consists of the intersection of the domains of \( f(x) \) and \( g(x) \). Under division, the domain of the new function is restricted by excluded values that cause the denominator to equal zero.

**Example 2 Multiply and Divide Functions**

Given \( f(x) = x^2 + 7x + 12 \) and \( g(x) = 3x - 4 \), find each function.

a. \((f \cdot g)(x)\)

\[
(f \cdot g)(x) = f(x) \cdot g(x) \\
= (x^2 + 7x + 12)(3x - 4) \\
= 3x^3 + 21x^2 + 36x - 4x^2 - 28x - 48 \\
= 3x^3 + 17x^2 + 8x - 48
\]

b. \(\left(\frac{f}{g}\right)(x)\)

\[
\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \\
= \frac{x^2 + 7x + 12}{3x - 4} \quad x \neq \frac{4}{3} \quad \text{Substitution}
\]

Because \( x = \frac{4}{3} \) makes the denominator \( 3x - 4 = 0 \), \( \frac{4}{3} \) is excluded from the domain of \( \left(\frac{f}{g}\right)(x) \).

**Guided Practice**

Given \( f(x) = x^2 - 7x + 2 \) and \( g(x) = x + 4 \), find each function.

2A. \((f \cdot g)(x)\)  
2B. \(\left(\frac{f}{g}\right)(x)\)
Composition of Functions

Another method used to combine functions is a composition of functions. In a composition of functions, the results of one function are used to evaluate a second function.

Key Concept Composition of Functions

Words Suppose \( f \) and \( g \) are functions such that the range of \( g \) is a subset of the domain of \( f \). Then the composition function \( f \circ g \) can be described by

\[
[f \circ g](x) = f(g(x)).
\]

Model

The composition of two functions may not exist. Given two functions \( f \) and \( g \), \( [f \circ g](x) \) is defined only if the range of \( g(x) \) is a subset of the domain of \( f \). Likewise, \( [g \circ f](x) \) is defined only if the range of \( f(x) \) is a subset of the domain of \( g \).

Example 3 Compose Functions

For each pair of functions, find \( [f \circ g](x) \) and \( [g \circ f](x) \), if they exist.

a. \( f = \{1, 8, 0, 13\}, \ (15, 11\}, \ (14, 9)\}, \ g = \{8, 15\}, \ (5, 1)\}, \ (10, 14\), \ (9, 0)\}

To find \( f \circ g \), evaluate \( g(x) \) first. Then use the range as the domain of \( f \) and evaluate \( f(x) \).

\[
\begin{align*}
[f(g(8))] = f(15) & \quad or \quad 11 & \quad g(8) = 15 \\
[f(g(5))] = f(1) & \quad or \quad 8 & \quad g(5) = 1 \\
f \circ g = \{(8, 11), (5, 1), (10, 9), (9, 13)\}
\end{align*}
\]

To find \( g \circ f \), evaluate \( f(x) \) first. Then use the range as the domain of \( g \) and evaluate \( g(x) \).

\[
\begin{align*}
g(f(1)) = g(8) & \quad or \quad 15 & \quad f(1) = 8 \\
g(f(0)) = g(13) & \quad g(13) \text{ is undefined.} & \quad g(11) \text{ is undefined.} \\
g(f(15)) = g(11) & \quad \text{11 is undefined.} & \quad g(14) = g(9) & \quad or \quad 0 & \quad f(14) = 0
\end{align*}
\]

Because 11 and 13 are not in the domain of \( g \), \( g \circ f \) is undefined for \( x = 11 \) and \( x = 13 \). However, \( g(f(1)) = 15 \) and \( g(f(14)) = 0 \), so \( g \circ f = \{(1, 15), (14, 0)\} \).

b. \( f(x) = 2a - 5 \), \( g(x) = 4a \)

\[
\begin{align*}
[f \circ g](x) = f[g(x)] & \quad \text{Composition of functions} & \quad [g \circ f](x) = g[f(x)] \\
= f(4a) & \quad \text{Substitute.} & \quad = g(2a - 5) \\
= 2(4a) - 5 & \quad \text{Substitute again.} & \quad = 4(2a - 5) \\
= 8a - 5 & \quad \text{Simplify.} & \quad = 8a - 20
\end{align*}
\]

Guided Practice

3A. \( f(x) = \{(3, -2), (-1, -5), (4, 7), (10, 8)\} \), \( g(x) = \{(4, 3), (2, -1), (9, 4), (3, 10)\} \)

3B. \( f(x) = x^2 + 2 \) and \( g(x) = x - 6 \)
Notice that in most cases, \( f \circ g \neq g \circ f \). Therefore, the order in which two functions are composed is important.

### Real-World Example 4: Use Composition of Functions

**SHOPPING** A new car dealer is discounting all new cars by 12%. At the same time, the manufacturer is offering a $1500 rebate on all new cars. Mr. Navarro is buying a car that is priced $24,500. Will the final price be lower if the discount is applied before the rebate or if the rebate is applied before the discount?

**Understand** Let \( x \) represent the original price of a new car, \( d(x) \) represent the price of a car after the discount, and \( r(x) \) the price of the car after the rebate.

**Plan** Write equations for \( d(x) \) and \( r(x) \).

- The original price is discounted by 12%.
  \[ d(x) = x - 0.12x \]
- There is a $1500 rebate on all new cars.
  \[ r(x) = x - 1500 \]

**Solve** If the discount is applied before the rebate, then the final price of Mr. Navarro’s new car is represented by \([r \circ d](24,500)\).

\[
[r \circ d](x) = r[d(x)]
\]
\[
[r \circ d](24,500) = r[24,500 - 0.12(24,500)]
\]
\[
= r(24,500 - 2940)
\]
\[
= r(21,560)
\]
\[
= 21,560 - 1500
\]
\[
= 20,060
\]

If the rebate is given before the discount is applied, then the final price of Mr. Navarro’s car is represented by \([d \circ r](24,500)\).

\[
[d \circ r](x) = d[r(x)]
\]
\[
[d \circ r](24,500) = d(24,500 - 1500)
\]
\[
= d(23,000)
\]
\[
= 23,000 - 0.12(23,000)
\]
\[
= 23,000 - 2760
\]
\[
= 20,240
\]

\[
[r \circ d](24,500) = 20,060 \text{ and } [d \circ r](24,500) = 20,240. \] So, the final price of the car is less when the discount is applied before the rebate.

**Check** The answer seems reasonable because the 12% discount is being applied to a greater amount. Thus, the dollar amount of the discount is greater.

### Guided Practice

4. **SHOPPING** Sounds-to-Go offers both an in-store $35 rebate and a 15% discount on a digital audio player that normally sells for $300. Which provides the better price: taking the discount before the rebate or taking the discount after the rebate?
Check Your Understanding

Examples 1–2 Find \((f + g)(x), (f - g)(x), (f \cdot g)(x),\) and \(\left(\frac{f}{g}\right)(x)\) for each \(f(x)\) and \(g(x)\).

1. \(f(x) = x + 2\)
   \(g(x) = 3x - 1\)
2. \(f(x) = x^2 - 5\)
   \(g(x) = -x + 8\)

Example 3
For each pair of functions, find \(f \circ g\) and \(g \circ f\), if they exist.

3. \(f = \{(2, 5), (6, 10), (12, 9), (7, 6)\}
   \(g = \{(9, 11), (6, 15), (10, 13), (5, 8)\}\)
4. \(f = \{(-5, 4), (14, 8), (12, 1), (0, -3)\}
   \(g = \{(-2, -4), (-3, 2), (-1, 4), (5, -6)\}\)

Find \([f \circ g](x)\) and \([g \circ f](x)\), if they exist.

5. \(f(x) = -3x\)
   \(g(x) = 5x - 6\)
6. \(f(x) = x + 4\)
   \(g(x) = x^2 + 3x - 10\)

Example 4
7. **FINANCIAL LITERACY** Dora has 8% of her earnings deducted from her paycheck for a college savings plan. She can choose to take the deduction either before taxes are withheld, which reduces her taxable income, or after taxes are withheld. Dora’s tax rate is 17.5%. If her pay before taxes and deductions is $950, will she save more money if the deductions are taken before or after taxes are withheld? Explain.

Practice and Problem Solving
Extra Practice begins on page 947.

Examples 1–2 Find \((f + g)(x), (f - g)(x), (f \cdot g)(x),\) and \(\left(\frac{f}{g}\right)(x)\) for each \(f(x)\) and \(g(x)\).

8. \(f(x) = 2x\)
   \(g(x) = -4x + 5\)
9. \(f(x) = x - 1\)
   \(g(x) = 5x - 2\)
10. \(f(x) = x^2\)
    \(g(x) = -x + 1\)
11. \(f(x) = 3x\)
    \(g(x) = -2x + 6\)
12. \(f(x) = x - 2\)
    \(g(x) = 2x - 7\)
13. \(f(x) = x^2\)
    \(g(x) = x - 5\)
14. \(f(x) = -x^2 + 6\)
    \(g(x) = 2x^2 + 3x - 5\)
15. \(f(x) = 3x^2 - 4\)
    \(g(x) = x^2 - 8x + 4\)

16. **WALKING** Isaac is walking on a moving walkway. His speed is given by the function \(I(x) = 3x - 4\), and the speed of the walkway is \(W(x) = 4x + 7\), where \(x\) is time in seconds.
   a. What is his total speed as he walks along the moving walkway?
   b. Isaac turns around because he left his cell phone at a restaurant. What is his speed as he walks against the moving walkway?

Example 3
For each pair of functions, find \(f \circ g\) and \(g \circ f\), if they exist.

17. \(f = \{(-8, -4), (0, 4), (2, 6), (-6, -2)\}\)
    \(g = \{(4, -4), (-2, -1), (-4, 0), (6, -5)\}\)
18. \(f = \{(-7, 0), (4, 5), (8, 12), (-3, 6)\}\)
    \(g = \{(6, 8), (-12, -5), (0, 5), (5, 1)\}\)
19. \(f = \{(5, 13), (-4, -2), (-8, -11), (3, 1)\}\)
    \(g = \{(-8, 2), (-4, 1), (3, -3), (5, 7)\}\)
20. \(f = \{(-4, -14), (0, -6), (-6, -18), (2, -2)\}\)
    \(g = \{(-6, 1), (-18, 13), (-14, 9), (-2, -3)\}\)
For each pair of functions, find \( f \circ g \) and \( g \circ f \), if they exist.

21. \( f = \{(−15, −5), (−4, 12), (1, 7), (3, 9)\} \)
   \( g = \{(3, −9), (7, 2), (8, −6), (12, 0)\} \)
22. \( f = \{(-1, 11), (2, -2), (5, -7), (4, -4)\} \)
   \( g = \{(5, -4), (4, -3), (-1, 2), (2, 3)\} \)
23. \( f = \{(7, -3), (-10, -3), (-7, -8), (-3, 6)\} \)
   \( g = \{(4, -3), (3, -7), (9, 8), (-4, -4)\} \)
24. \( f = \{(1, -1), (2, -2), (3, -3), (4, -4)\} \)
   \( g = \{(1, -4), (2, -3), (3, -2), (4, -1)\} \)
25. \( f = \{(-4, 1), (-2, 6), (-1, 10), (4, 11)\} \)
   \( g = \{(-1, 5), (3, -4), (6, 4), (10, 8)\} \)
26. \( f = \{(12, -3), (9, -2), (8, -1), (6, 3)\} \)
   \( g = \{(-1, 5), (-2, 6), (-3, -1), (-4, 8)\} \)

Find \( f \circ g \)(x) and \( g \circ f \)(x), if they exist.

27. \( f(x) = 2x \)
   \( g(x) = x + 5 \)
28. \( f(x) = -3x \)
   \( g(x) = -x + 8 \)
29. \( f(x) = x + 5 \)
   \( g(x) = 3x - 7 \)
30. \( f(x) = x - 4 \)
   \( g(x) = x^2 - 10 \)
31. \( f(x) = x^2 + 6x - 2 \)
   \( g(x) = x - 6 \)
32. \( f(x) = 2x^2 - x + 1 \)
   \( g(x) = 4x + 3 \)
33. \( f(x) = 4x - 1 \)
   \( g(x) = x^3 + 2 \)
34. \( f(x) = x^2 + 3x + 1 \)
   \( g(x) = x^2 \)
35. \( f(x) = 2x^2 \)
   \( g(x) = 8x^2 + 3x \)

36. **Finance** A ceramics store manufactures and sells coffee mugs. The revenue \( r(x) \) from the sale of \( x \) coffee mugs is given by \( r(x) = 6.5x \). Suppose the function for the cost of manufacturing \( x \) coffee mugs is \( c(x) = 0.75x + 1850 \).
   a. Write the profit function.
   b. Find the profit on 500, 1000, and 5000 coffee mugs.

37. **Shopping** Ms. Smith wants to buy an HDTV, which is on sale for 35% off the original price of $2299. The sales tax is 6.25%.
   a. Write two functions representing the price after the discount \( p(x) \) and the price after sales tax \( t(x) \).
   b. Which composition of functions represents the price of the HDTV, \( [p \circ t](x) \) or \( [t \circ p](x) \)? Explain your reasoning.
   c. How much will Ms. Smith pay for the HDTV?

Perform each operation if \( f(x) = x^2 + x - 12 \) and \( g(x) = x - 3 \). State the domain of the resulting function.

38. \((f - g)(x)\)
39. \(2(g \cdot f)(x)\)
40. \(\left(\frac{f}{g}\right)(x)\)

If \( f(x) = 5x, g(x) = -2x + 1 \), and \( h(x) = x^2 + 6x + 8 \), find each value.

41. \(f[g(-2)]\)
42. \(g[h(3)]\)
43. \(h[f(-5)]\)
44. \(h[g(2)]\)
45. \(f[h(-3)]\)
46. \(h[f(9)]\)
47. \(f[g(3a)]\)
48. \(f[h(a + 4)]\)
49. \(g[f(a^2 - a)]\)

50. **Multiple Representations** Let \( f(x) = x^2 \) and \( g(x) = x \).
   a. **Tabular** Make a table showing values for \( f(x), g(x), (f + g)(x), \) and \( (f - g)(x) \).
   b. **Graphical** Graph \( f(x), g(x), (f + g)(x), \) and \( (f - g)(x) \) on the same coordinate grid.
   c. **Graphical** Graph \( f(x), g(x), (f - g)(x) \) on the same coordinate grid.
   d. **Verbal** Describe the relationship among the graphs of \( f(x), g(x), (f + g)(x), \) and \( (f - g)(x) \).
51. **EMPLOYMENT** The number of women and men age 16 and over employed each year in the United States can be modeled by the following equations, where \( x \) is the number of years since 1994 and \( y \) is the number of people in thousands.

- Women: \( y = 1086.4x + 56,610 \)
- Men: \( y = 999.2x + 66,450 \)

   a. Write a function that models the total number of men and women employed in the United States during this time.

   b. If \( f \) is the function for the number of men, and \( g \) is the function for the number of women, what does \( (f - g)(x) \) represent?

If \( f(x) = x + 2 \), \( g(x) = -4x + 3 \), and \( h(x) = x^2 - 2x + 1 \), find each value.

52. \( (f \cdot g \cdot h)(3) \)

53. \( [(f + g) \cdot h](1) \)

54. \( \left[\frac{h}{g}\right](-6) \)

55. \( [f \circ (g \cdot h)](2) \)

56. \( [g \circ (h \cdot f)](-4) \)

57. \( [h \circ (f \circ g)](5) \)

58. **MULTIPLE REPRESENTATIONS** You will explore \( (f \cdot g)(x) \), \( \left[\frac{f}{g}\right](x) \), \( f \circ g \), \( g \circ f \), and \( g \circ f \) if \( f(x) = x^2 + 1 \) and \( g(x) = x - 3 \).

   a. **Tabular** Make a table showing values for \( (f \cdot g)(x) \), \( \left[\frac{f}{g}\right](x) \), \( f \circ g \), \( g \circ f \), and \( g \circ f \).

   b. **Graphical** Use a graphing calculator to graph \( (f \cdot g)(x) \) and \( \left[\frac{f}{g}\right](x) \) on the same coordinate plane.

   c. **Verbal** Explain the relationship between \( (f \cdot g)(x) \) and \( \left[\frac{f}{g}\right](x) \).

   d. **Graphical** Use a graphing calculator to graph \( f \circ g \) and \( g \circ f \) on the same coordinate plane.

   e. **Verbal** Explain the relationship between \( f \circ g \) and \( g \circ f \).

**H.O.T. Problems** Use Higher-Order Thinking Skills

59. **OPEN ENDED** Write two functions \( f(x) \) and \( g(x) \) such that \( (f \circ g)(4) = 0 \).

60. **ERROR ANALYSIS** Chris and Tobias are finding \( (f \circ g)(x) \), where \( f(x) = x^2 + 2x - 8 \) and \( g(x) = x^2 + 8 \). Is either of them correct? Explain your reasoning.

   **Chris**
   \[
   (f \circ g)(x) = f[g(x)] \\
   = (x^2 + 8)^2 + 2x - 8 \\
   = x^4 + 16x^2 + 64 + 2x - 8 \\
   = x^4 + 16x^2 + 2x + 58
   \]

   **Tobias**
   \[
   (f \circ g)(x) = f[g(x)] \\
   = (x^2 + 8)^2 + 2(x^2 + 8) - 8 \\
   = x^4 + 16x^2 + 64 + 2x^2 + 16 - 8 \\
   = x^4 + 18x^2 + 72
   \]

61. **CHALLENGE** Given \( f(x) = \sqrt[3]{x} \) and \( g(x) = \sqrt[5]{x} \), determine the domain for each of the following:

   a. \( g(x) \cdot g(x) \)

   b. \( f(x) \cdot f(x) \)

62. **REASONING** State whether each statement is sometimes, always, or never true. Explain.

   a. The domain of two functions \( f(x) \) and \( g(x) \) that are composed \( g[f(x)] \) is restricted by the domain of \( f(x) \).

   b. The domain of two functions \( f(x) \) and \( g(x) \) that are composed \( g[f(x)] \) is restricted by the domain of \( g(x) \).

63. **WRITING IN MATH** Explain why a person would perform a composition of functions. Include a real-world example that you could solve by using composition of functions.
### Standardized Test Practice

64. What is the value of \( x \) in the equation \( 7(x - 4) = 44 - 11x? \)
   - A 1
   - B 2
   - C 3
   - D 4

66. **GRIDDED RESPONSE** In his first three years of coaching basketball at North High School, Coach Lucas’ team won 8 games the first year, 17 games the second year, and 6 games the third year. How many games does the team need to win in the fourth year so the coach’s average will be 10 wins per year?

65. If \( g(x) = x^2 + 9x + 21 \) and \( h(x) = 2(x + 5)^2 \), which is an equivalent form of \( h(x) - g(x)? \)
   - F \( k(x) = -x^2 - 11x - 29 \)
   - G \( k(x) = x^2 + 11x + 29 \)
   - H \( k(x) = x + 4 \)
   - J \( k(x) = x^2 + 7x + 11 \)

### Spiral Review

Find all of the rational zeros of each function. **(Lesson 6-8)**

68. \( f(x) = 2x^3 - 13x^2 + 17x + 12 \)

69. \( f(x) = x^3 - 3x^2 - 10x + 24 \)

70. \( f(x) = x^4 - 4x^3 - 7x^2 + 34x - 24 \)

71. \( f(x) = 2x^3 - 5x^2 - 28x + 15 \)

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function. **(Lesson 6-7)**

72. \( f(x) = 2x^3 - x^3 + 5x^2 + 3x - 9 \)

73. \( f(x) = -4x^4 - x^2 - x + 1 \)

74. \( f(x) = 3x^4 - x^3 + 8x^2 + x - 7 \)

75. \( f(x) = 2x^4 - 3x^3 - 2x^2 + 3 \)

76. **MANUFACTURING** A box measures 12 inches by 16 inches by 18 inches. The manufacturer will increase each dimension of the box by the same number of inches and have a new volume of 5985 cubic inches. How much should be added to each dimension? **(Lesson 6-7)**

Solve each system of equations. **(Lesson 3-5)**

77. \[
\begin{align*}
x + 4y - z &= 6 \\
3x + 2y + 3z &= 16 \\
2x - y + z &= 3
\end{align*}
\]

78. \[
\begin{align*}
a + b - c &= 9 \\
a - b + 3c &= 9 \\
3a - 6c &= 6
\end{align*}
\]

79. \[
\begin{align*}
y + z &= 4 \\
2x + 4y - z &= -3 \\
3y &= -3
\end{align*}
\]

80. **INTERNET** A webmaster estimates that the time, in seconds, to connect to the server when \( n \) people are connecting is given by \( t(n) = 0.005n + 0.3 \). Estimate the time to connect when 50 people are connecting. **(Lesson 2-2)**

### Skills Review

Solve each equation or formula for the specified variable. **(Lesson 1-3)**

81. \( 5x - 7y = 12 \), for \( x \)

82. \( 3x^2 - 6xy + 1 = 4 \), for \( y \)

83. \( 4x + 8yz = 15 \), for \( x \)

84. \( D = mv \), for \( m \)

85. \( A = k^2 + b \), for \( k \)

86. \( (x + 2)^2 - (y + 5)^2 = 4 \), for \( y \)
Inverse Functions and Relations

1 Find Inverses

Recall that a relation is a set of ordered pairs. The inverse relation is the set of ordered pairs obtained by exchanging the coordinates of each ordered pair. The domain of a relation becomes the range of its inverse, and the range of the relation becomes the domain of its inverse.

**Key Concept Inverse Relations**

**Words**

Two relations are inverse relations if and only if whenever one relation contains the element \((a, b)\), the other relation contains the element \((b, a)\).

**Example**

\[ A \text{ and } B \text{ are inverse relations.} \]

\[ A = \{(1, 5), (2, 6), (3, 7)\} \quad B = \{(5, 1), (6, 2), (7, 3)\} \]

**Example 1 Find an Inverse Relation**

**GEOMETRY**

The vertices of \( \triangle ABC \) can be represented by the relation \{\((1, -2), (2, 5), (4, -1)\)\}. Find the inverse of this relation. Describe the graph of the inverse.

Graph the relation. To find the inverse, exchange the coordinates of the ordered pairs. The inverse of the relation is \{\((-2, 1), (5, 2), (-1, 4)\)\}.

Plotting these points shows that the ordered pairs describe the vertices of \( \triangle A'B'C' \) after a reflection in the line \( y = x \).

**Guided Practice**

1. **GEOMETRY**

The ordered pairs of the relation \{\((-8, -3), (-8, -6), (-3, -6)\)\} are the coordinates of the vertices of a right triangle. Find the inverse of this relation. Describe the graph of the inverse.

As with relations, the ordered pairs of inverse functions are also related. We can write the inverse of the function \( f(x) \) as \( f^{-1}(x) \).
Inverse Functions \( f^{-1} \) is read \( f \) inverse or the inverse of \( f \). Note that \(-1\) is not an exponent.

**Key Concept Property of Inverses**

Words
If \( f \) and \( f^{-1} \) are inverses, then \( f(a) = b \) if and only if \( f^{-1}(b) = a \).

Example
Let \( f(x) = x - 4 \) and represent its inverse as \( f^{-1}(x) = x + 4 \).

Evaluate \( f(6) \). Evaluate \( f^{-1}(2) \).

\[
f(x) = x - 4 \quad f^{-1}(x) = x + 4
\]
\[
f(6) = 6 - 4 \quad f^{-1}(2) = 2 + 4 \text{ or } 6
\]

Because \( f(x) \) and \( f^{-1}(x) \) are inverses, \( f(6) = 2 \) and \( f^{-1}(2) = 6 \).

When the inverse of a function is a function, the original function is one-to-one. Recall that the vertical line test can be used to determine whether a relation is a function. Similarly, the horizontal line test can be used to determine whether the inverse of a function is also a function.

No horizontal line can be drawn so that it passes through more than one point. The inverse of \( y = f(x) \) is a function.

A horizontal line can be drawn that passes through more than one point. The inverse of \( y = g(x) \) is not a function.

The inverse of a function can be found by exchanging the domain and the range.

**Example 2 Find and Graph an Inverse**

Find the inverse of each function. Then graph the function and its inverse.

a. \( f(x) = 2x - 5 \)

**Step 1** Rewrite the function as an equation relating \( x \) and \( y \).
\[
f(x) = 2x - 5 \quad \rightarrow \quad y = 2x - 5
\]

**Step 2** Exchange \( x \) and \( y \) in the equation.
\[
x = 2y - 5
\]

**Step 3** Solve the equation for \( y \).
\[
x = 2y - 5 \quad \text{Inverse of } y = 2x - 5
\]
\[
x + 5 = 2y \quad \text{Add 5 to each side.}
\]
\[
\frac{x + 5}{2} = y \quad \text{Divide each side by 2.}
\]

**Step 4** Replace \( y \) with \( f^{-1}(x) \).
\[
y = \frac{x + 5}{2} \quad \rightarrow \quad f^{-1}(x) = \frac{x + 5}{2}
\]

The inverse of \( f(x) = 2x - 5 \) is \( f^{-1}(x) = \frac{x + 5}{2} \).

The graph of \( f^{-1}(x) = \frac{x + 5}{2} \) is the reflection of the graph of \( f(x) = 2x - 5 \) in the line \( y = x \).
**StudyTip**

Functions  The inverse of the function in part b is not a function since it does not pass the vertical line test.

---

**GuidedPractice**

Find the inverse of each function. Then graph the function and its inverse.

2A. \( f(x) = \frac{x-3}{5} \)  
2B. \( f(x) = 3x^2 \)

---

**2 Verifying Inverses** You can determine whether two functions are inverses by finding both of their compositions. If both compositions equal the identity function \( I(x) = x \), then the functions are inverse functions.

---

**ReviewVocabulary**

Identity function  the function \( f(x) = x \) (Lesson 2-7)

---

**KeyConcept  Inverse Functions**

Words  Two functions \( f \) and \( g \) are inverse functions if and only if both of their compositions are the identity function.

Symbols  \( f(x) \) and \( g(x) \) are inverses if and only if \( [f \circ g](x) = x \) and \( [g \circ f](x) = x \).

---

**Example 3  Verify that Two Functions are Inverses**

Determine whether each pair of functions are inverse functions. Explain your reasoning.

a. \( f(x) = 3x + 9 \) and \( g(x) = \frac{1}{3}x - 3 \)

Verify that the compositions of \( f(x) \) and \( g(x) \) are identity functions.

\[
[f \circ g](x) = f[g(x)]  
[g \circ f](x) = g[f(x)]
\]

\[
= f\left(\frac{1}{3}x - 3\right)  
= g(3x + 9)
\]

\[
= 3\left(\frac{1}{3}x - 3\right) + 9  
= \frac{1}{3}(3x + 9) - 3
\]

\[
= x - 9 + 9 \text{ or } x  
= x + 3 - 3 \text{ or } x
\]

The functions are inverses because \( [f \circ g](x) = [g \circ f](x) = x \).

b. \( f(x) = 4x^2 \) and \( g(x) = 2\sqrt{x} \)

\[
[f \circ g](x) = f(2\sqrt{x})  
= 4(2\sqrt{x})^2  
= 4(4x) \text{ or } 16x
\]

Because \( [f \circ g](x) \neq x \), \( f(x) \) and \( g(x) \) are not inverses.

---

**GuidedPractice**

3A. \( f(x) = 3x - 3 \), \( g(x) = \frac{1}{3}x + 4 \)  
3B. \( f(x) = 2x^2 - 1 \), \( g(x) = \sqrt{x + \frac{1}{2}} \)
Check Your Understanding

Example 1 Find the inverse of each relation.
1. \{(-9, 10), (1, -3), (8, -5)\}  
2. \{(-2, 9), (4, -1), (-7, 9), (7, 0)\}

Example 2 Find the inverse of each function. Then graph the function and its inverse.
3. \(f(x) = -3x\)  
4. \(g(x) = 4x - 6\)  
5. \(h(x) = x^2 - 3\)

Example 3 Determine whether each pair of functions are inverse functions. Write yes or no.
6. \(f(x) = x - 7\)  
7. \(f(x) = \frac{1}{2}x + \frac{3}{4}\)  
8. \(f(x) = 2x^3\)  

Practice and Problem Solving

Example 1 Find the inverse of each relation.
9. \{(-8, 6), (6, -2), (7, -3)\}  
10. \{(7, 7), (4, 9), (3, -7)\}
11. \{(8, -1), (-8, -1), (-2, -8), (2, 8)\}  
12. \{(4, 3), (-4, -4), (-3, -5), (5, 2)\}
13. \{(1, -5), (2, 6), (3, -7), (4, 8), (5, -9)\}  
14. \{(3, 0), (5, 4), (7, -8), (9, 12), (11, 16)\}

Example 2 Find the inverse of each function. Then graph the function and its inverse.
15. \(f(x) = x + 2\)  
16. \(g(x) = 5x\)  
17. \(y = -2x + 1\)
18. \(h(x) = \frac{x - 4}{3}\)  
19. \(y = -\frac{5}{3}x - 8\)  
20. \(g(x) = x + 4\)
21. \(f(x) = 4x\)  
22. \(y = -8x + 9\)  
23. \(f(x) = 5x^2\)
24. \(h(x) = x^2 + 4\)  
25. \(f(x) = \frac{1}{2}x^2 - 1\)  
26. \(y = (x + 1)^2 + 3\)

Example 3 Determine whether each pair of functions are inverse functions. Write yes or no.
27. \(f(x) = 2x + 3\)  
28. \(f(x) = 4x + 6\)  
29. \(f(x) = -\frac{1}{3}x + 3\)  
30. \(f(x) = -6x\)  
31. \(f(x) = \frac{1}{2}x + 5\)  
32. \(f(x) = \frac{x + 10}{8}\)  
33. \(f(x) = 4x^2\)  
34. \(f(x) = \frac{1}{3}x^2 + 1\)  
35. \(f(x) = x^2 - 9\)  
36. \(f(x) = \frac{2}{3}x^3\)  
37. \(f(x) = (x + 6)^2\)  
38. \(f(x) = 2\sqrt{x - 5}\)  
39. \(f(x) = \sqrt{x - 5}\)

39. **FUEL** The average miles traveled for every gallon of gas consumed by Leroy’s car is represented by the function \(m(g) = 28g\).

a. Find a function \(c(g)\) to represent the cost per gallon of gasoline.

b. Use inverses to determine the function used to represent the cost per mile traveled in Leroy’s car.
40. **SHOES** The shoe size for the average U.S. teen or adult male can be determined using the formula \( M(x) = 3x - 22 \), where \( x \) is length of a foot in measured inches. The shoe size for the average U.S. teen or adult female can be found by using the formula \( F(x) = 3x - 21 \).
   a. Find the inverse of each function.
   b. If Lucy wears a size \( 7 \frac{1}{2} \) shoe, how long are her feet?

41. **GEOMETRY** The formula for the area of a circle is \( A = \pi r^2 \).
   a. Find the inverse of the function.
   b. Use the inverse to find the radius of a circle with an area of 36 square centimeters.

Use the horizontal line test to determine whether the inverse of each function is also a function.

42. \( f(x) = 2x^2 \)  
43. \( f(x) = x^3 - 8 \)  
44. \( g(x) = x^4 - 6x^2 + 1 \)  
45. \( h(x) = -2x^4 - x - 2 \)  
46. \( g(x) = x^5 + x^2 - 4x \)  
47. \( h(x) = x^3 + x^2 - 6x + 12 \)  

48. **SHOPPING** Felipe bought a used car. The sales tax rate was 7.25% of the selling price, and he paid $350 in processing and registration fees. Find the selling price if Felipe paid a total of $8395.75.

49. **TEMPERATURE** A formula for converting degrees Celsius to Fahrenheit is \( F(x) = \frac{9}{5}x + 32 \).
   a. Find the inverse \( F^{-1}(x) \). Show that \( F(x) \) and \( F^{-1}(x) \) are inverses.
   b. Explain what purpose \( F^{-1}(x) \) serves.

50. **MEASUREMENT** There are approximately 1.852 kilometers in a nautical mile.
   a. Write a function that converts nautical miles to kilometers.
   b. Find the inverse of the function that converts kilometers back to nautical miles.
   c. Using composition of functions, verify that these two functions are inverses.

51. **MULTIPLE REPRESENTATIONS** Consider the functions \( y = x^n \) for \( n = 0, 1, 2, \ldots \).
   a. **Graphing** Use a graphing calculator to graph \( y = x^n \) for \( n = 0, 1, 2, 3, \) and 4.
   b. **Tabular** For which values of \( n \) is the inverse a function? Record your results in a table.
   c. **Analytical** Make a conjecture about the values of \( n \) for which the inverse of \( f(x) = x^n \) is a function. Assume that \( n \) is a whole number.

**H.O.T. Problems** Use Higher-Order Thinking Skills

52. **REASONING** If a relation is not a function, then its inverse is sometimes, always, or never a function. Explain your reasoning.

53. **OPEN ENDED** Give an example of a function and its inverse. Verify that the two functions are inverses.

54. **CHALLENGE** Give an example of a function that is its own inverse.

55. **PROOF** Show that the inverse of a linear function \( y = mx + b \), where \( m \neq 0 \) and \( x \neq b \), is also a linear function.

56. **WRITING IN MATH** Suppose you have a composition of two functions that are inverses. When you put in a value of 5 for \( x \), why is the result always 5?
57. SHORT RESPONSE If the length of a rectangular television screen is 24 inches and its height is 18 inches, what is the length of its diagonal in inches?

58. GEOMETRY If the base of a triangle is represented by $2x + 5$ and the height is represented by $4x$, which expression represents the area of the triangle?

A. $(2x + 5) + (4x)$
B. $(2x + 5)(4x)$
C. $\frac{1}{2}(2x + 5) + (4x)$
D. $\frac{1}{2}(2x + 5)(4x)$

59. Which expression represents $f[g(x)]$ if $f(x) = x^2 + 3$ and $g(x) = -x + 1$?

F. $x^2 - x + 2$
G. $-x^2 - 2$
H. $-x^3 + x^2 - 3x + 3$
J. $x^2 - 2x + 4$

60. SAT/ACT Which of the following is the inverse of $f(x) = \frac{3x - 5}{2}$?

A. $g(x) = \frac{2x + 5}{3}$
B. $g(x) = \frac{2x - 5}{3}$
C. $g(x) = \frac{3x + 5}{2}$
D. $g(x) = 2x + 5$
E. $g(x) = \frac{3x - 5}{2}$

61. $g[f(3)]$
62. $f[h(-2)]$
63. $h[g(1)]$

64. CONSTRUCTION A picnic area has the shape of a trapezoid. The longer base is 8 more than 3 times the length of the shorter base, and the height is 1 more than 3 times the shorter base. What are the dimensions if the area is 4104 square feet? (Lesson 6-8)

Find the value of $c$ that makes each trinomial a perfect square. Then write the trinomial as a perfect square. (Lesson 5-5)

65. $x^2 + 34x + c$
66. $x^2 - 11x + c$

Simplify. (Lesson 5-4)

67. $(3 + 4i)(5 - 2i)$
68. $(\sqrt{6} + i)(\sqrt{6} - i)$
69. $\frac{1 + i}{1 - i}$
70. $\frac{4 - 3i}{1 + 2i}$

Refer to quadrilateral $QRST$ shown at the right. (Lesson 4-4)

71. Write the vertex matrix. Multiply the vertex matrix by $-1$.
72. Graph the preimage and image.
73. What type of transformation does the graph represent?

Skills Review

Graph each inequality. (Lesson 2-7)

74. $y > \frac{3}{4}x - 2$
75. $y \leq -3x + 2$
76. $y < -x - 4$
Graphing Technology Lab
Inverse Functions and Relations

You can use a TI-83/84 Plus graphing calculator to compare a function and its inverse using tables and graphs. Note that before you enter any values in the calculator, you should clear all lists.

**Activity 1  Graph Inverses with Ordered Pairs**

Graph \( f(x) = \{(1, 2), (2, 4), (3, 6), (4, 8), (5, 10), (6, 12)\} \) and its inverse.

**Step 1** Enter the \( x \)-values in L1 and the \( y \)-values in L2. Then graph the function.

**KEYSTROKES:**

```
STAT ENTER 1 ENTER 2 ENTER 3 ENTER 4 ENTER
5 ENTER 6 ENTER \( \downarrow \) 2 ENTER 4 ENTER 6 ENTER 8 ENTER
10 ENTER 12 ENTER \( \text{2nd} \) [STAT PLOT] ENTER ENTER GRAPH
```

Adjust the window to reflect the domain and range.

**Step 2** Define the inverse function by setting Xlist to L2 and Ylist to L1. Then graph the inverse function.

**KEYSTROKES:**

```
\( \text{2nd} \) [STAT PLOT] \( \downarrow \) ENTER \( \downarrow \) \( \downarrow \) \( \text{2nd} \) [L2]
\( \downarrow \) \( \text{2nd} \) [L1] GRAPH
```

**Step 3** Graph the line \( y = x \).

**KEYSTROKES:**

```
Y= \( x \), \( t \), \( \theta \), n GRAPH
```

**Activity 2  Graph Inverses with Function Notation**

Graph \( f(x) = 3x \) and its inverse \( g(x) = \frac{x}{3} \).

**Step 1** Clear the data from Activity 1.

**KEYSTROKES:**

```
\( \text{2nd} \) [STAT PLOT] ENTER \( \uparrow \) ENTER \( \uparrow \) \( \uparrow \) \( \text{2nd} \) [QUIT]
```

**Step 2** Enter \( f(x) \) as \( Y1 \), \( g(x) \) as \( Y2 \), and \( y = x \) as \( Y3 \). Then graph.

**KEYSTROKES:**

```
Y= 3 \( x \), \( t \), \( \theta \), n ENTER \( x \), \( t \), \( \theta \), n \( \div \) 3 ENTER \( x \), \( t \), \( \theta \), n GRAPH
```

**Exercises**

Graph each function \( f(x) \) and its inverse \( g(x) \). Then graph \( (f \circ g)(x) \).

1. \( f(x) = 5x \)
2. \( f(x) = x - 3 \)
3. \( f(x) = 2x + 1 \)
4. \( f(x) = \frac{1}{2}x + 3 \)
5. \( f(x) = x^2 \)
6. \( f(x) = x^2 - 3 \)

7. What is the relationship between the graphs of a function and its inverse?

8. **MAKE A CONJECTURE** For any function \( f(x) \) and its inverse \( g(x) \), what is \( (f \circ g)(x) \)?
Square Root Functions and Inequalities

1. **Square Root Functions** If a function contains the square root of a variable, it is called a square root function. The square root function is a type of radical function.

**KeyConcept** Parent Function of Square Root Functions

<table>
<thead>
<tr>
<th>Parent function:</th>
<th>$f(x) = \sqrt{x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain:</td>
<td>${x \mid x \geq 0}$</td>
</tr>
<tr>
<td>Range:</td>
<td>${f(x) \mid f(x) \geq 0}$</td>
</tr>
<tr>
<td>Intercepts:</td>
<td>$x = 0, f(x) = 0$</td>
</tr>
<tr>
<td>Not defined:</td>
<td>$x &lt; 0$</td>
</tr>
<tr>
<td>End behavior:</td>
<td>$x \to 0, f(x) \to 0$</td>
</tr>
<tr>
<td></td>
<td>$x \to +\infty, f(x) \to +\infty$</td>
</tr>
</tbody>
</table>

The domain of a square root function is limited to values for which the function is defined.

**Example 1** Identify Domain and Range

Identify the domain and range of $f(x) = \sqrt{x + 4}$.

The domain only includes values for which the radicand is nonnegative.

$x + 4 \geq 0$  \[\text{Write an inequality.}\]  $x \geq -4$  \[\text{Subtract 4 from each side.}\]

Thus, the domain is $\{x \mid x \geq -4\}$.

Find $f(-4)$ to determine the lower limit of the range.

$f(-4) = \sqrt{-4 + 4}$ or $0$

So, the range is $\{f(x) \mid f(x) \geq 0\}$.

**Guided Practice**

Identify the domain and range of each function.

1A. $f(x) = \sqrt{x - 3}$

1B. $f(x) = \sqrt{x + 6} + 2$
The same techniques used to transform the graph of other functions you have studied can be applied to the graphs of square root functions.

**Key Concept**  
Transformations of Square Root Functions  
\[ f(x) = a\sqrt{x} - h + k \]

<table>
<thead>
<tr>
<th>( h )—Horizontal Translation</th>
<th>( k )—Vertical Translation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>h</td>
</tr>
<tr>
<td>The domain is ( {x</td>
<td>x \geq h} ).</td>
</tr>
</tbody>
</table>

**Study Tip**  
**Domain and Range**  
The limits on the domain and range also represent the initial point of the graph of a square root function.

**Example 2**  
Graph Square Root Functions

Graph each function. State the domain and range.

a. \( y = \sqrt{x - 2} + 5 \)

The minimum point is at \((h, k) = (2, 5)\). Make a table of values for \(x \geq 2\), and graph the function. The graph is the same shape as \(f(x) = \sqrt{x}\), but is translated 2 units right and 5 units up. Notice the end behavior. As \(x\) increases, \(y\) increases.

The domain is \(\{x | x \geq 2\}\) and the range is \(\{y | y \geq 5\}\).

b. \( y = -2\sqrt{x + 3} - 1 \)

The minimum domain value is at \(h \) or -3. Make a table of values for \(x \geq -3\), and graph the function. Because \(a\) is negative, the graph is similar to \(f(x) = \sqrt{x}\), but is reflected in the line \(f(x) = -1\). Because \(|a| > 1\), the graph is vertically stretched. It is also translated 3 units left and 1 unit down.

The domain is \(\{x | x \geq -3\}\) and the range is \(\{y | y \leq -1\}\).

**Guided Practice**

2A. \( f(x) = 2\sqrt{x + 4} \)

2B. \( f(x) = \frac{1}{4}\sqrt{x - 5} + 3 \)
**Real-World Example 3** Use Graphs to Analyze Square Root Functions

**MUSIC** Refer to the application at the beginning of the lesson. The pitch, or frequency, measured in hertz (Hz) of a certain string can be determined by

\[ f(T) = \frac{1}{1.28} \sqrt{\frac{T}{0.0000708}} \]

where \( T \) is tension in kilograms.

**a.** Graph the function for tension in the domain \( 0 \leq T \leq 10 \).

Make a table of values for \( 0 \leq T \leq 10 \) and graph.

<table>
<thead>
<tr>
<th>( T )</th>
<th>( f(T) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>92.8</td>
</tr>
<tr>
<td>2</td>
<td>131.3</td>
</tr>
<tr>
<td>3</td>
<td>160.8</td>
</tr>
<tr>
<td>4</td>
<td>185.7</td>
</tr>
<tr>
<td>5</td>
<td>207.6</td>
</tr>
</tbody>
</table>

**b.** How much tension is needed for a pitch of over 200 Hz?

According to the graph and the table, more than 4.5 kilograms of tension is needed for a pitch of more than 200 hertz.

**Guided Practice**

3. **MUSIC** The frequency of vibrations for a certain guitar string when it is plucked can be determined by \( F = \frac{200}{\sqrt{T}} \), where \( F \) is the number of vibrations per second and \( T \) is the tension measured in pounds. Graph the function for \( 0 \leq T \leq 10 \). Then determine the frequency for \( T = 3, 6, \) and \( 9 \) pounds.

2 **Square Root Inequalities** A **square root inequality** is an inequality involving square roots. They are graphed using the same method as other inequalities.

**Example 4** Graph a Square Root Inequality

Graph \( y < \sqrt{x - 4} - 6 \).

Graph the boundary \( y = \sqrt{x - 4} - 6 \).

The domain is \( \{ x \mid x \geq 4 \} \). Because \( y \) is less than, the shaded region should be below the boundary and within the domain.

**CHECK** Select a point in the shaded region, and verify that it is a solution of the inequality.

Test \( (7, -5) \):

\(-5 < \sqrt{7 - 4} - 6 \)
\[-5 < \sqrt{3} - 6 \]
\[-5 < -4.27 \checkmark \]

**Guided Practice**

4A. \( f(x) \geq \sqrt{2x + 1} \)

4B. \( f(x) < -\sqrt{x + 2} - 4 \)
Check Your Understanding

Example 1 Identify the domain and range of each function.
1. \( f(x) = \sqrt{4x} \)  
2. \( f(x) = \sqrt{x - 5} \)  
3. \( f(x) = \sqrt{x + 8} - 2 \)

Example 2 Graph each function. State the domain and range.
4. \( f(x) = \sqrt{x} - 2 \)  
5. \( f(x) = 3\sqrt{x - 1} \)  
6. \( f(x) = \frac{1}{2}\sqrt{x + 4} - 1 \)  
7. \( f(x) = -\sqrt{3x - 5} + 5 \)

Example 3 8. OCEAN The speed that a tsunami, or tidal wave, can travel is modeled by the equation \( v = 356\sqrt{d} \), where \( v \) is the speed in kilometers per hour and \( d \) is the average depth of the water in kilometers. A tsunami is found to be traveling at 145 kilometers per hour. What is the average depth of the water? Round to the nearest hundredth of a kilometer.

Example 4 Graph each inequality.
9. \( f(x) \geq \sqrt{x} + 4 \)  
10. \( f(x) \leq \sqrt{x - 6} + 2 \)  
11. \( f(x) < -2\sqrt{x + 3} \)  
12. \( f(x) > \sqrt{2x - 1} - 3 \)

Practice and Problem Solving

Example 1 Identify the domain and range of each function.
13. \( f(x) = -\sqrt{2x} + 2 \)  
14. \( f(x) = \sqrt{x} - 6 \)  
15. \( f(x) = 4\sqrt{x - 2} - 8 \)  
16. \( f(x) = \sqrt{x + 2} + 5 \)  
17. \( f(x) = \sqrt{x - 4} - 6 \)  
18. \( f(x) = -\sqrt{x - 6} + 5 \)

Example 2 Graph each function. State the domain and range.
19. \( f(x) = \sqrt{6x} \)  
20. \( f(x) = -\sqrt{5x} \)  
21. \( f(x) = \sqrt{x - 8} \)  
22. \( f(x) = \sqrt{x + 1} \)  
23. \( f(x) = \sqrt{x + 3} + 2 \)  
24. \( f(x) = \sqrt{x - 4} - 10 \)  
25. \( f(x) = 2\sqrt{x - 5} - 6 \)  
26. \( f(x) = \frac{3}{4}\sqrt{x + 12} + 3 \)  
27. \( f(x) = -\frac{1}{2}\sqrt{x - 1} - 4 \)  
28. \( f(x) = -3\sqrt{x + 7} + 9 \)

Example 3 29. SKYDIVING The approximate time \( t \) in seconds that it takes an object to fall a distance of \( d \) feet is given by \( t = \sqrt{\frac{d}{16}} \). Suppose a parachutist falls 11 seconds before the parachute opens. How far does the parachutist fall during this time?

30. ROLLER COASTERS The velocity of a roller coaster as it moves down a hill is \( V = \sqrt{v^2 + 64h} \), where \( v \) is the initial velocity in feet per second and \( h \) is the vertical drop in feet. The designer wants the coaster to have a velocity of 90 feet per second when it reaches the bottom of the hill.

a. If the initial velocity of the coaster at the top of the hill is 10 feet per second, write an equation that models the situation.

b. How high should the designer make the hill?
Example 4  
Graph each inequality.

31. \( y < \sqrt{x - 5} \)  
32. \( y > \sqrt{x + 6} \)  
33. \( y \geq -4\sqrt{x + 3} \)  
34. \( y \leq -2\sqrt{x - 6} \)  
35. \( y > 2\sqrt{x + 7} - 5 \)  
36. \( y \geq 4\sqrt{x - 2} - 12 \)  
37. \( y \leq 6 - 3\sqrt{x - 4} \)  
38. \( y < \sqrt{4x - 12} + 8 \)

39. PHYSICS  The kinetic energy of an object is the energy produced due to its motion and mass. The formula for kinetic energy, measured in joules, is \( E = 0.5mv^2 \), where \( m \) is the mass in kilograms and \( v \) is the velocity of the object in meters per second.
   
a. Solve the above formula for \( v \).
   
b. If a 1500-kilogram vehicle is generating 1 million joules of kinetic energy, how fast is it traveling?
   
c. Escape velocity is the minimum velocity at which an object must travel to escape the gravitational field of a planet or other object. Suppose a 100,000-kilogram ship must have a kinetic energy of \( 3.624 \times 10^{14} \) joules to escape the gravitational field of Jupiter. Estimate the escape velocity of Jupiter.

40. DRIVING  After an accident, police can determine how fast a car was traveling before the driver put on his or her brakes by using the equation \( v = \sqrt{30fd} \). In this equation, \( v \) represents the speed in miles per hour, \( f \) represents the coefficient of friction, and \( d \) represents the length of the skid marks in feet. The coefficient of friction varies depending on road conditions. Assume that \( f = 0.6 \).
   
a. Find the speed of a car that skids 25 feet.
   
b. If your car is going 35 miles per hour, how many feet would it take you to stop?
   
c. If the speed of a car is doubled, will the skid be twice as long? Explain.

Write the square root function represented by each graph.

41.  
42.  
43.

44. MULTIPLE REPRESENTATIONS  In this problem, you will use the following functions to investigate transformations of square root functions.

\[ f(x) = 4\sqrt{x - 6} + 3 \]  
\[ g(x) = \sqrt{16x + 1} - 6 \]  
\[ h(x) = \sqrt{x + 3} + 2 \]

a. Graphical  Graph each function on the same set of axes.

b. Analytical  Identify the transformation on the graph of the parent function. What values caused each transformation?

c. Analytical  Which functions appear to be stretched or compressed vertically? Explain your reasoning.

d. Verbal  The two functions that are stretched appear to be stretched by the same magnitude. How is this possible?

e. Tabular  Make a table of the rate of change for all three functions between 8 and 12 as compared to 12 and 16. What generalization about rate of change in square root functions can be made as a result of your findings?
**PENDULUMS** The period of a pendulum can be represented by 

\[ T = 2\pi \sqrt{\frac{L}{g}} \]

where \( T \) is the time in seconds, \( L \) is the length in feet, and \( g \) is gravity, 32 feet per second squared.

a. Graph the function for \( 0 \leq L \leq 10 \).

b. What is the period for lengths of 2, 5, and 8 feet?

**PHYSICS** Using the function \( m = \frac{m_0}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}} \), Einstein’s theory of relativity states that the apparent mass \( m \) of a particle depends on its velocity \( v \). An object that is traveling extremely fast, close to the speed of light \( c \), will appear to have more mass compared to its mass at rest, \( m_0 \).

a. Use a graphing calculator to graph the function for a 10,000-kilogram ship for the domain \( 0 \leq v \leq 300,000,000 \). Use 300 million meters per second for the speed of light.

b. What viewing window did you use to view the graph?

c. Determine the apparent mass \( m \) of the ship for speeds of 100 million, 200 million, and 299 million meters per second.

**H.O.T. Problems** Use Higher-Order Thinking Skills

47. **CHALLENGE** Write an equation for a square root function with a domain of \( \{x \mid x \geq -4\} \), a range of \( \{y \mid y \leq 6\} \), and that passes through (5, 3).

48. **REASONING** For what positive values of \( a \) are the domain and range of \( f(x) = \sqrt{x} \) the set of real numbers?

49. **OPEN ENDED** Write a square root function for which the domain is \( \{x \mid x \geq 8\} \) and the range is \( \{y \mid y \leq 14\} \).

50. **WRITING IN MATH** Explain why there are limitations on the domain and range of square root functions.

51. **ERROR ANALYSIS** Molly and Cleveland are graphing \( y \leq \sqrt{5x} + 15 \). Is either of them correct? Explain your reasoning.

52. **WRITING IN MATH** Explain why \( y = \pm \sqrt{x} \) is not a function.

53. **OPEN ENDED** Write an equation of a relation that contains a radical and its inverse such that:
   a. the original relation is a function, and its inverse is not a function.
   b. the original relation is not a function, and its inverse is a function.
54. The expression \( \frac{-64x^6}{8x^3}, x \neq 0 \), is equivalent to
A. \( 8x^2 \)  
B. \( 8x^3 \)  
C. \( -8x^2 \)  
D. \( -8x^3 \)

55. **PROBABILITY** For a game, Patricia must roll a standard die and draw a card from a deck of 26 cards, each card having a letter of the alphabet on it. What is the probability that Patricia will roll an odd number and draw a letter in her name?
F. \( \frac{2}{3} \)  
H. \( \frac{1}{13} \)  
G. \( \frac{3}{26} \)  
J. \( \frac{1}{26} \)

56. **SHORT RESPONSE** What is the product of \((d + 6)\) and \((d - 3)\)?

57. **SAT/ACT** Given the graph of the square root function below, which must be true?
I. The domain is all real numbers.  
II. The function is \( y = \sqrt{x} + 3.5 \).  
III. The range is \( \{y \mid y \geq 3.5\} \).

58. \( f(x) = 2x \)  
59. \( f(x) = 3x - 7 \)  
60. \( f(x) = \frac{3x + 2}{5} \)

\[ g(x) = \frac{1}{2}x \]  
\[ g(x) = \frac{1}{3}x - \frac{7}{16} \]  
\[ g(x) = \frac{5x - 2}{3} \]

61. **TIME** The formula \( h = \frac{m}{60} \) converts minutes \( m \) to hours \( h \), and \( d = \frac{h}{24} \) converts hours \( h \) to days \( d \). Write a function that converts minutes to days. 

62. **CABLE TV** The number of households in the United States with cable TV after 1985 can be modeled by the function \( C(t) = -43.2t^2 + 1343t + 790 \), where \( t \) represents the number of years since 1985.

a. Graph this equation for the years 1985 to 2005.

b. Describe the turning points of the graph and its end behavior.

c. What is the domain of the function? Use the graph to estimate the range for the function.

d. What trends in households with cable TV does the graph suggest? Is it reasonable to assume that the trend will continue indefinitely?

63. 6.34  
64. 3.78787888…  
65. 5.333…  
66. 1.25

**Skills Review**

Determine whether each number is rational or irrational. 

Determine whether each pair of functions are inverse functions. Write yes or no. 

Determine whether each number is rational or irrational. 

Determine whether each number is rational or irrational.
### Simplify Radicals

Finding the square root of a number and squaring a number are inverse operations. To find the square root of a number $a$, you must find a number with a square of $a$. Similarly, the inverse of raising a number to the $n$th power is finding the $n$th root of a number.

<table>
<thead>
<tr>
<th>Powers</th>
<th>Factors</th>
<th>Words</th>
<th>Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^3 = 64$</td>
<td>$4 \cdot 4 \cdot 4 = 64$</td>
<td>4 is a cube root of 64.</td>
<td>$\sqrt[3]{64} = 4$</td>
</tr>
<tr>
<td>$x^4 = 625$</td>
<td>$5 \cdot 5 \cdot 5 \cdot 5 = 625$</td>
<td>5 is a fourth root of 625.</td>
<td>$\sqrt[4]{625} = 5$</td>
</tr>
<tr>
<td>$x^5 = 32$</td>
<td>$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$</td>
<td>2 is a fifth root of 32.</td>
<td>$\sqrt[5]{32} = 2$</td>
</tr>
<tr>
<td>$a^n = b$</td>
<td>$a \cdot a \cdot a \cdot \ldots \cdot a = b$</td>
<td>$a$ is an $n$th root of $b$.</td>
<td>$\sqrt[n]{b} = a$</td>
</tr>
</tbody>
</table>

This pattern suggests the following formal definition of an $n$th root.

#### Key Concept: Definition of $n$th Root

**Words**

For any real numbers $a$ and $b$, and any positive integer $n$, if $a^n = b$, then $a$ is an $n$th root of $b$.

**Example**

Because $(-3)^4 = 81$, $-3$ is a fourth root of 81 and 3 is a principal root.

The symbol $\sqrt[n]{b}$ indicates an $n$th root.

Some numbers have more than one real $n$th root. For example, 64 has two square roots, 8 and $-8$, since $8^2$ and $(-8)^2$ both equal 64. When there is more than one real root and $n$ is even, the nonnegative root is called the **principal root**.

Some examples of $n$th roots are listed below.

\[
\sqrt{25} = 5 \quad \text{and} \quad \sqrt{25} \text{ indicates the principal square root of } 25.
\]

\[
-\sqrt{25} = -5 \quad \text{and} \quad -\sqrt{25} \text{ indicates the opposite of the principal square root of } 25.
\]

\[
\pm \sqrt{25} = \pm 5 \quad \text{and} \quad \pm \sqrt{25} \text{ indicates both square roots of } 25.
\]
### Key Concept: Real nth Roots

Suppose \( n \) is an integer greater than 1, and \( a \) is a real number.

<table>
<thead>
<tr>
<th>( a )</th>
<th>( n ) is even.</th>
<th>( n ) is odd.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a &gt; 0 )</td>
<td>1 unique positive and 1 unique negative real root: ( \pm \sqrt[n]{a} ); positive root is principal root</td>
<td>1 unique positive and 0 negative real roots: ( \sqrt[n]{a} )</td>
</tr>
<tr>
<td>( a &lt; 0 )</td>
<td>0 real roots</td>
<td>0 positive and 1 negative real root: ( \sqrt[n]{a} )</td>
</tr>
<tr>
<td>( a = 0 )</td>
<td>1 real root: ( \sqrt[1]{0} = 0 )</td>
<td>1 real root: ( \sqrt[1]{0} = 0 )</td>
</tr>
</tbody>
</table>

### Example 1: Find Roots

**Simplify.**

**a.** \( \pm \sqrt[4]{16y^4} \)

\[
\pm \sqrt[4]{16y^4} = \pm (4y^2)^\frac{1}{4} = \pm 4y^\frac{1}{2} = \pm 2y
\]

The square roots of \( 16y^4 \) are \( \pm 2y \).

**b.** \( -\sqrt[8]{(x^2 - 6)^8} \)

\[
-\sqrt[8]{(x^2 - 6)^8} = -\sqrt[8]{(x^2 - 6)^8} = -(x^2 - 6)^\frac{8}{8} = -(x^2 - 6)
\]

The opposite of the principal square root of \( (x^2 - 6)^8 \) is \( -(x^2 - 6)^4 \).

**c.** \( \sqrt[5]{243a^{20}b^{25}} \)

\[
\sqrt[5]{243a^{20}b^{25}} = \sqrt[5]{(3a^4b^5)^5} = 3a^4b^5
\]

The fifth root of \( 243a^{20}b^{25} \) is \( 3a^4b^5 \).

**d.** \( \sqrt[16]{16x^4y^8} \)

\[
\sqrt[16]{16x^4y^8} = 2x^\frac{4}{16}y^\frac{8}{16} = 2x^{\frac{1}{4}}y^{\frac{1}{2}}
\]

There are no real roots since \( \sqrt{-16} \) is not a real number. However, there are two imaginary roots, \( 4ix^2y^4 \) and \( -4ix^2y^4 \).

### Guided Practice

1A. \( \pm \sqrt[10]{36x^{10}} \)

1B. \( -\sqrt[16]{(y + 7)^{16}} \)

When you find an even root of an even power and the result is an odd power, you must use the absolute value of the result to ensure that the answer is nonnegative.

### Example 2: Simplify Using Absolute Value

**Simplify.**

**a.** \( \sqrt[16]{y^8} \)

\[
\sqrt[16]{y^8} = |y|
\]

Since \( y \) could be negative, you must take the absolute value of \( y \) to identify the principal root.

**b.** \( \sqrt[18]{64(x^2 - 3)^{18}} \)

\[
\sqrt[18]{64(x^2 - 3)^{18}} = 2\sqrt[18]{(x^2 - 3)^3}
\]

Since the index 6 is even and the exponent 3 is odd, you must use absolute value.

### Guided Practice

2A. \( \sqrt[6]{36y^5} \)

2B. \( \sqrt[12]{16(x - 3)^{12}} \)
Approximate Radicals with a Calculator  Recall that real numbers that cannot be expressed as terminating or repeating decimals are irrational numbers. Approximations for irrational numbers are often used in real-world problems.

Real-World Example 3  Approximate Radicals

INJURY PREVENTION  Refer to the beginning of the lesson.

a. If \( c = \sqrt[5]{b^2} \) represents the number of collisions and \( b \) represents the number of bicycle riders per intersection, estimate the number of collisions at an intersection that has 1000 bicycle riders per week.

**Understand**  You want to find out how many collisions there are.

**Plan**  Let 1000 be the number of bicycle riders. The number of collisions is \( c \).

**Solve**

\[
\begin{align*}
    c &= \sqrt[5]{b^2} \\
    &= \sqrt[5]{1000^2} \\
    &\approx 15.85
\end{align*}
\]

Use a calculator.

There are about 16 collisions per week at the intersection.

**Check**

\[
\begin{align*}
    15.85 \pm \sqrt[5]{b^2} &\approx c = 15.85 \\
    15.85^5 \pm b^2 &\approx \text{Raise each side to the fifth power.} \\
    1,000,337 \approx b^2 &\approx \text{Simplify.} \\
    1000 &\approx b \checkmark \text{Take the positive square root of each side.}
\end{align*}
\]

b. If the total number of collisions reported in one week is 21, estimate the number of bicycle riders that passed through that intersection.

\[
\begin{align*}
    c &= \sqrt[5]{b^2} \quad \text{Original formula} \\
    21 &= \sqrt[5]{b^2} \quad c = 27 \\
    21^5 &= b^2 \quad \text{Raise each side to the fifth power.} \\
    4,084,101 &= b^2 \quad \text{Simplify.} \\
    2021 &\approx b \quad \text{Take the positive square root of each side.}
\end{align*}
\]

Guided Practice

3A. The surface area of a sphere can be determined from the volume of the sphere using the formula \( S = \sqrt[3]{36\pi V^2} \), where \( V \) is the volume. Determine the surface area of a sphere with a volume of 200 cubic inches.

3B. If the surface area of a sphere is about 214.5 square inches, determine the volume.

---

**Check Your Understanding**

### Examples 1–2  Simplify.

1. \( \pm \sqrt{100y^8} \)

3. \( \sqrt{(y - 6)^8} \)

5. \( \sqrt{-16y^4} \)

2. \( -\sqrt[4]{49y^{12}} \)

4. \( \sqrt[4]{16x^{16}y^{24}} \)

6. \( \sqrt[6]{64(2y + 1)^{18}} \)

### Example 3  Use a calculator to approximate each value to three decimal places.

7. \( \sqrt{58} \)

8. \( -\sqrt{76} \)

9. \( \sqrt{-43} \)

10. \( \sqrt{71} \)

11. **TELEVISION**  The radius \( r \) of the orbit of a television satellite is given by \( r = \sqrt[3]{\frac{GMt^2}{4\pi^2}} \), where \( G \) is the universal gravitational constant, \( M \) is the mass of Earth, and \( t \) is the time it takes the satellite to complete one orbit. Find the radius of the satellite’s orbit if \( G = 6.67 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2 \), \( M = 5.98 \times 10^{24} \text{ kg} \), and \( t = 2.6 \times 10^6 \text{ seconds} \).
Practice and Problem Solving

**Examples 1–2  Simplify.**

12. \( \pm \sqrt[3]{121x^4y^{16}} \)

13. \( \pm \sqrt[3]{225a^{16}b^{36}} \)

14. \( \pm \sqrt[3]{49x^4} \)

15. \( -\sqrt[3]{16c^4d^2} \)

16. \( -\sqrt[3]{81a^{16}b^{20}c^{12}} \)

17. \( -\sqrt[3]{400x^{32}y^{40}} \)

18. \( \sqrt[3]{(x + 15)^4} \)

19. \( \sqrt[3]{(x^2 + 6)^{16}} \)

20. \( \sqrt[3]{(a^2 + 4d)^{12}} \)

21. \( \sqrt[3]{8a^{6}b^{12}} \)

22. \( \sqrt[3]{d^{24}x^{36}} \)

23. \( \sqrt[3]{27b^{18}c^{12}} \)

24. \( -\sqrt[3]{(2x + 1)^6} \)

25. \( \sqrt[3]{-(x + 2)^8} \)

26. \( \sqrt[3]{-(y - 9)^9} \)

27. \( \sqrt[3]{x^{18}} \)

28. \( \sqrt[3]{a^{12}} \)

29. \( \sqrt[3]{a^{12}} \)

30. \( \sqrt[3]{81(x + 4)^4} \)

31. \( \sqrt[3]{(4x - 7)^{24}} \)

32. \( \sqrt[3]{(y^3 + 5)^{18}} \)

33. \( \sqrt[3]{256(5x - 2)^{12}} \)

34. \( \sqrt[3]{a^{16}y^{8}} \)

35. \( \sqrt[3]{32a^{15}b^{10}} \)

**Example 3**

**36. SHIPPING** An online book store wants to increase the size of the boxes it uses to ship orders. The new volume \( N \) is equal to the old volume \( V \) times the scale factor \( F \) cubed, or \( N = V \cdot F^3 \). What is the scale factor if the old volume was 0.8 cubic feet and the new volume is 21.6 cubic feet?

**37. GEOMETRY** The side length of a cube is determined by \( r = \sqrt[3]{V} \), where \( V \) is the volume in cubic units. Determine the side length of a cube with a volume of 512 cm\(^3\).

Use a calculator to approximate each value to three decimal places.

38. \( \sqrt[3]{92} \)
39. \( -\sqrt[3]{150} \)
40. \( \sqrt[3]{0.43} \)
41. \( \sqrt[3]{0.62} \)

42. \( \sqrt[3]{168} \)
43. \( \sqrt[3]{-4382} \)
44. \( \sqrt[3]{(8912)^2} \)
45. \( \sqrt[3]{(4756)^2} \)

**46. GEOMETRY** The radius \( r \) of a sphere with volume \( V \) can be found using the formula \( r = \sqrt[3]{\frac{3V}{4\pi}} \).

a. Determine the radius for volumes of 1000 cm\(^3\), 8000 cm\(^3\), and 64,000 cm\(^3\).

b. How does the volume of the sphere change if the radius is doubled? Explain.

**Simplify.**

47. \( \sqrt[3]{196c^6d^4} \)
48. \( \sqrt[3]{-64y^6z^6} \)
49. \( \sqrt[3]{-27a^{15}b^9} \)
50. \( \sqrt[3]{-16x^{16}y^8} \)

51. \( \sqrt[3]{400x^{16}y^6} \)
52. \( \sqrt[3]{8c^3d^{12}} \)
53. \( \sqrt[3]{64(x + y)^6} \)
54. \( \sqrt[3]{-(y - z)^{15}} \)

**55. PHYSICS** Johannes Kepler developed the formula \( d = \sqrt{61t^2} \), where \( d \) is the distance of a planet from the Sun in millions of miles and \( t \) is the number of Earth-days that it takes for the planet to orbit the Sun. If the length of a year on Mars is 687 Earth-days, how far from the Sun is Mars?

**56. CHEMISTRY** All matter is composed of atoms. The nucleus of an atom is the center portion of the atom that contains most of the mass of the atom. A theoretical formula for the radius \( r \) of the nucleus of an atom is \( r = (1.3 \times 10^{-15})\sqrt{A} \) meters, where \( A \) is the mass number of the nucleus. Find the radius of the nucleus for each atom in the table.

<table>
<thead>
<tr>
<th>Atom</th>
<th>Mass Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>carbon</td>
<td>6</td>
</tr>
<tr>
<td>oxygen</td>
<td>8</td>
</tr>
<tr>
<td>sodium</td>
<td>11</td>
</tr>
<tr>
<td>aluminum</td>
<td>13</td>
</tr>
<tr>
<td>chlorine</td>
<td>17</td>
</tr>
</tbody>
</table>
57. **BIOLOGY** Kleiber’s Law, \( P = 73.3\sqrt[4]{m^3} \), shows the relationship between the mass \( m \) in kilograms of an organism and its metabolism \( P \) in Calories per day. Determine the metabolism for each of the animals listed at the right.

<table>
<thead>
<tr>
<th>Animal</th>
<th>Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>bald eagle</td>
<td>4.5</td>
</tr>
<tr>
<td>golden retriever</td>
<td>30</td>
</tr>
<tr>
<td>komodo dragon</td>
<td>72</td>
</tr>
<tr>
<td>bottlenose dolphin</td>
<td>156</td>
</tr>
<tr>
<td>Asian elephant</td>
<td>2300</td>
</tr>
</tbody>
</table>

58. **MULTIPLE REPRESENTATIONS** In this problem, you will use \( f(x) = x^n \) and \( g(x) = \sqrt[n]{x} \) to explore inverses.

a. **Tabular** Make tables for \( f(x) \) and \( g(x) \) using \( n = 3 \) and \( n = 4 \).

b. **Graphical** Graph the equations.

c. **Analytical** Which equations are functions? Which functions are one-to-one?

d. **Analytical** For what values of \( n \) are \( g(x) \) and \( f(x) \) inverses of each other?

e. **Verbal** What conclusions can you make about \( g(x) = \sqrt[n]{x} \) and \( f(x) = x^n \) for all positive even values of \( n \)? for odd values of \( n \)?

### H.O.T. Problems Use Higher-Order Thinking Skills

59. **ERROR ANALYSIS** Ashley and Kimi are simplifying \( \sqrt[4]{16x^4y^8} \). Is either of them correct? Explain your reasoning.

- **Ashley**
  \[
  \sqrt[4]{16x^4y^8} = \sqrt[4]{(2xy^2)^4} = 2|xy^2| 
  \]

- **Kimi**
  \[
  \sqrt[4]{16x^4y^8} = \sqrt[4]{(2xy^2)^4} = 2y^2|x| 
  \]

60. **CHALLENGE** Under what conditions is \( \sqrt{x^2 + y^2} = x + y \) true?

61. **REASONING** Determine whether the statement \( \sqrt[4]{(-x)^4} = x \) is sometimes, always, or never true.

62. **CHALLENGE** For what real values of \( x \) is \( \sqrt[3]{x^3} > x \)?

63. **OPEN ENDED** Write a number for which the principal square root and cube root are both integers.

64. **WRITING IN MATH** Explain when and why absolute value symbols are needed when taking an \( n \)th root.

65. **CHALLENGE** Write an equivalent expression for \( \sqrt{2x} \cdot \sqrt{8y} \). Simplify the radical.

### CHALLENGE Simplify each expression.

66. \( \sqrt{0.0016} \)

67. \( \sqrt[4]{-0.000001} \)

68. \( \sqrt{-0.00032} \cdot \sqrt[4]{-0.027} \)

69. **CHALLENGE** Solve \( \frac{5}{\sqrt{a}} = -125 \) for \( a \).
## Standardized Test Practice

**70.** What is the value of \( w \) in the equation \( \frac{1}{2}(4w + 36) = 3(4w - 3) \)?

- A 2
- B 2.7
- C 27
- D 36

**71.** What is the product of the complex numbers \((5 + i)\) and \((5 - i)\)?

- F 24
- G 26
- H 25 - \( i \)
- J 26 - 10\( i \)

**72.** **EXTENDED RESPONSE** A cylindrical cooler has a diameter of 9 inches and a height of 11 inches. Tate plans to use it for soda cans that have a diameter of 2.5 inches and a height of 4.75 inches.

- a. Tate plans to place two layers consisting of 9 cans each into the cooler. What is the volume of the space that will not be filled with the cans?
- b. Find the ratio of the volume of the cooler to the volume of the cans in part a.

**73.** **SAT/ACT** Which of the following is closest to \( \sqrt[7]{32} \)?

- A 1.8
- B 1.9
- C 2.0
- D 2.1
- E 2.2

## Spiral Review

**Graph each function.** (Lesson 7-3)

74. \( y = \sqrt{x} - 5 \)  
75. \( y = \sqrt{x} - 2 \)  
76. \( y = 3\sqrt{x} + 4 \)

**77.** **HEALTH** The average weight of a baby born at a certain hospital is \( 7\frac{1}{2} \) pounds and the average length is 19.5 inches. One kilogram is about 2.2 pounds and 1 centimeter is about 0.3937 inches. Find the average weight in kilograms and the length in centimeters. (Lesson 7-2)

**Simplify.** (Lesson 6-1)

78. \( (4c - 5) - (c + 11) + (-6c + 17) \)  
79. \( (11x^2 + 13x - 15) - (7x^2 - 9x + 19) \)  
80. \( (d - 5)(d + 3) \)  
81. \( (2a^2 + 6)^2 \)  

**82.** **GAS MILEAGE** The gas mileage \( y \) in miles per gallon for a certain vehicle is given by the equation \( y = 10 + 0.9x - 0.01x^2 \), where \( x \) is the speed of the vehicle between 10 and 75 miles per hour. Find the range of speeds that would give a gas mileage of at least 25 miles per gallon. (Lesson 5-8)

**Write each equation in vertex form, if not already in that form. Identify the vertex, axis of symmetry, and direction of opening. Then graph the function.** (Lesson 5-7)

83. \( y = -6(x + 2)^2 + 3 \)  
84. \( y = -\frac{1}{3}x^2 + 8x \)  
85. \( y = (x - 2)^2 - 2 \)  
86. \( y = 2x^2 + 8x + 10 \)

## Skills Review

**Find each product.** (Lesson 6-1)

87. \( (x + 4)(x + 5) \)  
88. \( (y - 3)(y + 4) \)  
89. \( (a + 2)(a - 9) \)  
90. \( (a - b)(a - 3b) \)  
91. \( (x + 2y)(x - y) \)  
92. \( 2(w + z)(w - 4z) \)
You can use a graphing calculator to graph $n$th root functions.

**Example 1** Graph an $n$th Root Function

Graph $y = \sqrt[n]{x}$.

Enter the equation as $Y_1$ and graph.

**KEYSTROKES:**

```
Y = 5 MATH 5 X,T,\theta,n GRAPH
```

Another way to enter the equation is to use $y = x^{\frac{1}{n}}$. You will learn about this later in Chapter 7.

**Example 2** $n$th Root Functions with Different Roots

Graph and compare $y = \sqrt[n]{x}$ and $y = \sqrt[n]{x-2}$.

Enter $y = \sqrt[n]{x}$ as $Y_1$ and $y = \sqrt[n]{x-2}$ as $Y_2$. Change the viewing window. Then graph.

**KEYSTROKES:**

```
Y = CLEAR 2nd [\sqrt] (X,T,\theta,n) ENTER 4
MATH 5 X,T,\theta,n GRAPH
WINDOW (-) 2 ENTER 10 ENTER 1 ENTER
(-) 2 ENTER 10 ENTER 1
```

**Example 3** $n$th Root Functions with Different Radicands

Graph and compare $y = \sqrt[3]{x}$, $y = \sqrt[3]{x+4}$, and $y = \sqrt[3]{x-4}$.

Enter $y = \sqrt[3]{x}$ as $Y_1$, $y = \sqrt[3]{x+4}$ as $Y_2$, and $y = \sqrt[3]{x-4}$ as $Y_3$. Then graph in the standard viewing window.

**KEYSTROKES:**

```
Y = CLEAR 3 MATH 5 X,T,\theta,n ENTER CLEAR 3
MATH 5 (X,T,\theta,n + 4) ENTER 3 MATH
5 X,T,\theta,n + 4 ENTER ZOOM 6
```

**Exercises**

Graph each function.

1. $y = \sqrt{x}$
2. $y = \sqrt[4]{x+2}$
3. $y = \sqrt{x} + 2$
4. $y = \sqrt[4]{x}$
5. $y = \sqrt[3]{x-5}$
6. $y = \sqrt[4]{x} - 5$
7. What is the effect of adding or subtracting a constant under the radical sign?
8. What is the effect of adding or subtracting a constant outside the radical sign?
Mid-Chapter Quiz
Lessons 7-1 through 7-4

Given \( f(x) = 2x^2 + 4x - 3 \) and \( g(x) = 5x - 2 \), find each function. (Lesson 7-1)

1. \((f + g)(x)\)
2. \((f - g)(x)\)
3. \((f \cdot g)(x)\)
4. \(\left(\frac{f}{g}\right)(x)\)
5. \([f \circ g](x)\)
6. \([g \circ f](x)\)

7. **SHOPPING** Mrs. Ross is shopping for her children's school clothes. She has a coupon for 25% off her total. The sales tax of 6% is added to the total after the coupon is applied. (Lesson 7-1)

a. Express the total price after the discount and the total price after the tax using function notation. Let \( x \) represent the price of the clothing, \( p(x) \) represent the price after the 25% discount, and \( g(x) \) represent the price after the tax is added.

b. Which composition of functions represents the final price, \( p \circ g(x) \) or \( g \circ p(x) \)? Explain your reasoning.

Determine whether each pair of functions are inverse functions. Write yes or no. (Lesson 7-2)

8. \( f(x) = 2x + 16 \) \hspace{1cm} \( g(x) = \frac{1}{2}x - 8 \)
9. \( f(x) = 4x + 15 \) \hspace{1cm} \( h(x) = \frac{1}{4}x - 15 \)
10. \( f(x) = x^2 - 5 \) \hspace{1cm} \( g(x) = 5 + x^{-2} \)
11. \( f(x) = -6x + 8 \) \hspace{1cm} \( h(x) = \frac{8 - x}{6} \)

Find the inverse of each function, if it exists. (Lesson 7-2)

12. \( h(x) = \frac{2}{5}x + 8 \)
13. \( f(x) = \frac{4}{9}(x - 3) \)
14. \( h(x) = -\frac{10}{3}(x + 5) \)
15. \( f(x) = \frac{x + 12}{7} \)

16. **JOBS** Louise runs a lawn care service. She charges $25 for supplies plus $15 per hour. The function \( f(h) = 15h + 25 \) gives the cost \( f(h) \) for \( h \) hours of work. (Lesson 7-2)

a. Find \( f^{-1}(h) \). What is the significance of \( f^{-1}(h) \)?

b. If Louise charges a customer $85, how many hours did she work?

Graph each inequality. (Lesson 7-3)

17. \( y < \sqrt{x - 5} \)
18. \( y \leq -2\sqrt{x} \)
19. \( y > \sqrt{x + 9} + 3 \)
20. \( y \geq \sqrt{x + 4} - 5 \)

Graph each function. State the domain and range of each function. (Lesson 7-3)

21. \( y = 2 + \sqrt{x} \)
22. \( y = \sqrt{x + 4} - 1 \)

23. **MULTIPLE CHOICE** What is the domain of \( f(x) = \sqrt{2x + 5} \)? (Lesson 7-3)

A \( \{ x | x > \frac{5}{2} \} \)
B \( \{ x | x > -\frac{5}{2} \} \)
C \( \{ x | x \geq \frac{5}{2} \} \)
D \( \{ x | x \geq -\frac{5}{2} \} \)

Simplify. (Lesson 7-4)

24. \( \pm \sqrt{121a^4b^{18}} \)
25. \( \sqrt{(x^4 + 3)^{12}} \)
26. \( \sqrt[3]{27(2x - 5)^{15}} \)
27. \( \sqrt[5]{-(y - 6)^{20}} \)
28. \( \sqrt[3]{8(x + 4)^6} \)
29. \( \sqrt[4]{16(y + x)^6} \)

30. **MULTIPLE CHOICE** The radius of the cylinder below is equal to the height of the cylinder. The radius \( r \) can be found using the formula \( r = \sqrt[3]{\frac{V}{\pi}} \). Find the radius of the cylinder if the volume is 500 cubic inches. (Lesson 7-4)

\[ \text{F} \quad 2.53 \text{ inches} \]
\[ \text{G} \quad 5.42 \text{ inches} \]
\[ \text{H} \quad 7.94 \text{ inches} \]
\[ \text{J} \quad 24.92 \text{ inches} \]

31. **PRODUCTION** The cost in dollars of producing \( x \) cell phones in a factory is represented by \( C(p) = 5p + 60 \). The number of cell phones produced in \( h \) hours is represented by \( P(h) = 40h \). (Lesson 7-1)

a. Find the composition function.

b. Determine the cost of producing cell phones for 8 hours.
7-5 Operations with Radical Expressions

**Then**
- You simplified expressions with \( n \)th roots. (Lesson 7-4)

**Now**
1. Simplify radical expressions.
2. Add, subtract, multiply, and divide radical expressions.

**Why?**
- Golden rectangles have been used by artists and architects to create beautiful designs. Many golden rectangles appear in the Parthenon in Athens, Greece. The ratio of the lengths of the sides of a golden rectangle is \( \frac{2}{\sqrt{5} - 1} \).
- In this lesson, you will learn to simplify radical expressions like \( \frac{2}{\sqrt{5} - 1} \).

**NewVocabulary**
- Rationalizing the denominator
- Like radical expressions
- Conjugate

**Tennessee Curriculum Standards**
- \( 3103.1 \) Perform operations on algebraic expressions and justify the procedures.
- \( 3103.3.6 \) Simplify expressions and solve equations containing radicals.

**KeyConcept** **Product Property of Radicals**

**Words**
For any real numbers \( a \) and \( b \) and any integer \( n > 1 \), \( \sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b} \), if \( n \) is even and \( a \) and \( b \) are both nonnegative or if \( n \) is odd.

**Examples**
\[
\begin{align*}
\sqrt{2} \cdot \sqrt{8} & = \sqrt{16} \text{ or } 4 \\
\sqrt[3]{3} \cdot \sqrt[9]{9} & = \sqrt[27]{27} \text{ or } 3
\end{align*}
\]

In order for a radical to be in simplest form, the radicand must contain no factors that are \( n \)th powers of an integer or polynomial.

**Example 1** **Simplify Expressions with the Product Property**

**Simplify.**

a. \( \sqrt{32x^8} \)
\[
\begin{align*}
\sqrt{32x^8} & = \sqrt{2^5 \cdot (x^4)^2} \\
& = \sqrt{2^2 \cdot (x^4)^2 \cdot 2} \\
& = 4x^4 \sqrt{2}
\end{align*}
\]

b. \( \sqrt[4]{16a^{24}b^{13}} \)
\[
\begin{align*}
\sqrt[4]{16a^{24}b^{13}} & = \sqrt[4]{2^4 \cdot (a^6)^4 \cdot (b^3)^4 \cdot b} \\
& = \sqrt[4]{2^4} \cdot \sqrt[4]{(a^6)^4} \cdot \sqrt[4]{(b^3)^4} \cdot \sqrt[4]{b} \\
& = 2a^6 b^3 \sqrt[4]{b}
\end{align*}
\]

In this case, the absolute value symbols are not necessary because in order for \( \sqrt[4]{16a^{24}b^{13}} \) to be defined, \( b \) must be nonnegative.

Thus, \( \sqrt[4]{16a^{24}b^{13}} = 2a^6 b^3 \sqrt[4]{b} \).

**GuidedPractice**

1A. \( \sqrt{12c^6d^3} \)
1B. \( \sqrt[3]{27y^{12}z^7} \)
The Quotient Property of Radicals is another property used to simplify radicals.

### Key Concept Quotient Property of Radicals

**Words**
For any real numbers $a$ and $b \neq 0$ and any integer $n > 1$,
\[
\sqrt[\,\,n]{\frac{a}{b}} = \frac{\sqrt[\,\,n]{a}}{\sqrt[\,\,n]{b}} \quad \text{if all roots are defined.}
\]

**Examples**
\[
\frac{\sqrt[\,\,3]{27}}{\sqrt[\,\,3]{3}} = \sqrt[\,\,3]{9} = 3 \\
\frac{\sqrt[\,\,4]{x^6}}{\sqrt[\,\,4]{8}} = \frac{\sqrt[\,\,4]{x^6}}{\sqrt[\,\,4]{(2^3)}} = \frac{x^2}{2} \quad \text{or} \quad \frac{1}{2}x^2
\]

To eliminate radicals from a denominator or fractions from a radicand, you can use a process called **rationalizing the denominator**. To rationalize a denominator, multiply the numerator and denominator by a quantity so that the radicand has an exact root.

<table>
<thead>
<tr>
<th>If the denominator is:</th>
<th>Multiply the numerator and denominator by:</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{b}$</td>
<td>$\sqrt{b}$</td>
<td>$\frac{2}{\sqrt{3}} = \frac{2 \cdot \sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$</td>
</tr>
<tr>
<td>$\sqrt{b^x}$</td>
<td>$\sqrt[,,x]{b^n-x}$</td>
<td>$\frac{5}{\sqrt{2}} = \frac{5 \cdot \sqrt{2}}{\sqrt{2} = \frac{5\sqrt{4}}{2}}$</td>
</tr>
</tbody>
</table>

### Example 2 Simplify Expressions with the Quotient Property

**Simplify.**

**a.** $\sqrt[\,\,3]{\frac{x^6}{y^2}}$

\[
\sqrt[\,\,3]{\frac{x^6}{y^2}} = \frac{\sqrt[\,\,3]{x^6}}{\sqrt[\,\,3]{y^2}} \quad \text{Quotient Property}
\]

\[
= \frac{\sqrt[\,\,3]{(x^2)^2}}{\sqrt[\,\,3]{y^2} \cdot y} \quad \text{Factor into squares.}
\]

\[
= \frac{\sqrt[\,\,3]{(x^2)^2}}{\sqrt[\,\,3]{y^2} \cdot \sqrt[\,\,3]{y}} \quad \text{Product Property}
\]

\[
= \frac{x^2}{y^2} \sqrt[\,\,3]{\frac{y}{y^2}} \quad \text{Simplify.}
\]

\[
= \frac{x^2}{y^2} \cdot \sqrt[\,\,3]{\frac{y}{y^2}} \quad \text{Rationalize the denominator.}
\]

\[
= \frac{x^2 \sqrt[\,\,3]{y}}{y^2} \quad \sqrt[\,\,3]{y} \cdot \sqrt[\,\,3]{y} = y
\]

**b.** $\sqrt[\,\,4]{\frac{6}{5x}}$

\[
\sqrt[\,\,4]{\frac{6}{5x}} = \frac{\sqrt[\,\,4]{6}}{\sqrt[\,\,4]{5x}} \quad \text{Quotient Property}
\]

\[
= \frac{\sqrt[\,\,4]{6 \cdot 5^3x^3}}{\sqrt[\,\,4]{5x \cdot 5^3x^3}} \quad \text{Rationalize the denominator.}
\]

\[
= \frac{\sqrt[\,\,4]{750x^3}}{\sqrt[\,\,4]{5^4x^4}} \quad \text{Product Property}
\]

\[
= \frac{\sqrt[\,\,4]{750x^3}}{5x} \quad \text{Multiply.}
\]

\[
= \frac{\sqrt[\,\,4]{750x^3}}{5} \quad \sqrt[\,\,4]{5^4x^4} = 5x
\]

### Guided Practice

2A. $\frac{\sqrt{a^9}}{\sqrt{b^5}}$

2B. $\sqrt[\,\,4]{\frac{3}{4y}}$
Here is a summary of the rules used to simplify radicals.

### Concept Summary: Simplifying Radical Expressions

A radical expression is in simplified form when the following conditions are met:

- The index $n$ is as small as possible.
- The radicand contains no factors (other than 1) that are $n$th powers of an integer or polynomial.
- The radicand contains no fractions.
- No radicals appear in a denominator.

#### Operations with Radicals

You can use the Product and Quotient Properties to multiply and divide some radicals.

### Example 3: Multiply Radicals

**Simplify $5\sqrt{-12ab^4} \cdot 3\sqrt{18a^2b^2}$.**

$$
5\sqrt{-12ab^4} \cdot 3\sqrt{18a^2b^2} = 5 \cdot 3 \cdot \sqrt{-12ab^4 \cdot 18a^2b^2}
$$

$ = 15 \cdot \sqrt{-2^2 \cdot 3 \cdot a^4 \cdot 2 \cdot 3^3 \cdot b^6}

$ = 15 \cdot (-2) \cdot a \cdot b^2

$ = -90ab^2$

#### Guided Practice

**Simplify.**

3A. $6\sqrt{8c^3d^5} \cdot 4\sqrt{2cd^3}$

3B. $2\sqrt{8x^3y^2} \cdot 3\sqrt{2x^3y^2}$

Radicals can be added and subtracted in the same manner as monomials. In order to add or subtract, the radicals must be like terms. Radicals are **like radical expressions** if both the index and the radicand are identical.

### Example 4: Add and Subtract Radicals

**Simplify $\sqrt{98} - 2\sqrt{32}$.**

$$
\sqrt{98} - 2\sqrt{32} = \sqrt{2^2 \cdot 7^2} - 2\sqrt{2^2 \cdot 2^2}
$$

$ = 7\sqrt{2} - 2 \cdot 2 \cdot \sqrt{2}

$ = 7\sqrt{2} - 8\sqrt{2}

$ = -\sqrt{2}$

**Guided Practice**

4A. $4\sqrt{8} + 3\sqrt{50}$

4B. $5\sqrt{12} + 2\sqrt{27} - \sqrt{128}$
Just as you can add and subtract radicals like monomials, you can multiply radicals using the FOIL method as you do when multiplying binomials.

**Example 5 Multiply Radicals**

Simplify \((4\sqrt{3} + 5\sqrt{2})(3\sqrt{2} - 6)\).

\[
(4\sqrt{3} + 5\sqrt{2})(3\sqrt{2} - 6) = 4\sqrt{3} \cdot 3\sqrt{2} + 4\sqrt{3} \cdot (-6) + 5\sqrt{2} \cdot 3\sqrt{2} + 5\sqrt{2} \cdot (-6)
\]

\[
= 12\sqrt{3} \cdot 2 - 24\sqrt{3} + 15\sqrt{2}^2 - 30\sqrt{2}
\]

\[
= 12\sqrt{6} - 24\sqrt{3} + 30 - 30\sqrt{2}
\]

**Guided Practice**

Simplify.

5A. \((6\sqrt{3} - 5)(2\sqrt{5} + 4\sqrt{2})\)

5B. \((7\sqrt{2} - 3\sqrt{3})(7\sqrt{2} + 3\sqrt{3})\)

Binomials of the form \(a\sqrt{b} + c\sqrt{d}\) and \(a\sqrt{b} - c\sqrt{d}\), where \(a, b, c,\) and \(d\) are rational numbers, are called **conjugates** of each other. You can use conjugates to rationalize denominators.

**Real-World Example 6 Use a Conjugate to Rationalize a Denominator**

**ARCHITECTURE** Refer to the beginning of the lesson. Use a conjugate to rationalize the denominator and simplify \(\frac{2}{\sqrt{5} - 1}\).

\[
\frac{2}{\sqrt{5} - 1} = \frac{2}{\sqrt{5} - 1} \cdot \frac{\sqrt{5} + 1}{\sqrt{5} + 1}
\]

\[
= \frac{2\sqrt{5} + 2(1)}{(\sqrt{5})^2 + 1(\sqrt{5}) - 1(\sqrt{5}) - 1(1)}
\]

\[
= \frac{2\sqrt{5} + 2}{5 + \sqrt{5} - \sqrt{5} - 1}
\]

\[
= \frac{2\sqrt{5} + 2}{4}
\]

\[
= \frac{\sqrt{5} + 1}{2}
\]

**Guided Practice**

6. **GEOMETRY** The area of the rectangle at the right is 900 ft². Write and simplify an equation for \(L\) in terms of \(x\).
Examples 1–5 Simplify.
1. \(\sqrt{36ab^4c^5}\)  
2. \(\sqrt{144x^2y^5}\)  
3. \(\sqrt{\frac{c^5}{d^9}}\)  
4. \(\sqrt[4]{\frac{5x}{8y}}\)  
5. \(5\sqrt{2x} \cdot 3\sqrt{8x}\)  
6. \(4\sqrt{5a^3} \cdot \sqrt{125a^3}\)  
7. \(3\sqrt{36xy} \cdot 2\sqrt{6x^2y^2}\)  
8. \(\sqrt{3x^3y^2} \cdot \sqrt{27xy^2}\)  
9. \(5\sqrt{32} + \sqrt{27} + 2\sqrt{75}\)  
10. \(4\sqrt{40} + 3\sqrt{28} - \sqrt{200}\)  
11. \((4 + 2\sqrt{5})(3\sqrt{3} + 4\sqrt{5})\)  
12. \((8\sqrt{3} - 2\sqrt{2})(8\sqrt{3} + 2\sqrt{2})\)  
13. \(\frac{5}{\sqrt{2} + 3}\)  
14. \(\frac{8}{\sqrt{6} - 5}\)  
15. \(\frac{4 + \sqrt{2}}{\sqrt{2} - 3}\)  
16. \(\frac{6 - \sqrt{3}}{\sqrt{3} + 4}\)

Example 6 17. GEOMETRY Find the altitude of the triangle if the area is \(189 + 4\sqrt{3}\) square centimeters.

Practice and Problem Solving

Examples 1–4 Simplify.
18. \(\sqrt{72a^8b^5}\)  
19. \(\sqrt{9a^{15}b^3}\)  
20. \(\sqrt{24a^{16}b^8c}\)  
21. \(\sqrt{18a^6b^3c^5}\)  
22. \(\frac{\sqrt{5a^5}}{\sqrt{b^{13}}}\)  
23. \(\sqrt{\frac{7x}{10y^3}}\)  
24. \(\sqrt{\frac{6x^2}{\sqrt{5y}}}\)  
25. \(\sqrt{\frac{7x^3}{4b^2}}\)  
26. \(3\sqrt{5y} \cdot 8\sqrt{10yz}\)  
27. \(2\sqrt{32a^3b^5} \cdot \sqrt{8a^7b^2}\)  
28. \(6\sqrt{3ab} \cdot 4\sqrt{24ab^3}\)  
29. \(5\sqrt{x^8y^3} \cdot 5\sqrt{2x^5y^4}\)  
30. \(3\sqrt{90} + 4\sqrt{20} + \sqrt{162}\)  
31. \(9\sqrt{12} + 5\sqrt{32} - \sqrt{72}\)  
32. \(4\sqrt{28} - 8\sqrt{810} + \sqrt{44}\)  
33. \(3\sqrt{54} + 6\sqrt{288} - \sqrt{147}\)  
34. GEOMETRY Find the perimeter of the rectangle.
35 GEOMETRY Find the area of the rectangle.
36. GEOMETRY Find the exact surface area of a sphere with radius of \(4 + \sqrt{5}\) inches.

Examples 5–6 Simplify.
37. \((7\sqrt{2} - 3\sqrt{3})(4\sqrt{6} + 3\sqrt{12})\)  
38. \((8\sqrt{5} - 6\sqrt{3})(8\sqrt{5} + 6\sqrt{3})\)  
39. \((12\sqrt{10} - 6\sqrt{5})(12\sqrt{10} + 6\sqrt{5})\)  
40. \((6\sqrt{5} + 5\sqrt{2})(2\sqrt{6} + 3\sqrt{8})\)  
41. \(\frac{6}{\sqrt{3} - \sqrt{2}}\)  
42. \(\frac{\sqrt{2}}{\sqrt{5} - \sqrt{3}}\)  
43. \(\frac{9 - 2\sqrt{3}}{\sqrt{3} + 6}\)  
44. \(\frac{2\sqrt{2} + 2\sqrt{5}}{\sqrt{5} + \sqrt{2}}\)
Simplify.

45. \( \sqrt{16y^{12}} \)
46. \( \sqrt{-54x^6y^{11}} \)
47. \( \sqrt[3]{162a^6b^{13}c} \)

48. \( \sqrt{48a^9b^3c^{16}} \)
49. \( \sqrt{\frac{12x^3y^2}{5a^2b}} \)
50. \( \sqrt{\frac{36xy^2}{10xz}} \)

51. \( \frac{x + 1}{\sqrt{x} - 1} \)
52. \( \frac{x - 2}{\sqrt{x^2 - 4}} \)
53. \( \frac{\sqrt{x}}{\sqrt{x^2 - 1}} \)

54. **APPLES** The diameter of an apple is related to its weight and can be modeled by the formula \( d = \sqrt[3]{3w} \), where \( d \) is the diameter in inches and \( w \) is the weight in ounces. Find the diameter of an apple that weighs 6.47 ounces.

Simplify each expression if \( b \) is an even number.

55. \( \sqrt[5]{a^b} \)
56. \( \sqrt[4]{a^{4b}} \)
57. \( \sqrt[3]{a^b} \)
58. \( \sqrt[5]{a^3b} \)

59. **MULTIPLE REPRESENTATIONS** In this problem, you will explore operations with like radicals.

   a. **Numerical** Copy the diagram at the right on dot paper. Use the Pythagorean Theorem to prove that the length of the red segment is \( \sqrt[2]{2} \) units.

   b. **Graphical** Extend the segment to represent \( \sqrt[2]{2} + \sqrt[2]{2} \).

   c. **Analytical** Use your drawing to show that \( \sqrt[2]{2} + \sqrt[2]{2} \neq \sqrt[2]{2 + 2} \) or 2.

   d. **Graphical** Use the dot paper to draw a square with side lengths \( \sqrt[2]{2} \) units.

   e. **Numerical** Prove that the area of the square is \( \sqrt[2]{2} \cdot \sqrt[2]{2} = 2 \) square units.

---

**H.O.T. Problems** Use Higher-Order Thinking Skills

60. **ERROR ANALYSIS** Twyla and Ben are simplifying \( 4\sqrt{32} + 6\sqrt{18} \). Is either of them correct? Explain your reasoning.

   **Twyla**
   
   \[
   4\sqrt{32} + 6\sqrt{18} = 4\sqrt{4^2 \cdot 2} + 6\sqrt{3^2 \cdot 2} = 16\sqrt{2} + 18\sqrt{2} = 34\sqrt{2}
   \]

   **Ben**
   
   \[
   4\sqrt{32} + 6\sqrt{18} = 4\sqrt{16 \cdot 2} + 6\sqrt{9 \cdot 2} = 64\sqrt{2} + 54\sqrt{2} = 118\sqrt{2}
   \]

61. **CHALLENGE** Show that \( -\frac{1 - i\sqrt{3}}{2} \) is a cube root of 1.

62. **REASONING** For what values of \( a \) is \( \sqrt{a} \cdot \sqrt{-a} \) a real number? Explain.

63. **CHALLENGE** Find four combinations of whole numbers that satisfy \( \sqrt{256} = b \).

64. **OPEN ENDED** Find a number other than 1 that has a positive whole number for a square root, cube root, and fourth root.

65. **WRITING IN MATH** Explain why absolute values may be unnecessary when an \( n \)th root of an even power results in an odd power.
66. **PROBABILITY** A six-sided number cube has faces with the numbers 1 through 6 marked on it. What is the probability that a number less than 4 will occur on one toss of the number cube?

A $\frac{1}{2}$  
B $\frac{1}{3}$  
C $\frac{1}{4}$  
D $\frac{1}{5}$

67. When the number of a year is divisible by 4, the year is a leap year. However, when the year is divisible by 100, the year is not a leap year, unless the year is divisible by 400. Which is not a leap year?

F 1884  
H 1904  
G 1900  
J 1940

68. **SHORT RESPONSE** Which property is illustrated by $4x + 0 = 4x$?

69. **SAT/ACT** The expression $\sqrt{180a^2b^8}$ is equivalent to which of the following?

A $3\sqrt{10} \left| a \right|b^4$  
B $5\sqrt{6} \left| a \right|b^4$  
C $6\sqrt{5} \left| a \right|b^4$  
D $18\sqrt{10} \left| a \right|b^4$  
E $36\sqrt{5} \left| a \right|b^4$

---

**Spiral Review**

Simplify. (Lesson 7-4)

70. $\sqrt{81x^6}$  
71. $\sqrt[3]{729a^3b^9}$  
72. $\sqrt{(g + 5)^2}$

73. Graph $y \leq \sqrt{x} - 2$. (Lesson 7-3)

Solve each equation. (Lesson 6-5)

74. $x^4 - 34x^2 + 225 = 0$  
75. $x^4 - 15x^2 - 16 = 0$  
76. $x^4 + 6x^2 - 27 = 0$  
77. $x^3 + 64 = 0$  
78. $27x^3 + 1 = 0$  
79. $8x^3 - 27 = 0$

80. **MODELS** A model car builder is building a display table for model cars. He wants the perimeter of the table to be 26 feet, but he wants the area of the table to be no more than 30 square feet. What could be the width of the table? (Lesson 5-8)

81. **CONSTRUCTION** Cho charges $1500 to build a small deck and $2500 to build a large deck. During the spring and summer, she built 5 more small decks than large decks. If she earned $23,500 how many of each type of deck did she build? (Lesson 4-6)

82. **FOOD** The Hot Dog Grille offers the lunch combinations shown. Assume that the price of a combo meal is the same price as purchasing each item separately. Find the prices for a hot dog, a soda, and a bag of potato chips. (Lesson 3-5)

**Lunch Combo Meals**

1. Two hot dogs, one soda .................. $5.40  
2. One hot dog, potato chips, one soda.............. $4.35  
3. Two hot dogs, two bags of chips .................. $5.70

---

**Skills Review**

Evaluate each expression.

83. $2 \left( \frac{1}{6} \right)$  
84. $3 \left( \frac{1}{8} \right)$  
85. $\frac{1}{4} + \frac{1}{3}$

86. $\frac{1}{2} + \frac{3}{8}$  
87. $\frac{2}{3} - \frac{1}{4}$  
88. $\frac{5}{6} - \frac{2}{5}$
1 Rational Exponents and Radicals

You know that squaring a number and taking the square root of a number are inverse operations. But how would you evaluate an expression that contains a fractional exponent such as the one above? You can investigate such an expression by assuming that fractional exponents behave as integral exponents.

\[
\left( \frac{1}{b^2} \right)^2 = b^{\frac{1}{2}} \cdot b^{\frac{1}{2}} \]

Write as a multiplication expression.

\[
= b^{\frac{1}{2} + \frac{1}{2}} \]

Add the exponents.

\[
= b^1 \text{ or } b \]

Simplify.

Thus, \( b^{\frac{1}{2}} \) is a number with a square equal to \( b \). So \( b^{\frac{1}{2}} = \sqrt{b} \).

---

Example 1 Radical and Exponential Forms

Simplify.

a. Write \( x^6 \) in radical form.

\[
x^6 = \sqrt[6]{x} \quad \text{Definition of } b^{\frac{1}{6}}
\]

b. Write \( \sqrt{z} \) in exponential form.

\[
\sqrt{z} = z^{\frac{1}{2}} \quad \text{Definition of } b^{\frac{1}{2}}
\]

Guided Practice

1A. Write \( a^3 \) in radical form.

1B. Write \( \sqrt{c} \) in exponential form.

1C. Write \( d^4 \) in radical form.

1D. Write \( \sqrt[3]{c^{-5}} \) in exponential form.
The rules for negative exponents also apply to negative rational exponents.

### Example 2 Evaluate Expressions with Rational Exponents

Evaluate each expression.

a. \(81^{-\frac{1}{4}}\)

\[
81^{-\frac{1}{4}} = \frac{1}{81^{\frac{1}{4}}} = \frac{1}{\sqrt[4]{81}} = \frac{1}{3}
\]

b. \(216^{\frac{2}{3}}\)

\[
216^{\frac{2}{3}} = (6^3)^{\frac{2}{3}} = 6^{3 \cdot \frac{2}{3}} = 6^{\frac{2}{1}} = 6^2
\]

### Guided Practice

2A. \(-3125^{-\frac{1}{5}}\)

2B. \(256^{\frac{3}{8}}\)

Examples 2a and 2b use the definition of \(b^{-\frac{1}{n}}\) and the properties of powers to evaluate an expression. Both methods suggest the following general definition of rational exponents

### Key Concept Rational Exponents

Words

For any real nonzero number \(b\), and any integers \(x\) and \(y\), with \(y > 1\), \(b^\frac{x}{y} = \sqrt[y]{b^x} = (\sqrt[y]{b})^x\), except when \(b < 0\) and \(y\) is even. When \(b < 0\) and \(y\) is even, a complex root may exist.

Examples

\(27^\frac{2}{3} = (\sqrt[3]{27})^2 = 3^2\) or 9

\((-16)^\frac{3}{2} = (\sqrt[2]{-16})^3 = (4i)^3\) or \(-64i\)

### Real-World Example 3 Solve Equations with Rational Exponents

**FINANCIAL LITERACY** Refer to the beginning of the lesson. Suppose a video game system costs $390 now. How much would the price increase in six months with an annual inflation rate of 5.3%?

\[
C = c(1 + r)^n
\]

\[
= 390(1 + 0.053)^{\frac{1}{2}}
\]

\[
= 390(1 + 0.053)^{\frac{6}{12}}
\]

\[
= 400.20
\]

In six months the price of the video game system will be $400.20 − $390.00 or $10.20 more than its current price.

### Guided Practice

3. Suppose a gallon of milk costs $2.99 now. How much would the price increase in 9 months with an annual inflation rate of 5.3%?
Simplify Expressions

All of the properties of powers you learned in Lesson 6-1 apply to rational exponents. Write each expression with all positive exponents. Also, any exponents in the denominator of a fraction must be positive integers. So, it may be necessary to rationalize a denominator.

Example 4 Simplify Expressions with Rational Exponents

Simplify each expression.

a. \( \frac{2}{3} \cdot \frac{4}{7} \)
\[ \frac{a^{\frac{2}{3}} \cdot a^{\frac{4}{7}}}{a^{\frac{2}{3}} + a^{\frac{4}{7}}} = \frac{2}{3} + \frac{4}{7} \quad \text{Add powers.} \]
\[ = \frac{6}{3} \quad \text{Add exponents.} \]

b. \( b^{-\frac{5}{6}} \)
\[ b^{-\frac{5}{6}} = \frac{1}{b^{\frac{5}{6}}} \quad \text{Why use } \frac{b^5}{b^6} \text{?} \]
\[ = \frac{\frac{1}{b^{\frac{5}{6}}}}{b^{\frac{5}{6}}} \quad \text{Rational exponents} \]
\[ = \frac{b^{\frac{1}{6}}}{b^{\frac{5}{6}}} \quad \text{Power of a Power} \]
\[ = \frac{b^{\frac{1}{6}}}{b} \quad \text{Quotient of Powers} \]
\[ = \frac{b^{\frac{1}{6}}}{b} \quad \text{Simplify.} \]
\[ = \frac{b^{\frac{1}{6}}}{b} \quad \text{Rewrite in radical form.} \]

Guided Practice

4A. \( p^{\frac{1}{4}} \cdot p^{\frac{9}{4}} \)
4B. \( r^{-\frac{4}{3}} \)

When simplifying a radical expression, always use the least index possible. Using rational exponents makes this process easier, but the answer should be written in radical form.

Example 5 Simplify Radical Expressions

Simplify each expression.

a. \( \sqrt[4]{27} \)
\[ \sqrt[4]{27} = \frac{27^{\frac{1}{4}}}{3^{\frac{1}{2}}} \quad \text{Rational exponents} \]
\[ = \frac{(3^3)^{\frac{1}{4}}}{3^{\frac{1}{2}}} \quad 27 = 3^3 \]
\[ = \frac{3^{\frac{3}{4}}}{3^{\frac{1}{2}}} \quad \text{Power of a Power} \]
\[ = 3^{\frac{3}{4} - \frac{1}{2}} \quad \text{Quotient of Powers} \]
\[ = 3^{\frac{1}{4}} \quad \text{Simplify.} \]
\[ = \sqrt[4]{3} \quad \text{Rewrite in radical form.} \]

b. \( \sqrt[6]{64z^{-6}} \)
\[ \sqrt[6]{64z^{-6}} = (64z^{-6})^{\frac{1}{3}} \quad \text{Rational exponents} \]
\[ = (8^2 \cdot z^6)^{\frac{1}{3}} \quad 64 = 8^2 \]
\[ = 8^{\frac{2}{3}} \cdot z^{6 \cdot \frac{1}{3}} \quad \text{Power of a Power} \]
\[ = 8^{\frac{2}{3}} \cdot z^2 \quad \text{Simplify.} \]
\[ = 4z^2 \quad \text{Rewrite in radical form.} \]
Radical Expressions

Write the simplified expression in the same form as the beginning expression. When you start with a radical expression, end with a radical expression. When you start with an expression with rational exponents, end with an expression with rational exponents.

c. \( \frac{x^2 - 2}{3x^2 + 2} \)

\[
\frac{1}{3x^2} - \frac{2}{3x^2 + 2} = \frac{x^2 - 2}{3x^2 + 2} \cdot \frac{3x^2 - 2}{3x^2 + 2}
\]

\[
= \frac{3x^2 - 8x^2/3 + 4}{9x^2/3 - 4}
\]

\[
= \frac{3x - 8x^2/3 + 4}{9x - 4}
\]

\( \text{Multiply.} \)

\( \text{Simplify.} \)

Study Tip

Radical Expressions Write the simplified expression in the same form as the beginning expression. When you start with a radical expression, end with a radical expression. When you start with an expression with rational exponents, end with an expression with rational exponents.

Guided Practice

5A. \( \frac{\sqrt{32}}{\sqrt{2}} \)

5B. \( \sqrt{16x^4} \)

5C. \( \frac{y^{\frac{1}{3}} + 2}{y^{\frac{1}{3}} - 2} \)

Concept Summary

Expressions with Rational Exponents

An expression with rational exponents is simplified when all of the following conditions are met.

- It has no negative exponents.
- It has no exponents that are not positive integers in the denominator.
- It is not a complex fraction.
- The index of any remaining radical is the least number possible.

Check Your Understanding

Example 1

Write each expression in radical form, or write each radical in exponential form.

1. \( 10^{\frac{1}{4}} \)

2. \( x^{\frac{3}{5}} \)

3. \( \sqrt[5]{15} \)

4. \( \sqrt[3]{7x^2y^9} \)

Example 2

Evaluate each expression.

5. \( 343^{\frac{1}{3}} \)

6. \( 32^{-\frac{1}{3}} \)

7. \( 125^{\frac{2}{3}} \)

8. \( \frac{24}{4^{\frac{3}{2}}} \)

Example 3

GARDENING If the area \( A \) of a square is known, then the lengths of its sides \( \ell \) can be computed using \( \ell = A^{\frac{1}{2}} \). You have purchased a 169 ft\(^2\) share in a community garden for the season. What is the length of one side of your square garden?

Example 4–5

Simplify each expression.

10. \( a^{\frac{3}{4}} \cdot a^{\frac{1}{2}} \)

11. \( \frac{x^{\frac{4}{3}}}{x^{\frac{1}{3}}} \)

12. \( \frac{b^3}{c^{\frac{1}{2}}} \cdot \frac{c}{b^3} \)

13. \( \sqrt[4]{9x^2} \)

14. \( \frac{\sqrt[4]{64}}{\sqrt[4]{4}} \)

15. \( \frac{g^{\frac{1}{2}} - 1}{g^{\frac{1}{2}} + 1} \)
Example 1  Write each expression in radical form, or write each radical in exponential form.

16. \(8^{\frac{1}{5}}\)  
17. \(4^{\frac{2}{7}}\)  
18. \(a^{\frac{3}{4}}\)  
19. \((x^{\frac{3}{2}})^{2}\)

20. \(\sqrt{17}\)  
21. \(\sqrt[3]{63}\)  
22. \(\sqrt[3]{5xy^2}\)  
23. \(\sqrt[4]{625x^2}\)

Example 2  Evaluate each expression.

24. \(27^{\frac{1}{3}}\)  
25. \(256^{\frac{1}{4}}\)  
26. \(16^{-\frac{1}{2}}\)  
27. \(81^{-\frac{1}{4}}\)

Example 3  28. **BASKETBALL**  A women's regulation-sized basketball is slightly smaller than a men's basketball. The radius \(r\) of the ball that holds \(V\) cubic units of air is \(\left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}\).

a. Find the radius of a women's basketball.

b. Find the radius of a men's basketball.

29. **GEOMETRY**  The radius \(r\) of a sphere with volume \(V\) is given by \(r = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}\). Find the radius of a ball with a volume of 77 cm\(^3\).

Examples 4–5  Simplify each expression.

30. \(x^{\frac{1}{3}} \cdot x^{\frac{2}{3}}\)  
31. \(a^{\frac{4}{9}} \cdot a^{\frac{1}{4}}\)  
32. \(b^{-\frac{3}{4}}\)  
33. \(y^{-\frac{4}{5}}\)  
34. \(\sqrt[3]{\frac{81}{3}}\)

35. \(\sqrt[3]{27}\)  
36. \(\sqrt[3]{25x^2}\)  
37. \(\sqrt[3]{818^3}\)  
38. \(\frac{h^2 + 1}{h^2 - 1}\)  
39. \(\frac{x^\frac{1}{2} + 2}{x^\frac{1}{4} - 2}\)

**GEOMETRY**  Find the area of each figure.

40.  
41.  
42. Find the simplified form of \(18^{\frac{1}{2}} + 2^{\frac{1}{2}} - 32^{\frac{1}{2}}\).

43. What is the simplified form of \(64^\frac{1}{3} - 32^\frac{1}{3} + 8^\frac{1}{3}\)?

Simplify each expression.

44. \(a^\frac{2}{3} \cdot a^\frac{5}{4}\)  
45. \(x^{-\frac{3}{4}} \cdot x^{\frac{2}{3}}\)  
46. \(\left(b^4\right)^{\frac{1}{3}}\)  
47. \(\left(y^{-\frac{3}{5}}\right)^{-\frac{1}{4}}\)

48. \(\sqrt[5]{64}\)  
49. \(\sqrt[5]{216}\)  
50. \(d^{-\frac{5}{6}}\)  
51. \(w^{-\frac{7}{8}}\)
52. **WILDLIFE** A population of 100 deer is reintroduced to a wildlife preserve. Suppose the population does extremely well and the deer population doubles in two years. Then the number \( D \) of deer after \( t \) years is given by \( D = 100 \cdot 2^{t/2} \).

a. How many deer will there be after \( 4\frac{1}{2} \) years?

b. Make a table that charts the population of deer every year for the next five years.

c. Make a graph using your table.

d. Using your table and graph, decide whether this is a reasonable trend over the long term. Explain.

**Simplify each expression.**

53. \( \frac{\frac{f^{-\frac{1}{4}}}{4f^{\frac{1}{2}} \cdot f^{-\frac{1}{3}}}}{\frac{g^{\frac{5}{3}}}{g^{\frac{3}{5}} + 2}} \)

54. \( \frac{\frac{2}{3}}{\frac{c^{\frac{1}{3}}}{c^{6}}} \)

55. \( \frac{\frac{\sqrt[3]{2}}{\sqrt[3]{x^2}}}{\sqrt[3]{3 + \sqrt[3]{2}}} \)

56. \( \frac{\sqrt[4]{2}}{z^{\frac{1}{2}}} \)

57. \( \sqrt{23} \cdot \sqrt[4]{23^2} \)

58. \( \sqrt[4]{364^4} \)

59. \( \sqrt[4]{81} \)

60. \( \sqrt[4]{256} \)

61. \( \frac{ab}{\sqrt{c^2}} \)

62. \( \frac{xy}{\sqrt{z^2}} \)

63. \( \frac{\frac{1}{8} - \frac{9}{4}}{\sqrt{3} + \sqrt{2}} \)

64. \( \frac{\sqrt[4]{x^2} - \frac{1}{2} \cdot \frac{4}{3}}{x^{\frac{2}{3}} + z^{\frac{2}{3}}} \)

65. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the functions \( f(x) = x^3 \) and \( g(x) = x^{\frac{1}{3}} \).

a. **Tabular** Copy and complete the table to the right.

b. **Graphical** Graph \( f(x) \) and \( g(x) \).

c. **Verbal** Explain the transformation between \( f(x) \) and \( g(x) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
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</table>

**H.O.T. Problems** Use Higher-Order Thinking Skills

66. **REASONING** Determine whether \( -x^{-2} = (-x)^{-2} \) is always, sometimes, or never true. Explain your reasoning.

67. **CHALLENGE** Consider \( \sqrt[4]{(-16)^3} \).

a. Explain why the expression is not a real number.

b. Find \( n \) such that \( n^{\frac{1}{4}}(-16)^3 \) is a real number.

68. **OPENEnded** Find two different expressions that equal 2 in the form \( x^{\frac{1}{3}} \).

69. **WRITING IN MATH** Explain how it might be easier to simplify an expression using rational exponents rather than using radicals.

70. **ERROR ANALYSIS** Ayana and Kenji are simplifying \( \frac{x^{\frac{3}{4}}}{x^{\frac{1}{2}}} \). Is either of them correct? Explain your reasoning.

**Ayana**

\[
\frac{3}{4} \cdot \frac{1}{2} = \frac{3}{4} + \frac{1}{2} = x^\frac{3}{4} + \frac{2}{4} = x^\frac{3}{4} + \frac{1}{2} = x\frac{3}{4}
\]

**Kenji**

\[
\frac{3}{4} \cdot \frac{1}{2} = x^{\frac{3}{4}} + \frac{1}{2} = x^\frac{3}{4} + \frac{1}{2} = x^\frac{3}{4} + \frac{1}{2} = x\frac{3}{4}
\]
Standardized Test Practice

71. The expression $\sqrt{56} - c$ is equivalent to a positive integer when $c$ is equal to
A $-10$ B $-8$ C $36$ D $56$

72. SAT/ACT Which of the following sentences is true about the graphs of $y = 2(x - 3)^2 + 1$ and $y = 2(x + 3)^2 + 1$?
F Their vertices are maximums.
G The graphs have the same shape with different vertices.
H The graphs have different shapes with the same vertices.
J The graphs have different shapes with different vertices.
K One graph has a vertex that is a maximum; the other has a vertex that is a minimum.

73. GEOMETRY What is the converse of the statement?
If it is summer, then it is hot outside.
A If it is not hot outside, then it is not summer.
B If it is not summer, then it is not hot outside.
C If it is hot outside, then it is summer.
D If it is hot outside, it is not summer.

74. SHORT RESPONSE If $3^5 \cdot p = 3^3$, then find $p$.

Spiral Review

Simplify. (Lesson 7-5)
75. $\sqrt{243}$
76. $\sqrt[3]{16y^3}$
77. $3\sqrt[3]{56y^2z^3}$

78. PHYSICS The speed of sound in a liquid is $s = \sqrt{\frac{B}{d}}$, where $B$ is the bulk modulus of the liquid and $d$ is its density. For water, $B = 2.1 \times 10^9$ N/m$^2$ and $d = 10^3$ kg/m$^3$.
Find the speed of sound in water to the nearest meter per second. (Lesson 7-4)

Find $p(-4)$ and $p(x + h)$ for each function. (Lesson 6-3)
79. $p(x) = x - 2$
80. $p(x) = -x + 4$
81. $p(x) = 6x + 3$
82. $p(x) = x^2 + 5$
83. $p(x) = x^2 - x$
84. $p(x) = 2x^3 - 1$

Solve each equation by factoring. (Lesson 5-3)
85. $x^2 - 11x = 0$
86. $x^2 + 6x - 16 = 0$
87. $4x^2 - 13x = 12$
88. $x^2 - 14x = -49$
89. $x^2 + 9 = 6x$
90. $x^2 - 3x = -\frac{9}{4}$

91. GEOMETRY A rectangle is inscribed in an isosceles triangle as shown. Find the dimensions of the inscribed rectangle with maximum area. (Hint: Use similar triangles.) (Lesson 5-1)

Skills Review

Find each power. (Lesson 7-5)
92. $(\sqrt{x - 3})^2$
93. $(\sqrt[3]{3x - 4})^3$
94. $(\sqrt[3]{7x - 1})^4$
95. $(\sqrt{x - 4})^2$
96. $(2\sqrt{x} - 5)^2$
97. $(3\sqrt{x} + 1)^2$
Solving Radical Equations

Radical equations include radical expressions. You can solve a radical equation by raising each side of the equation to a power.

**Key Concept** Solving Radical Equations

1. **Step 1** Isolate the radical on one side of the equation.
2. **Step 2** Raise each side of the equation to a power equal to the index of the radical to eliminate the radical.
3. **Step 3** Solve the resulting polynomial equation. Check your results.

When solving radical equations, the result may be a number that does not satisfy the original equation. Such a number is called an **extraneous solution**.

**Example 1** Solve Radical Equations

Solve each equation.

a. \( \sqrt{x + 2} + 4 = 7 \)

Original equation

\[ \sqrt{x + 2} + 4 = 7 \]

Subtract 4 from each side to isolate the radical.

\[ \sqrt{x + 2} = 3 \]

Square each side to eliminate the radical.

\[ (\sqrt{x + 2})^2 = 3^2 \]

Find the squares.

\[ x + 2 = 9 \]

Subtract 2 from each side.

\[ x = 7 \]

**CHECK**

\[ \sqrt{x + 2} + 4 = 7 \]

Original equation

\[ \sqrt{7 + 2} + 4 = 7 \]

Replace \( x \) with 7.

\[ 7 = 7 \checkmark \]

Simplify.

b. \( \sqrt{x - 12} = 2 - \sqrt{x} \)

Original equation

\[ \sqrt{x - 12} = 2 - \sqrt{x} \]

Square each side.

\[ (\sqrt{x - 12})^2 = (2 - \sqrt{x})^2 \]

Find the squares.

\[ x - 12 = 4 - 4\sqrt{x} + x \]

Isolate the radical.

\[ -16 = -4\sqrt{x} \]

Divide each side by \(-4\).

\[ 4 = \sqrt{x} \]

Square each side.

\[ 16 = x \]
Solution Check You can use a graphing calculator to check solutions of equations. Graph each side of the original equation, and examine the intersection.

\[
\sqrt{x - 12} = 2 - \sqrt{x}
\]
\[
\frac{\sqrt{16 - 12}}{\frac{\sqrt{16}}{2}} = \frac{2 - \sqrt{16}}{2}
\]
\[
\sqrt{4} = 2 - 4
\]
\[
2 \neq -2 \quad \text{X}
\]

The solution does not check, so the equation has an extraneous solution. The graphs of \( y = \sqrt{x - 12} \) and \( y = 2 - \sqrt{x} \) do not intersect, which confirms that there is no real solution.

Guided Practice

1A. \( \sqrt{x - 2} - 1 = 5 \)

1B. \( \sqrt{x + 15} = 5 + \sqrt{x} \)

Guided Practice

Solve each equation.

2A. \( (3n + 2)^\frac{1}{3} + 1 = 0 \)

2B. \( 3(5y - 1)^\frac{1}{3} - 2 = 0 \)
You can apply the methods used to solve square and cube root equations to solving equations with roots of any index. To undo an $n$th root, raise to the $n$th power.

**Example 4  Solve a Radical Inequality**

Solve $3 + \sqrt{5x - 10} \leq 8$.

**Step 1** Since the radicand of a square root must be greater than or equal to zero, first solve $5x - 10 \geq 0$ to identify the values of $x$ for which the left side of the inequality is defined.

- $5x - 10 \geq 0$ \hspace{1cm} Set the radicand $\geq 0.$
- $5x \geq 10$ \hspace{1cm} Add 10 to each side.
- $x \geq 2$ \hspace{1cm} Divide each side by 5.

**Step 2** Solve $3 + \sqrt{5x - 10} \leq 8$.

- $3 + \sqrt{5x - 10} \leq 8$ \hspace{1cm} Original inequality
- $\sqrt{5x - 10} \leq 5$ \hspace{1cm} Isolate the radical.
- $5x - 10 \leq 5$ \hspace{1cm} Eliminate the radical.
- $5x \leq 15$ \hspace{1cm} Add 10 to each side.
- $x \leq 3$ \hspace{1cm} Divide each side by 5.
Step 3 It appears that \(2 \leq x \leq 7\). You can test some \(x\)-values to confirm the solution. Use three test values: one less than 2, one between 2 and 7, and one greater than 7. Organize the test values in a table.

<table>
<thead>
<tr>
<th>(x = 0)</th>
<th>(x = 4)</th>
<th>(x = 9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3 + \sqrt{5(0)} - 10 \leq 8)</td>
<td>(3 + \sqrt{5(4)} - 10 \leq 8)</td>
<td>(3 + \sqrt{5(9)} - 10 \leq 8)</td>
</tr>
<tr>
<td>(3 + \sqrt{-10} \leq 8)</td>
<td>(6.16 \leq 8)</td>
<td>(8.92 \leq 8)</td>
</tr>
<tr>
<td>Since (\sqrt{-10}) is not a real number, the inequality is not satisfied.</td>
<td>Since (6.16 \leq 8), the inequality is satisfied.</td>
<td>Since (8.92 \leq 8), the inequality is not satisfied.</td>
</tr>
</tbody>
</table>

The solution checks. Only values in the interval \(2 \leq x \leq 7\) satisfy the inequality. You can summarize the solution with a number line.

Guided Practice

Solve each inequality.

4A. \(\sqrt{2x + 2} + 1 \geq 5\)  
4B. \(\sqrt{4x - 4} - 2 < 4\)

Check Your Understanding

Examples 1–2 Solve each equation.

1. \(\sqrt{x - 4} + 6 = 10\)
2. \(\sqrt{x + 13} - 8 = -2\)
3. \(8 - \sqrt{x + 12} = 3\)
4. \(\sqrt{x - 8} + 5 = 7\)
5. \(\sqrt{x - 2} = 3\)
6. \((x - 5)^\frac{1}{3} - 4 = -2\)
7. \((4y)^\frac{1}{3} + 3 = 5\)
8. \(\sqrt[3]{x + 8} - 6 = -3\)
9. \(\sqrt{y} - 7 = 0\)
10. \(2 + 4z^2 = 0\)
11. \(5 + \sqrt{4y - 5} = 12\)
12. \(\sqrt{2t - 7} = \sqrt{t + 2}\)

13. PHYSICS The time \(T\) in seconds that it takes a pendulum to make a complete swing back and forth is given by the formula \(T = 2\pi \sqrt{\frac{L}{g}}\), where \(L\) is the length of the pendulum in feet and \(g\) is the acceleration due to gravity, 32 feet per second squared.

a. In Tokyo, Japan, a huge pendulum in the Shinjuku building measures 73 feet 9.75 inches. How long does it take for the pendulum to make a complete swing?

b. A clockmaker wants to build a pendulum that takes 20 seconds to swing back and forth. How long should the pendulum be?

Example 3 14. MULTIPLE CHOICE Solve \((2y + 6)^\frac{1}{4} - 2 = 0\).

A. \(y = 1\)  
B. \(y = 5\)  
C. \(y = 11\)  
D. \(y = 15\)

Example 4 Solve each inequality.

15. \(\sqrt{3x + 4} - 5 \leq 4\)
16. \(\sqrt{b - 7} + 6 \leq 12\)
17. \(2 + \sqrt{4y - 4} \leq 6\)
18. \(\sqrt{3t + 3} - 1 \leq 2\)
19. \(1 + \sqrt{7x - 3} > 3\)
20. \(\sqrt{3x + 6} + 2 \leq 5\)
21. \(-2 + \sqrt{9 - 5x} \geq 6\)
22. \(6 - \sqrt{2y + 1} < 3\)
Example 1  Solve each equation. Confirm by using a graphing calculator.

23. \( \sqrt{2x + 5} - 4 = 3 \)  

24. \( 6 + \sqrt{3x + 1} = 11 \)

25. \( \sqrt{x + 6} = 5 - \sqrt{x + 1} \)  

26. \( \sqrt{x - 3} = \sqrt{x + 4} - 1 \)

27. \( \sqrt{x - 15} = 3 - \sqrt{x} \)  

28. \( \sqrt{x - 10} = 1 - \sqrt{x} \)

29. \( 6 + \sqrt{4x + 8} = 9 \)  

30. \( 2 + \sqrt{3y - 5} = 10 \)

31. \( \sqrt{x - 4} = \sqrt{2x - 13} \)  

32. \( \sqrt{7a - 2} = \sqrt{a + 3} \)

33. \( \sqrt{x - 5} - \sqrt{x} = -2 \)  

34. \( \sqrt{b - 6} + \sqrt{b} = 3 \)

35. GRAVITY  Isabel accidentally dropped her keys from the top of a Ferris wheel. The formula \( t = \frac{1}{4} \sqrt{d - h} \) describes the time \( t \) in seconds at which the keys are \( h \) meters above the ground and Isabel is \( d \) meters above the ground. If Isabel was 65 meters high when she dropped the keys, how many meters above the ground will the keys be after 2 seconds?

Example 2  Solve each equation.

36. \( (5n - 6)^{\frac{1}{3}} + 3 = 4 \)  

37. \( (5p - 7)^{\frac{1}{3}} + 3 = 5 \)

38. \( (6q + 1)^{\frac{1}{4}} + 2 = 5 \)  

39. \( (3x + 7)^{\frac{1}{4}} - 3 = 1 \)

40. \( (3y - 2)^{\frac{1}{5}} + 5 = 6 \)  

41. \( (4z - 1)^{\frac{1}{8}} - 1 = 2 \)

42. \( 2(x - 10)^{\frac{1}{3}} + 4 = 0 \)  

43. \( 3(x + 5)^{\frac{1}{3}} - 6 = 0 \)

44. \( \sqrt{5x + 10} - 5 = 0 \)  

45. \( \sqrt{4n - 8} - 4 = 0 \)

46. \( \frac{1}{7}(14n)^{\frac{1}{3}} = 1 \)  

47. \( \frac{1}{4}(32b)^{\frac{1}{3}} = 1 \)

Example 3  

48. MULTIPLE CHOICE  Solve \( \sqrt{y + 2} + 9 = 14 \).

A 23  B 53  C 123  D 623

49. MULTIPLE CHOICE  Solve \( (2x - 1)^{\frac{1}{4}} - 2 = 1 \).

F 41  G 28  H 13  J 1

Example 4  Solve each inequality.

50. \( 1 + \sqrt{5x - 2} > 4 \)  

51. \( \sqrt{2x + 14} - 6 \geq 4 \)  

52. \( 10 - \sqrt{2x + 7} \leq 3 \)

53. \( 6 + \sqrt{3y + 4} < 6 \)  

54. \( \sqrt{2x + 5} - \sqrt{9 + x} > 0 \)  

55. \( \sqrt{d + 3} + \sqrt{d + 7} > 4 \)

56. \( \sqrt{3x + 9} - 2 < 7 \)  

57. \( \sqrt{2y + 5} + 3 \leq 6 \)  

58. \( -2 + \sqrt{8 - 4z} \geq 8 \)

59. \( -3 + \sqrt{6a + 1} > 4 \)  

60. \( \sqrt{2} - \sqrt{b} + 6 \leq -\sqrt{b} \)  

61. \( \sqrt{c} + 9 - \sqrt{c} > \sqrt{3} \)

62. PENDULUMS  The formula \( s = 2\pi\sqrt{\frac{\ell}{32}} \) represents the swing of a pendulum, where \( s \) is the time in seconds to swing back and forth, and \( \ell \) is the length of the pendulum in feet. Find the length of a pendulum that makes one swing in 1.5 seconds.

63. FISH  The relationship between the length and mass of certain fish can be approximated by the equation \( L = 0.46\sqrt{M} \), where \( L \) is the length in meters and \( M \) is the mass in kilograms. Solve this equation for \( M \).
64. **HANG TIME** Refer to the information at the beginning of the lesson regarding hang time. Describe how the height of a jump is related to the amount of time in the air. Write a step-by-step explanation of how to determine the height of Jordan’s 0.98-second jump.

**CONCERTS** The organizers of a concert are preparing for the arrival of 50,000 people in the open field where the concert will take place. Each person is allotted 5 square feet of space, so the organizers rope off a circular area of 250,000 square feet. Using the formula \( A = \pi r^2 \), where \( A \) represents the area of the circular region and \( r \) represents the radius of the region, find the radius of this region.

66. **WEIGHTLIFTING** The formula \( M = 512 - 146,230B^{-\frac{8}{5}} \) can be used to estimate the maximum total mass that a weightlifter of mass \( B \) kilograms can lift using the snatch and the clean and jerk. According to the formula, how much does a person weigh who can lift at most 470 kilograms?

**H.O.T. Problems** Use Higher-Order Thinking Skills

67. **WHICH ONE DOESN’T BELONG?** Which equation does not have a solution?

\[
\sqrt{x - 1} + 3 = 4 \\
\sqrt{x + 1} + 3 = 4 \\
\sqrt{x - 2} + 7 = 10 \\
\sqrt{x + 2} - 7 = -10
\]

68. **CHALLENGE** Lola is working to solve \((x + 5)^{\frac{1}{4}} = -4\). She said that she could tell there was no real solution without even working the problem. Is Lola correct? Explain your reasoning.

69. **REASONING** Determine whether \( \sqrt{(x^2)^2} = x \) is sometimes, always, or never true when \( x \) is a real number. Explain your reasoning.

70. **OPEN ENDED** Select a whole number. Now work backward to write two radical equations that have that whole number as solutions. Write one square root equation and one cube root equation. You may need to experiment until you find a whole number you can easily use.

71. **WRITING IN MATH** Explain the relationship between the index of the root of a variable in an equation and the power to which you raise each side of the equation to solve the equation.

72. **OPEN ENDED** Write an equation that can be solved by raising each side of the equation to the given power.

\[ \begin{align*}
a. \quad & \frac{3}{2} \text{ power} \\
b. \quad & \frac{5}{4} \text{ power} \\
c. \quad & \frac{7}{8} \text{ power}
\end{align*} \]

73. **CHALLENGE** Solve \( 7^{3x - 1} = 49x + 1 \) for \( x \). (Hint: \( b^x = b^y \) if and only if \( x = y \).)

**REASONING** Determine whether the following statements are sometimes, always, or never true for \( x^\frac{1}{n} = a \). Explain your reasoning.

74. If \( n \) is odd, there will be extraneous solutions.

75. If \( n \) is even, there will be extraneous solutions.
76. What is an equivalent form of \( \frac{4}{5 + i} \)?

A \( \frac{10 - 2i}{13} \)  
B \( \frac{5 - i}{6} \)  
C \( \frac{6 - i}{6} \)  
D \( \frac{6 - i}{13} \)

77. Which set of points describes a function?

F \( \{(3, 0), (-2, 5), (2, -1), (2, 9)\} \)  
G \( \{(-3, 5), (-2, 3), (-1, 5), (0, 7)\} \)  
H \( \{(2, 5), (2, 4), (2, 3), (2, 2)\} \)  
J \( \{(3, 1), (-3, 2), (3, 3), (-3, 4)\} \)

78. SHORT RESPONSE The perimeter of an isosceles triangle is 56 inches. If one leg is 20 inches long, what is the measure of the base of the triangle?

79. SAT/ACT If \( \sqrt{x + 5} + 1 = 4 \), what is the value of \( x \)?

A 4  
B 10  
C 11  
D 12  
E 20

80. \( 27 - \frac{2}{3} \)  
81. \( \frac{1}{9^3} \cdot 9^{\frac{5}{3}} \)  
82. \( \left( \frac{8}{27} \right)^{-\frac{2}{3}} \)

83. GEOMETRY The measures of the legs of a right triangle can be represented by the expressions \( 4x^2y^2 \) and \( 8x^2y^2 \). Use the Pythagorean Theorem to find a simplified expression for the measure of the hypotenuse. (Lesson 7-5)

Find the inverse of each function. (Lesson 7-2)

84. \( y = 3x - 4 \)  
85. \( y = -2x - 3 \)  
86. \( y = x^2 \)  
87. \( y = (2x + 3)^2 \)

For each graph,

a. describe the end behavior,  
b. determine whether it represents an odd-degree or an even-degree polynomial function, and  
c. state the number of real zeros. (Lesson 6-3)

88. [Graph of a function]  
89. [Graph of a function]  
90. [Graph of a function]

Skills Review

Solve each equation. Write in simplest form. (Lesson 1-3)

91. \( \frac{8}{5}x = \frac{4}{15} \)  
92. \( \frac{27}{14}y = \frac{6}{7} \)  
93. \( \frac{3}{10} = \frac{12}{25} \)  
94. \( \frac{6}{7} = 9m \)

95. \( \frac{9}{8}b = 18 \)  
96. \( \frac{6}{7}n = \frac{3}{4} \)  
97. \( \frac{1}{3}p = \frac{5}{6} \)  
98. \( \frac{2}{3}y = 7 \)
You can use a TI-83/84 Plus graphing calculator to solve radical equations and inequalities. One way to do this is to rewrite the equation or inequality so that one side is 0. Then use the zero feature on the calculator.

**Example 1 Radical Equation**

Solve $\sqrt{x} + \sqrt{x + 2} = 3$.

**Step 1** Rewrite the equation.

- Subtract 3 from each side of the equation to get $\sqrt{x} + \sqrt{x + 2} - 3 = 0$.
- Enter the function $y = \sqrt{x} + \sqrt{x + 2} - 3$ in the Y= list.

**KEYSTROKES:**

```
\begin{array}{l}
\text{Y= 2nd } \sqrt{x, T, \theta, n} + \sqrt{x, T, \theta, n} + 2 - 3 \text{ ENTER}
\end{array}
```

**Step 2** Use a table.

- You can use the **TABLE** function to locate intervals where the solution(s) lie. First, enter the starting value and the interval for the table.

**KEYSTROKES:**

```
2nd [TBLSET] 0 ENTER 1 ENTER
```

**Step 3** Estimate the solution.

- Complete the table and estimate the solution(s).

**KEYSTROKES:**

```
2nd [TABLE]
```

Since the function changes sign from negative to positive between $x = 1$ and $x = 2$, there is a solution between 1 and 2.

**Step 4** Use the **ZERO** feature.

- Graph the function in the standard viewing window; then select **ZERO** from the **CALC** menu.

**KEYSTROKES:**

```
ZOOM 6 2nd [CALC] 2
```

Place the cursor on a point at which $y < 0$ and press **ENTER** for the **LEFT BOUND**. Then place the cursor on a point at which $y > 0$ and press **ENTER** for the **RIGHT BOUND**. You can use the same point for the **GUESS**. The solution is about 1.36. This is consistent with the estimate made by using the **TABLE**.
**Example 2** Radical Inequality

Solve \(2\sqrt{x} > \sqrt{x + 2} + 1\).

**Step 1** Graph each side of the inequality and use the TRACE feature.

- In the Y= list, enter \(y_1 = 2\sqrt{x}\) and \(y_2 = \sqrt{x + 2} + 1\). Then press GRAPH.

- Press TRACE. You can use \(\uparrow\) or \(\downarrow\) to switch the cursor between the two curves.

The calculator screen above shows that, for points to the left of where the curves cross, \(Y_1 < Y_2\) or \(2\sqrt{x} < \sqrt{x + 2} + 1\). To solve the original inequality, you must find points for which \(Y_1 > Y_2\). These are the points to the right of where the curves cross.

**Step 2** Use the intersect feature.

- You can use the intersect feature on the CALC menu to approximate the x-coordinate of the point at which the curves cross.

**KEYSTROKES:**

| 2nd [CALC] 5 |

- Press ENTER for each of FIRST CURVE?, SECOND CURVE?, and GUESS?.

The calculator screen shows that the x-coordinate of the point at which the curves cross is about 2.40. Therefore, the solution of the inequality is about \(x > 2.40\). Use the symbol > in the solution because the symbol in the original inequality is >.

**Step 3** Use the TABLE feature to check your solution.

- Start the table at 2 and show x-values in increments of 0.1. Scroll through the table.

**KEYSTROKES:**

| 2nd [TBLSET] 2 ENTER .1 [ENTER] 2nd [TABLE] |

Notice that when \(x\) is less than or equal to 2.4, \(Y_1 < Y_2\). This verifies the solution \(\{x \mid x > 2.40\}\).

**Exercises**

Use a graphical method to solve each equation or inequality.

1. \(\sqrt{x + 4} = 3\)
2. \(\sqrt{3x - 5} = 1\)
3. \(\sqrt{x + 5} = \sqrt{3x + 4}\)
4. \(\sqrt{x + 3} + \sqrt{x - 2} = 4\)
5. \(\sqrt{3x - 7} = \sqrt{2x - 2} - 1\)
6. \(\sqrt{x + 8} - 1 = \sqrt{x + 2}\)
7. \(\sqrt{x - 3} \geq 2\)
8. \(\sqrt{x + 3} > 2\sqrt{x}\)
9. \(\sqrt{x} + \sqrt{x - 1} < 4\)

10. **WRITING IN MATH** Explain how you could apply the technique in the first example to solving an inequality.
**Key Concepts**

### Operations on Functions (Lesson 7-1)

<table>
<thead>
<tr>
<th>Operation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum</td>
<td>$(f + g)(x) = f(x) + g(x)$</td>
</tr>
<tr>
<td>Difference</td>
<td>$(f - g)(x) = f(x) - g(x)$</td>
</tr>
<tr>
<td>Product</td>
<td>$(f \cdot g)(x) = f(x) \cdot g(x)$</td>
</tr>
<tr>
<td>Quotient</td>
<td>$(\frac{f}{g})(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$</td>
</tr>
<tr>
<td>Composition</td>
<td>$<a href="x">f \circ g</a> = f(g(x))$</td>
</tr>
</tbody>
</table>

### Inverse and Square Root Functions (Lessons 7-2 and 7-3)

- Two functions are inverses if and only if both their compositions are the identity function.

### Roots of Real Numbers (Lesson 7-4)

**Real $n$th roots of $b$, $\sqrt[n]{b}$, or $-\sqrt[n]{b}$**

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\sqrt[n]{b}$ if $b &gt; 0$</th>
<th>$\sqrt[n]{b}$ if $b &lt; 0$</th>
<th>$\sqrt[n]{b}$ if $b = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>even</td>
<td>one positive root</td>
<td>no real roots</td>
<td>one real root, 0</td>
</tr>
<tr>
<td>odd</td>
<td>one positive root, one negative root</td>
<td>no negative roots</td>
<td></td>
</tr>
</tbody>
</table>

### Radicals (Lessons 7-5 through 7-7)

For any real numbers $a$ and $b$ and any integers $n$, $x$, and $y$, with $b \neq 0$, $n > 1$, and $y > 1$, the following are true.

- Product Property: $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
- Quotient Property: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
- Rational Exponents: $b^\frac{x}{y} = \sqrt[y]{b^x} = (\sqrt[y]{b})^x$, $b \geq 0$

**Study Organizer**

Be sure the Key Concepts are noted in your Foldable.

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**Key Vocabulary**

- composition of functions (p. 411)
- principal root (p. 431)
- conjugates (p. 442)
- radical equation (p. 453)
- extraneous solution (p. 453)
- radical function (p. 424)
- index (p. 431)
- radical inequality (p. 455)
- radical sign (p. 431)
- radicand (p. 431)
- rationalizing the denominator (p. 440)
- square root function (p. 424)
- square root inequality (p. 426)

**Vocabulary Check**

Choose a word or term that best completes each statement.

1. If both compositions result in the ________, then the functions are inverse functions.
2. Radicals are ________ if both the index and the radicand are identical.
3. In a(n) ________, the results of one function are used to evaluate a second function.
4. When there is more than one real root, the nonnegative root is called the ________.
5. To eliminate radicals from a denominator or fractions from a radicand, you use a process called ________.
6. Equations with radicals that have variables in the radicands are called ________.
7. Two relations are ________ if and only if one relation contains the element $(b, a)$ when the other relation contains the element $(a, b)$.
8. When solving a radical equation, sometimes you will obtain a number that does not satisfy the original equation. Such a number is called a(n) ________.
9. The square root function is a type of ________.
7-1 Operations on Functions (pp. 409–416)

Find \([f \circ g](x)\) and \([g \circ f](x)\).

10. \(f(x) = 2x + 1\)  
   \(g(x) = 4x - 5\)

11. \(f(x) = x^2 + 1\)  
   \(g(x) = x - 7\)

12. \(f(x) = x^2 + 4\)  
   \(g(x) = -2x + 1\)

13. \(f(x) = 4x\)  
   \(g(x) = 5x - 1\)

14. \(f(x) = x^3\)  
   \(g(x) = x - 1\)

15. \(f(x) = x^2 + 2x - 3\)  
   \(g(x) = x + 1\)

16. MEASUREMENT  The formula \(f = 3y\) converts yards \(y\) to feet \(f\) and \(f = \frac{n}{12}\) converts inches \(n\) to feet \(f\). Write a composition of functions that converts yards to inches.

Example 1

If \(f(x) = x^2 + 3\) and \(g(x) = 3x - 2\), find \(g[f(x)]\) and \(f[g(x)]\).

\[
g[f(x)] = 3(x^2 + 3) - 2 = 3x^2 + 9 - 2 = 3x^2 + 7\]

Replace \(f(x)\) with \(x^2 + 3\).

\[
f[g(x)] = (3x - 2)^2 + 3 = 9x^2 - 12x + 4 + 3 = 9x^2 - 12x + 7\]

Replace \(g(x)\) with \(3x - 2\).


7-2 Inverse Functions and Relations (pp. 417–422)

Find the inverse of each function. Then graph the function and its inverse.

17. \(f(x) = 5x - 6\)

18. \(f(x) = -3x - 5\)

19. \(f(x) = \frac{1}{2}x + 3\)

20. \(f(x) = \frac{4x + 1}{5}\)

21. \(f(x) = x^2\)

22. \(f(x) = (2x + 1)^2\)

23. SHOPPING  Samuel bought a computer. The sales tax rate was 6% of the sale price, and he paid $50 for shipping. Find the sale price if Samuel paid a total of $1322.

Use the horizontal line test to determine whether the inverse of each function is also a function.

24. \(f(x) = 3x^2\)

25. \(h(x) = x^3 - 3\)

26. \(g(x) = -3x^4 + 2x - 1\)

27. \(g(x) = 4x^3 - 5x\)

28. \(f(x) = -3x^5 + x^3 - 3\)

29. \(h(x) = 4x^4 + 7x\)

30. FINANCIAL LITERACY  During the last month, Jonathan has made two deposits of $45, made a deposit of double his original balance, and has withdrawn $35 five times. His balance is now $189. Write an equation that models this problem. How much money did Jonathan have in his account at the beginning of the month?

Example 2

Find the inverse of \(f(x) = -2x + 7\).

Rewrite \(f(x)\) as \(y = -2x + 7\). Then interchange the variables and solve for \(y\).

\[
x = -2y + 7 \quad \text{Interchange the variables.}
\]

\[
2y = -x + 7 \quad \text{Solve for } y.
\]

\[
y = \frac{-x + 7}{2} \quad \text{Divide each side by 2.}
\]

\[
f^{-1}(x) = \frac{-x + 7}{2} \quad \text{Rewrite using function notation.}
\]

Example 3

Use the horizontal line test to determine whether the inverse of \(f(x) = 2x^3 + 1\) is also a function.

Graph the function.

No horizontal line can be drawn so that it passes through more than one point. The inverse of this function is a function.
### 7-3 Square Root Functions and Inequalities

**Graph each function. State the domain and range.**

31. \( f(x) = \sqrt{3x} \)
32. \( f(x) = -\sqrt{6x} \)
33. \( f(x) = \sqrt{x - 7} \)
34. \( f(x) = \sqrt{x + 5} - 3 \)
35. \( f(x) = \frac{3}{4}\sqrt{x - 1} + 5 \)
36. \( f(x) = \frac{1}{3}\sqrt{x + 4} - 1 \)

37. **GEOMETRY** The area of a circle is given by the formula \( A = \pi r^2 \). What is the radius of a circle with an area of 300 square inches?

**Graph each inequality.**

38. \( y \geq \sqrt{x} + 3 \)
39. \( y < 2\sqrt{x - 5} \)
40. \( y > -\sqrt{x - 1} + 2 \)

**Example 4**

Graph \( f(x) = \sqrt{x + 1} - 2 \). State the domain and range.

Identify the domain.

\( x + 1 \geq 0 \)

Write the radicand as greater than or equal to 0.

\( x \geq -1 \)

Subtract 1 from each side.

Make a table of values for \( x \geq -1 \) and graph the function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>-0.59</td>
</tr>
<tr>
<td>2</td>
<td>-0.27</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.24</td>
</tr>
<tr>
<td>5</td>
<td>0.45</td>
</tr>
</tbody>
</table>

The domain is \( \{x \mid x \geq -1\} \), and the range is \( \{f(x) \mid f(x) \geq -2\} \).

### 7-4 nth Roots

**Simplify.**

41. \( \pm \sqrt{121} \)
42. \( \sqrt{-125} \)
43. \( \sqrt{(-6)^2} \)
44. \( \sqrt{-\left(x + 3\right)^4} \)
45. \( \sqrt[6]{(x^2 + 2)^{18}} \)
46. \( \sqrt[3]{27(x + 3)^3} \)
47. \( \sqrt[12]{a^8b^{12}} \)
48. \( \sqrt[25]{243x^{10}y^{25}} \)

49. **PHYSICS** The velocity \( v \) of an object can be defined as \( v = \sqrt{\frac{2K}{m}} \) where \( m \) is the mass of an object and \( K \) is the kinetic energy in joules. Find the velocity in meters per second of an object with a mass of 17 grams and a kinetic energy of 850 joules.

**Example 5**

Simplify \( \sqrt[6]{64x^6} \).

\[ \sqrt[6]{64x^6} = \sqrt{(8x^3)^2} = 8|x^3| \]

Use absolute value symbols because \( x \) could be negative.

**Example 6**

Simplify \( \sqrt[4]{4096x^{12}y^{24}} \).

\[ \sqrt[4]{4096x^{12}y^{24}} = \sqrt[4]{(4x^3y^6)^4} = 4x^3y^6 \]

Simplify.
### 7-5 Operations with Radical Expressions (pp. 439-445)

**Example 7**

Simplify \(2\sqrt[3]{18a^2b} \cdot 3\sqrt[3]{12ab^5}\).

\[
2\sqrt[3]{18a^2b} \cdot 3\sqrt[3]{12ab^5} = (2 \cdot 3)\sqrt[3]{18a^2b \cdot 12ab^5} \\
= 6\sqrt[3]{233a^3b^6} \\
= 6 \cdot \sqrt[3]{2^3 \cdot 3 \cdot a^3 \cdot b^6} \\
= 6 \cdot 2 \cdot 3 \cdot a \cdot b^2 \\
= 36ab^2
\]

**Example 8**

Simplify \(\sqrt[3]{x^4 \cdot y^5}\).

\[
\sqrt[3]{x^4 \cdot y^5} = \frac{\sqrt[3]{(x^2)^2}}{\sqrt[3]{(y^3)^2 \cdot \sqrt{y}}} \\
= \frac{x^2}{y^3} \cdot \frac{\sqrt{y}}{\sqrt{y}} \\
= \frac{x^2\sqrt{y}}{y^3} \\
= \sqrt{y} \cdot \sqrt{y} = y
\]

### 7-6 Rational Exponents (pp. 446-452)

**Example 9**

Simplify \(a^{\frac{2}{3}} \cdot a^{\frac{1}{5}}\).

\[
a^{\frac{2}{3}} \cdot a^{\frac{1}{5}} = a^{\frac{2}{3} + \frac{1}{5}} \\
= a^{\frac{10}{15} + \frac{3}{15}} \\
= a^{\frac{13}{15}}
\]

**Example 10**

Simplify \(\frac{2a}{\sqrt{b}}\).

\[
\frac{2a}{\sqrt{b}} = \frac{2a}{b^{\frac{1}{2}}} = \frac{2a}{b^{\frac{1}{2}}} \cdot \frac{b^{\frac{1}{2}}}{b^{\frac{1}{2}}} \\
= \frac{2a b^{\frac{1}{2}}}{b} \\
= \frac{2ab^{\frac{1}{2}}}{b} \\
= \frac{2ab^{\frac{1}{2}}}{b} \\
= 2ab^{\frac{1}{2}}
\]

**Example 11**

Find \(a^2\) if \(a^3 = 8\).

\[
a^3 = 8 \\
a = \sqrt[3]{8} \\
a = 2
\]

### 58. GEOMETRY

What are the perimeter and the area of the rectangle?

![Rectangle Diagram]

**Example 12**

Find the area and perimeter of the right triangle.

**Example 13**

Find the area and perimeter of the parallelogram.

**Example 14**

Find the area and perimeter of the trapezoid.

**Example 15**

Find the area and perimeter of the circle.

**Example 16**

Find the area and perimeter of the ellipse.

**Example 17**

Find the area and perimeter of the rectangle.
### 7-7 Solving Radical Equations and Inequalities (pp. 453–459)

#### Solve each equation.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>66. ( \sqrt{x - 3} + 5 = 15 )</td>
<td>( x = 16 )</td>
</tr>
<tr>
<td>68. ( 4 + \sqrt{3x - 1} = 8 )</td>
<td>( x = 9 )</td>
</tr>
<tr>
<td>70. ( \sqrt{2x + 3} = 3 )</td>
<td>( x = 2 )</td>
</tr>
<tr>
<td>72. ( a^2 - 4 = 0 )</td>
<td>( a = 2 )</td>
</tr>
</tbody>
</table>

#### Example 11

Solve \( \sqrt{2x + 9} - 2 = 5 \).

1. Original equation: \( \sqrt{2x + 9} - 2 = 5 \)
2. Add 2 to each side: \( \sqrt{2x + 9} = 7 \)
3. Square each side: \( (\sqrt{2x + 9})^2 = 7^2 \)
4. Evaluate the squares: \( 2x + 9 = 49 \)
5. Subtract 9 from each side: \( 2x = 40 \)
6. Divide each side by 2: \( x = 20 \)

#### Example 12

Solve \( \sqrt{2x - 5} + 2 > 5 \).

1. Original inequality: \( \sqrt{2x - 5} + 2 > 5 \)
2. Subtract 2 from each side: \( \sqrt{2x - 5} > 3 \)
3. Square each side: \( (\sqrt{2x - 5})^2 > 3^2 \)
4. Evaluate the squares: \( 2x - 5 > 9 \)
5. Add 5 to each side: \( 2x > 14 \)
6. Divide each side by 2: \( x > 7 \)

Since \( x \geq 2.5 \) contains \( x > 7 \), the solution of the inequality is \( x > 7 \).

#### Solve each inequality.

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>75. ( 2 + \sqrt{3x - 1} &lt; 5 )</td>
<td>( x &lt; 4 )</td>
</tr>
<tr>
<td>77. ( 6 - \sqrt{3x + 5} \leq 3 )</td>
<td>( x \geq 1 )</td>
</tr>
<tr>
<td>79. ( 5 + \sqrt{2y - 7} &lt; 5 )</td>
<td>( y &lt; 2 )</td>
</tr>
<tr>
<td>81. ( \sqrt{3x + 1} - \sqrt{6 + x} &gt; 0 )</td>
<td>( x &gt; 1 )</td>
</tr>
</tbody>
</table>
Determine whether each pair of functions are inverse functions. Write yes or no. Explain your reasoning.

1. \( f(x) = 3x + 8 \), \( g(x) = \frac{x - 8}{3} \)
2. \( f(x) = \frac{1}{3}x + 5 \), \( g(x) = 3x - 15 \)
3. \( f(x) = x + 7 \), \( g(x) = x - 7 \)
4. \( g(x) = 3x - 2 \), \( f(x) = \frac{x - 2}{3} \)

5. **MULTIPLE CHOICE** Which inequality represents the graph below?

![Graph](image)

A \( y \geq \sqrt{x + 4} \)  
B \( y \leq \sqrt{x + 4} \)  
C \( y \geq \sqrt{x - 4} \)  
D \( y \leq \sqrt{x - 4} \)

If \( f(x) = 3x + 2 \) and \( g(x) = x^2 - 2x + 1 \), find each function.

6. \((f + g)(x)\)  
7. \((f \cdot g)(x)\)  
8. \((f - g)(x)\)  
9. \(\left(\frac{f}{g}\right)(x)\)

Solve each equation.

10. \(\sqrt{a + 12} = \sqrt{5a} - 4\)  
11. \(\sqrt{3x} = \sqrt{x} - 2\)  
12. \(4(\sqrt{3x + 1}) - 8 = 0\)  
13. \(\sqrt{5m + 6} + 15 = 21\)  
14. \(\sqrt{3x + 21} = \sqrt{5x + 27}\)  
15. \(1 + \sqrt{x + 11} = \sqrt{2x + 15}\)  
16. \(\sqrt{x - 5} = \sqrt{2x - 4}\)  
17. \(\sqrt{x - 6} - \sqrt{x} = 3\)

18. **MULTIPLE CHOICE** Which expression is equivalent to \(125^{\frac{1}{3}}\)?

- F \( -5 \)  
- G \( \frac{1}{5} \)  
- H \( \frac{1}{5} \)  
- J \( 5 \)

Simplify.

19. \((2 + \sqrt{5})(6 - 3\sqrt{5})\)  
20. \((3 - 2\sqrt{2})(-7 + \sqrt{2})\)

21. \(\frac{12}{2 - \sqrt{3}}\)  
22. \(\frac{m^2 - 1}{2m^2 + 1}\)

23. \(4\sqrt{3} - 8\sqrt{48}\)  
24. \(\frac{2}{5} \cdot \frac{1}{5} \cdot 5\)

25. \(\sqrt[4]{729a^9b^{24}}\)  
26. \(\sqrt[5]{32x^{15}y^{10}}\)

27. \(w - \frac{4}{5}\)  
28. \(\frac{\frac{3}{5}}{\frac{1}{r}}\)

29. \(\frac{a - \frac{1}{2}}{6a^3 - a - \frac{1}{4}}\)  
30. \(\frac{\frac{3}{y^2}}{y^2 + 2}\)

31. **MULTIPLE CHOICE** What is the area of the rectangle?

![Rectangle](image)

- A \( 2\sqrt{3} + 3\sqrt{2} \) units\(^2\)  
- B \( 4 + 2\sqrt{6} + 2\sqrt{3} \) units\(^2\)  
- C \( 2\sqrt{3} + \sqrt{6} \) units\(^2\)  
- D \( 2\sqrt{3} + 3 \) units\(^2\)

Solve each inequality.

32. \(\sqrt{4x - 3} < 5\)  
33. \(-2 + \sqrt{3m - 1} < 4\)

34. \(2 + \sqrt{4x - 4} \leq 6\)  
35. \(\sqrt{2x} + 3 - 4 \leq 5\)

36. \(\sqrt{b + 12} - \sqrt{b} > 2\)  
37. \(\sqrt{y - 7} + 5 \geq 10\)

38. \(\sqrt{a - 5} - \sqrt{a + 7} \leq 4\)  
39. \(\sqrt{c + 5} + \sqrt{c + 10} > 2\)

40. **GEOMETRY** The area of a triangle with sides of length \(a\), \(b\), and \(c\) is given by \(A = \sqrt{s(s - a)(s - b)(s - c)}\), where \(s = \frac{1}{2}(a + b + c)\). What is the area of the triangle expressed in radical form?

![Triangle](image)
Work Backward

In certain math problems, you are given information about an end result, but you need to find out something that happened earlier. You can work backward to solve problems like this.

Strategies for Working Backward

Step 1
Read the problem statement carefully.

Ask yourself:
- What information am I given?
- What am I being asked to solve?
- Does any of the information given relate to an end result?
- Am I being asked to solve for a quantity that occurred “earlier” in the problem statement?
- What operations are being used in the problem?

Step 2
Model the problem situation with an equation, an inequality, or a graph as appropriate. Then work backward to solve the problem.

- If needed, sketch a flow of events to show the sequence described in the problem statement.
- Use inverse operations to undo any operations while working backward until you arrive at your answer.

Step 3
Check by beginning with your answer and seeing if you arrive at the same result given in the problem statement.

Test Practice Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

Maria bought a used car. The sales tax rate was 6.75% of the selling price, and she had to pay $450 in processing, title, and registration fees. If Maria paid a total of $15,768.63, what was the sale price of the car? Show your work.

Read the problem carefully. You know the total amount that Maria paid for the car after sales tax was applied and after she paid all of the other fees. You need to find the sale price of the car before taxes and fees.
Let $x$ represent the sale price of the car and set up an equation. Use the work backward strategy to solve the problem.

**Words**
The sale price of the car plus the sales tax and other fees is equal to the final price.

**Variable**
Let $x = $ sale price.

**Equation**
\[
1.0675x + 450 = 15,768.63
\]

Using the work backward strategy results in a simple equation. Use inverse operations to solve for $x$.

\[
1.0675x + 450 = 15,768.63 \quad \text{Original equation}
\]

\[
1.0675x = 15,318.63 \quad \text{Subtract 450 from each side.}
\]

\[
x \approx 14,350 \quad \text{Divide each side by 1.0675.}
\]

Check your answer by working the problem forward. Begin with your answer and see if you get the same result as in the problem statement.

\[
14,350(1.0675) \approx 15,318.63 \quad \text{Compute the sales tax.}
\]

\[
15,318.63 + 450 = 15,768.63 \quad \text{Add the other fees.}
\]

\[
15,768.63 = 15,768.63 \quad \text{The result is the same.}
\]

So, the sale price of the car was $14,350.

**Exercises**

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

1. The equation \( d = \frac{s^2}{30f} \) can be used to model the length of the skid marks left by a car when a driver applies the brakes to come to a sudden stop. In the equation, \( d \) is the length (in feet) of the skid marks left on the road, \( s \) is the speed of the car in miles per hour, and \( f \) is a coefficient of friction that describes the condition of the road. Suppose a car left skid marks that are 120 feet long.

   a. Solve the equation for \( s \), the speed of the car.

   b. If the coefficient of friction for the road is 0.75, about how fast was the car traveling?

   c. How fast was the car traveling if the coefficient of friction for the road is 1.1?

2. An object is shot straight upward into the air with an initial speed of 800 feet per second. The height \( h \) that the object will be after \( t \) seconds is given by the equation \( h = -16t^2 + 800t \). When will the object reach a height of 10,000 feet?

   A 10 seconds
   B 25 seconds
   C 100 seconds
   D 625 seconds

3. Pedro is creating a scale drawing of a car. He finds that the height of the car in the drawing is \( \frac{1}{32} \) of the actual height of the car \( x \). Which equation best represents this relationship?

   F \( y = x - \frac{1}{32} \)
   H \( y = \frac{1}{32}x \)
   G \( y = -\frac{1}{32}x \)
   J \( y = x + \frac{1}{32} \)
Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. A sporting goods store is discounting all camping equipment by 20% during the off-season. Charles also has a coupon good for $5.00 off his next purchase from the store. If the coupon is applied after the store discount, which of the following functions can be used to find the final price of a tent that originally cost \( d \) dollars?
   A \( P(d) = 0.8 \times (d + 5) \)
   B \( P(d) = (0.8 \times d) - 5 \)
   C \( P(d) = 0.2 \times (d - 5) \)
   D \( P(d) = 0.8 \times (d - 5) \)

2. Find the equation that can be used to determine the total area of the composite figure below.

   \[ A = \\frac{1}{2} \pi \ell w + \pi \left( \frac{1}{2} \ell \right)^2 \]

3. Which expression is equivalent to \( 3a(2a + 1) - (2a - 2)(a + 3) \)?
   A \( 2a^2 + 6a + 7 \)
   B \( 4a^2 - a + 6 \)
   C \( 4a^2 + 6a - 6 \)
   D \( 4a^2 - 3a + 7 \)

4. You know the final price but need to know the sale price. Work backward to find the solution.

5. Simplify \( \sqrt[3]{-27b^6c^{12}} \).
   A \(-3b^2c^4\)
   B \(-3b^3c^4\)
   C \(-3b^4c^3\)
   D \(-3b^6c^2\)

6. Kay bought a used car. The sales tax rate was 6.5% of the selling price, and she also had to pay $325 in registration fees. Find the selling price if Kay spent a total of $15,501.25.
   F \$13,850\)
   G \$14,120\)
   H \$14,250\)
   J \$14,650\)

7. Which equation will produce the narrowest parabola when graphed?
   A \( y = 3x^2\)
   B \( y = \frac{3}{4}x^2\)
   C \( y = -6x^2\)
   D \( y = -\frac{3}{4}x^2\)

8. Find the inverse of \( f(x) = x - 5 \).
   F \( f(x) = x + 5 \)
   G \( f(x) = 5x \)
   H \( f(x) = \frac{x}{5} \)
   J \( f(x) = 5 - x \)

9. The equations of two lines are \( 2x - y = 6 \) and \( 4x - y = -2 \). Which of the following describes their point of intersection?
   A \( (2, -2) \)
   B \( (-8, -38) \)
   C \( (-4, -14) \)
   D no intersection

**Test-Taking Tip**

**Question 4** You know the final price but need to know the sale price. Work backward to find the solution.
Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. Which pair of polygons is congruent?

11. Suppose a projectile is launched into the air from a platform. The formula \( h = -16t^2 + 40t + 70 \) relates the height \( h \) of the object (in feet) and the time \( t \) since it was launched (in seconds). What is the maximum height the object reaches?

12. The radius of a sphere with volume \( V \) can be found using the formula \( r = \sqrt[3]{\frac{3V}{4\pi}} \).

   a. What is the radius of the sphere at the right? Round to the nearest tenth.

   b. Solve the formula for \( V \) to find the formula for the volume of a sphere, given its radius.

   c. What is the volume of a basketball that has a diameter of 9 inches? Round to the nearest tenth.

13. GRIDDED RESPONSE The perimeter of the quadrilateral below is 160. What is the value of \( m \)?

   \[
   4m^2 + 1 \quad 2m + 6 \quad 3m - 4 \quad 4m^2 + 2m + 1
   \]

Extended Response

Record your answers on a sheet of paper. Show your work.

14. The amount that a retailer charges for shipping an electronics purchase is determined by the weight of the package. The charges for several different weights are given in the table.

<table>
<thead>
<tr>
<th>Weight (lb)</th>
<th>Shipping ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.58</td>
</tr>
<tr>
<td>3</td>
<td>6.76</td>
</tr>
<tr>
<td>4</td>
<td>7.35</td>
</tr>
<tr>
<td>7</td>
<td>9.12</td>
</tr>
<tr>
<td>10</td>
<td>10.89</td>
</tr>
<tr>
<td>13</td>
<td>12.66</td>
</tr>
<tr>
<td>15</td>
<td>13.84</td>
</tr>
</tbody>
</table>

   a. Find the rate of change of the shipping charge per pound.

   b. Write an equation that could be used to find the shipping charge \( y \) for a package that weighs \( x \) pounds.

   c. Find the shipping charge for a package that weighs 19 pounds.

15. Suppose \( f(x) \) and \( g(x) \) are inverse functions.

   a. Describe how the graphs of \( f(x) \) and \( g(x) \) would appear on a coordinate grid.

   b. What is the value of the composition \( f \circ g(2) \)? Explain.